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# REFRACTION AND ACCOMMODATION OF THE EYE.

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# THE

# Refraction and Accommodation of the Eye

## AND THEIR ANOMALIES.

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BY

Edmund
E. LANDOLT, M.D., 1846
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*TRANSLATED, UNDER THE AUTHOR'S SUPERVISION,*

BY

C. M. CULVER, M.A., M.D.,
FORMERLY CLINICAL ASSISTANT TO THE AUTHOR; MEMBER OF THE ALBANY INSTITUTE, ALBANY, N.Y.

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# INTRODUCTION.

THE fact that the eye is an optical instrument is being constantly brought to our notice; we read it everywhere, we hear it discussed on all sides; and it is universally acknowledged. On the other hand, it cannot be denied that Physiology forms the basis of Medical Science, and that a thorough knowledge of the former on the part of the Practitioner constitutes an essential element of success in whatever department of Medicine he may be occupied.

We therefore venture to draw the conclusion that, in order to treat the eyes successfully, an intimate knowledge of their optical construction and their functions is indispensable.

Facts confirm our conclusions. At least two-thirds of the patients who consult an oculist suffer from optical disturbances. But even the Surgery of the Eye is closely connected with Physical and Physiological Optics. The most skilful operator will fail to realise the object of an operation for cataract, unless he is able to substitute a proper glass for the removed crystalline lens. A strabismus operation, when undertaken without an exact knowledge of the optical and muscular functions of the Eye, is but a rude and even dangerous experiment.

Manual dexterity alone no longer suffices in the treatment of Diseases of the Eye. In order to be a perfect operator, it is necessary *besides* to be able to correctly diagnose a case, and to crown one's work by the adaptation of the organ to its functions.

It will not do to commence the most unimportant article with the assurance that Ophthalmology has made enormous strides during the last three decades, and there to let the matter rest. In spite of all progress, Practical Ophthalmology is still in its infancy. It belongs to us to follow our predecessors in a manner worthy of them, and to render productive their wonderful scientific achievements. This may be brought about not so much by the introduction of new ointments, collyria, and other remedies, as by a more intimate knowledge of the Physiology of the Eye, which deals in great measure with Optics and Physics.

It cannot be denied that the practice of the present day leaves much to be desired in this respect. This is no doubt due to the defective Physical and Mathematical education which is frequently received by the Practitioner, and explains why he instinctively avoids the study of anything bearing a resemblance to Algebra, and looks upon it as something very difficult to be mastered,—something quite without the confines of his sphere.

With regard to the study of the Refraction of the Eye, the amount of Mathematical knowledge which is essential in practice is, however, very easily acquired. Indeed, even in following the course of the rays of light through the entire dioptric system of the eye, as far as the retina—a calculation which, towards the end, becomes somewhat complicated—the Author has not found it necessary to go beyond the elementary knowledge of Mathematics which he acquired at college, when studying the obligatory Greek and Latin.

In order, however, to spare the reader even these formulæ, he has introduced a new plan into this book, and has separated the Mathematical portion from the rest of the work. The first chapter is specially devoted to those who take a particular interest in the scientific solution of the questions under consideration; its perusal, however, may be entirely omitted; the reader may begin with the second chapter, in and after which he will find no formulæ, and yet will easily understand the meaning of the text.

Even the Mathematical chapter is written in a simple and elementary way, not demanding of the reader any of that preliminary knowledge which is so quickly forgotten. The Author begins at the beginning, with the luminous point emitting its undulations, which radiate in all directions. The latter are followed from one medium to the other through a plane, and then through spherical surfaces, which form systems more and more complex.

The work, however, is not intended to be a general treatise on Optics. In all that has been considered, the view of our special object—namely, the explanation of facts indispensable to the Ophthalmologist—has never been lost. Each theorem is followed by its practical application; the single convex surface represents the cornea, the second medium the aqueous (or the vitreous) humor, the system formed by two spherical surfaces finds its analogue in the crystalline lens or in the spectacle glass; and the two systems combined form the dioptric apparatus of the Eye.

Thus it is to be hoped that the first chapter will enable the reader to dispense with other books on Physics, as far as concerns the resolution of the most essential problems in ocular Optics.

The indispensable Mathematical questions being thus treated, we do not return to them. In this manner the other chapters of the work form an elementary manual on Refraction and Accommodation stripped of analyses and formulæ.

It has appeared to the Author that, in practice, Accommodation and Convergence, and the relations existing between these two functions, so necessary to binocular vision, have not sufficiently been taken into account. These, however, have become extremely simple to manage since the introduction of the metric system into Ophthalmology, which has given to the former as a standard the *Dioptry*, and to the latter the *Metre-Angle*. He has, therefore, not hesitated to develop these subjects to a greater extent than is usual in text-books.

A chapter on the Methods of Determination ought not to be wanting in this treatise on Refraction. But, as the Author is writing for practice rather than for science, he has only given those methods really applicable to the former. All the principles of Optometry are explained, but he has abstained from entering into a description of all the different kinds of Optometers. If the principle has been understood, it is easy to comprehend the manner of working and the practical value, not only of the Optometers of the present, but also of those of the future.

The clinical portion of the work comprises all that, from a practical point of view, the Author has found to be of importance in his own practice in the clinics of the different countries which he has had the advantage of visiting, and in the writings of competent authors. That there is still much to be added, to be perfected, and even altered, follows from the fact that the science of Ophthalmology is still in its infancy.

This work was first published in the French language. The present English edition has, however, been considerably modified by the Author's experience since its first publication.

The Author is greatly indebted to Dr. C. M. Culver, of Albany, U.S.A., who has spared no pains in the translation of this work, a service which he is happy thus publicly to acknowledge.

LANDOLT.

PARIS, *May* 1886.

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# REFRACTION AND ACCOMMODATION OF THE EYE.

# THE REFRACTION AND ACCOMMODATION OF THE EYE.

## CHAPTER I.

## PHYSICAL PORTION.

### LUMINOUS POINT AND RAYS.

IF a stone be thrown into water, the surface of which was hitherto quiet and smooth, a series of circles, concentric about the point where the stone strikes, will be seen to form upon this surface. These circles remain concentric, because the disturbance communicated by the shock at one point to the aqueous molecules is propagated in the form of ripples, in the plane of the surface and with equal rapidity in every direction.

A luminous point produces analogous phenomena in space. It causes, everywhere around it, an undulatory movement of the ether. These undulations, however, are not limited solely to the horizontal plane, as appears to be the case in the water; they are not merely circles. They are propagated in every plane and in every direction, and thus form around the luminous point, as a centre, larger and larger spheres.

Light is therefore propagated from the luminous point in all directions in straight lines, that is to say, following the radii of these spheres of undulation of the ether. These lines of direction are commonly called *luminous rays*. Thus it may be said that a luminous point gives off rays, which are propagated in all directions, in straight lines and with equal rapidity as long as they remain in the same medium.

### REFRACTION BY A PLANE SURFACE.

When one of such rays meets another transparent medium, separated from the first by a plane surface, its course is modified according to the nature of the medium and the angle at which the ray falls upon this separating surface. If the ray strike the surface perpendicularly, it continues its course in the same direction; the rapidity of propagation of the light is, however, increased or diminished according to the nature of the second medium.

When the ray falls upon the surface at any other than a right angle, it is deviated from its primitive direction, though remaining in the same plane—the *plane of incidence.* This is called the *refraction of light.*[1]

We give the name *incident ray* to the ray before its passage into the second medium, and that of *emergent ray* to the ray after it has penetrated the second medium.

## INDEX OF REFRACTION.

Not all transparent media refract light equally. We distinguish, from an optical point of view, between denser and rarer media. In the former the light is propagated less rapidly, in the latter more rapidly.

When a luminous ray passes from a rarer into a denser medium, it is deviated toward a perpendicular let fall at the point where it strikes the surface; and it will approach nearer to this perpendicular in proportion as the difference of refractive power (density) between the two media is greater.

Conversely, when a ray passes from a refractive medium into one of less refractive power, it is deviated in the opposite direction; *i.e.*, it deviates from the perpendicular in pursuing its course, and that, too, proportionately to the difference, as to density, between the two media. This follows directly, moreover, from what we have just said. We have only to invert the case, and consider the emergent ray as the incident. The incident ray will then become the emergent, according to the well-known law that the course followed by a luminous ray, emanating from a point A of one medium, and passing through any number of other media to reach a point C, is exactly the same as that taken by a ray passing from C to A. This is a law with which we shall very frequently have to deal further on.

---

[1] In strict exactness it should be said that a portion of the light is reflected in the plane of incidence, at an angle equal to the angle of incidence, whenever the luminous ray strikes the surface at a certain angle. Only a part of the light, then, penetrates the second medium.

In relation to the portion refracted, the reflected part is greater in proportion as the angle of incidence is greater. As this angle increases, there comes a time when all the light is reflected, and when none of it penetrates the second medium. This is the *total reflection* of which we shall have occasion to speak again, later on.

The refraction to which a luminous ray is subjected, in passing from one medium to another, is evidently as much greater as the ray is more deviated from its primitive direction.

To express the relation of the refractive powers of two media, we employ, therefore, the relation of the angle of incidence with the angle of refraction.

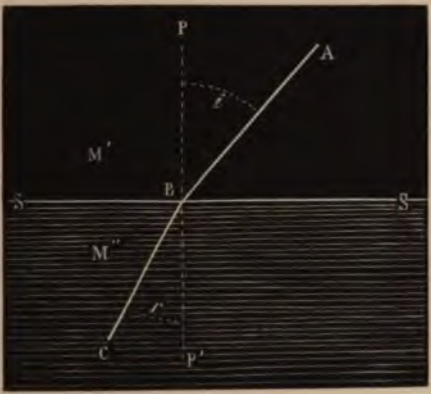

Fig. 1.

Let M' (Fig. 1) be the first, M" the second medium, and S S the plane surface which separates them. From the luminous point A is given off a ray which strikes the surface at B. This is the incident ray. Erect at the point B a perpendicular P P'. The plane of the paper will be the plane of incidence, and the angle A B P the angle of incidence ($i$).

Instead of continuing its course in the same direction, the ray A B is deflected toward the point C. C B P' is the angle of refraction ($r$).

In order to deal more clearly with the fundamental laws of refraction, let us take, instead of a single ray, a pencil of luminous rays $a\,a'\,b\,b'$ (Fig. 2).

They have come sufficiently far from the luminous source, that the surface of the wave, of which $c\,b'$ represents a section, may be regarded as plane, and the rays as parallel to each other. The light of this pencil has been propagated in the medium M' with a rapidity ($v'$), up to the time when it meets the plane surface S S, which separates the two media M' and M". The latter medium is the denser; that is to say, that light is propagated in it with less rapidity. The surface $c\,b'$ of the last wave, which is entirely in the medium M', forms, with the surface, the angle $c\,b'\,a'$. We designate this angle by $i$. While the portion of the wave corresponding to the point $b$ enters the medium M", and is there propagated with lessened speed only to the point $d$, the portion $c$ of the same wave still remains in the medium M′, and takes a longer course to $a'$. Hence $a'\,d$ is the first wave-surface which contains all the light of the pencil. It forms, in the second medium, with the surface S S, the angle $b'\,a'\,d$. This we call $r$. Now, $b'\,c$ being perpendicular to $a\,a'$, and $a'\,d$ perpendicular to $b'\,b''$, it will be seen that—

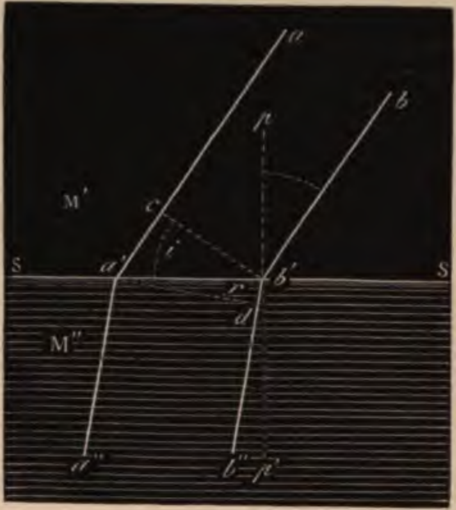

Fig. 2.

$$\sin i = \frac{a'c}{a'b'}$$

$$\sin r = \frac{b'd}{a'b'}$$

Hence

$$\frac{\sin i}{\sin r} = \frac{a'c}{b'd}$$

Now the angle $i$ is equal to the angle $p\,b'\,b$; that is, to the angle of incidence, since their sides are perpendicular to each other. For the same reason, the angle $r$ is equal to the angle $b''\,b'\,p'$; that is, to the angle of refraction.

The length of $a'\,c$ is evidently proportional to the rapidity of propagation of the light in the medium $M' = v'$. The length of $b'\,d$ is proportional to the rapidity of propagation of the light in the medium $M'' = v''$. Hence we have

$$\frac{\sin i}{\sin r} = \frac{v'}{v''}$$

and we say that *the sine of the angle of incidence is to the sine of the angle of refraction as the rapidity of propagation of light in the first medium is to the rapidity of propagation of light in the second medium.*

The relation of the rapidities of propagation from one medium to the other is constant. It is given the sign $n$, and called the *index of refraction.*

It is then more simply written—

$$\frac{\sin i}{\sin r} = n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1.$$

Hence $n$ indicates how much more or less rapidly light is propagated in the second than in the first medium, and how much the sine of the angle of refraction is smaller or greater than that of the angle of incidence.

The *absolute index of refraction* is that which is found when light passes from a vacuum into a given medium.

The *relative index of refraction* is that which is found when light passes from atmospheric air into another medium.

The difference of rapidity of propagation is, moreover, not very great between a vacuum and atmospheric air at 0° (centigrade), or 32 degrees Fahrenheit, and 760 millimetres of barometric pressure. They are to each other as 1 to 1,000,294. Hence we may neglect their difference, and regard light passing from the air into another medium as coming from a vacuum.

Letting $v$ represent the speed of light in a vacuum, $v'$ in a medium I, $v''$ in a medium I I, the absolute index of refraction of the medium I will be—

$$n' = \frac{v}{v'};$$

that of the medium I I will be—

$$n'' = \frac{v}{v''};$$

whence

$$\frac{n'}{n''} = \frac{v''}{v'}.$$

In accordance with formula 1 we may unite—

$$\frac{\sin i}{\sin r} = \frac{v'}{v''} = \frac{n''}{n'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1\,a,$$

or, again—

$$n' \times \sin i = n'' \times \sin r, \quad . \quad . \quad . \quad . \quad 1\,b.$$

That is to say, that *the product of the sine of the angle of incidence and the index of refraction of the first medium is equal to the product of the sine of the angle of refraction and the index of the second medium.*

When the first medium, M′, in which the luminous point is situated, is less refractive than the second, M″, the rapidity ($v'$) of the former is greater than that ($v''$) of the second; $n = \frac{v'}{v''}$, becomes greater than 1, and sin $i$ greater than sin $r$. Hence the refracted ray B C (Fig. 3) approaches the perpendicular P P. The converse is the case when the light passes from the second into the first medium. We then have $\frac{v''}{v'} = \frac{1}{n}$.

The index of refraction of the second medium is thus the reciprocal value of the index of the first one. The refracted ray C D (Fig. 3) recedes from the perpendicular P′ P′ in the same proportion that the ray B C approached it. From this it follows that, when a dioptric medium is limited by plane and parallel surfaces which separate it from the same medium on both sides, the incident rays are so refracted that they leave the medium in a direction parallel to their primitive one. They are only more or less displaced laterally. See the rays A B and C D (Fig. 3).

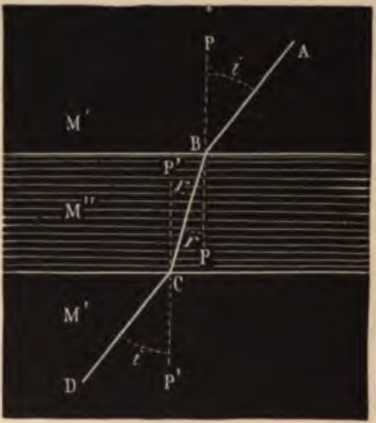

Fig. 3.

The case is the same for an infinite number of media of different indices of refraction, separated by plane and parallel surfaces, provided that, after having passed through all of them, the luminous ray re-enter the first medium: it will then be parallel to its primitive direction.

We take, as measure of the refractive power of a substance, the deviation to which a ray of light coming from the air (or rather from a vacuum) is subject when it enters this substance, *i.e.*, the relation between the sine of the angle of incidence and the sine of the angle of refraction, when light passes from a vacuum into this substance. This is, in other words, the absolute index of refraction. It is called simply the index of refraction.

The following are the absolute indices of refraction for certain substances:—

### I.—*Solids.*

| | |
|---|---|
| Crown glass, | 1·5 |
| Flint glass, | 1·57 to 1·58 |
| Ice, | 1·310 |
| Rock crystal, | 1·562 |
| Quartz (ordinary index), | 1·547 |
| Diamond, | 2·48 to 2·75 |
| Cornea, | 1·3365 ✓ |
| Crystalline lens, cortical layer, | 1·393 |
| Do. intermediate layer, | 1·419 |
| Do. nucleus, | 1·431 |

### II.—*Liquids.*

| | |
|---|---|
| Water, | 1·336 |
| White of egg, | 1·351 |
| Human blood, | 1·354 |
| Sulphuric ether, | 1·358 |
| Rectified alcohol, | 1·372 |
| Canada balsam, | 1·532 |
| Saturated solution of sea salt, | 1·575 |
| Bi-sulphide of carbon, | 1·678 |

### III.—*Gases.*

| | |
|---|---|
| Air, | 1·000294 |
| Oxygen, | 1·000272 |
| Nitrogen, | 1·000300 |
| Carbonic acid, | 1·000439[1] |

### *Example.*

Let us take water as an example. The table shows that the index of refraction of water is 1·33 or $\frac{4}{3}$. This signifies that when a luminous ray penetrates a surface of water at an angle $i$, the sine of this angle, divided by the sine of the angle of refraction, gives the quotient 1·33.

Letting 10° equal the angle of incidence, what is the angle of refraction? We may solve the problem either by calculation or by construction. Let us first calculate. We write—

---

[1] The indices of refraction of the crystalline are those found by Woinow in a person sixteen years of age. The others are taken from Wundt's *Physique médicale*, translated by Monoyer, and from Pouillet-Müller's *Lehrbuch der Physik*.

$$\frac{\sin 10^\circ}{\sin r} = 1{\cdot}33$$

$$\sin r = \frac{\sin 10^\circ}{1{\cdot}33}$$

$$\begin{array}{r} \log \sin 10^\circ = 0{\cdot}2396 - 1 \\ \log 1{\cdot}33 = 0{\cdot}1238 \\ \hline \sin \log 0{\cdot}1158 - 1 = 7^\circ\ 30'\ 10'' \end{array}$$

Dividing 10° by 7° 30′ 10″ we obtain in fact 1·33.

Let us solve the problem again by construction, which is no less instructive. The number 1·33 is equivalent to $\frac{4}{3}$, and if we say that the sine of the angle of incidence is to that of the angle of refraction as 4 is to 3, this signifies that the second is always less by $\frac{1}{4}$ than the first.

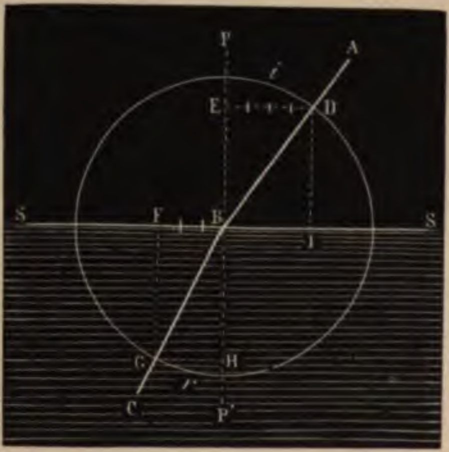

Fig. 4.

In Figure 4, let A B represent the incident ray which meets the surface S S in B, at an angle A B P, P P′ being the vertical erected at B. In order to find first its sine, we place one point of a pair of dividers at B, and describe a circle of any diameter around this point as a centre. From the point D, where the circle intersects the ray A B, we draw a perpendicular D E to B P. This is the sine of the angle of incidence. Now divide this length E D into four equal parts. Measure off from B toward F, a distance equal to three of these parts. Draw from this point a line parallel to B P, and the point where it intersects the circle will mark the point through which the refracted ray B C should pass. Indeed G H is the sine of the angle of refraction C B P′, for G H = F P = $\frac{3}{4}$ of E D.

The task may be rendered still simpler if we commence by measuring off from B, upon the horizontal, four divisions, B I, to the right in our examples, and three divisions of equal value, B F, toward the left. From the extremity of this line, to the right, erect a vertical line until it intersects the ray at D; at the left let fall the vertical F G.

A circle whose centre is B, and having a radius B D, will indicate, at G, the point through which the refracted ray passes.

There are still several other methods of constructing the refracted ray, but we do not wish to waste time in details.

## LIMIT ANGLE—TOTAL REFLECTION.

The greatest value that the angle of incidence can have is evidently 90°, which is the case when light passes along the refracting surface in a direction parallel to the plane of that surface. The sine of 90° is 1.

For this case the formula—

$$\frac{\sin i}{\sin r} = n$$

becomes, therefore,

$$\frac{1}{\sin r} = n$$

or

$$\sin r = \frac{1}{n}.$$

The value which results, from this, for the angle of refraction $r$, is called the *limit angle*. This is evidently the greatest value that the angle of refraction can have.

We know that, for water,

$$n = \frac{4}{3}$$

hence

$$\frac{1}{n} = \frac{3}{4} = 0{\cdot}75.$$

0·75 corresponds to the sine of 48° 35′. This is the limit angle for water.

If we consider the matter from another point of view, we may say that, a luminous point being in the water, all rays emanating from it, up to and including those which form with the surface an angle of 48° 35′, emerge from the water. The latter rays will pass along the surface of the water. But rays forming with the surface an angle greater than 48° 35′ will no longer emerge from the water. They all remain in it, and are reflected by its surface at the same angle at which they meet the surface. Since all the rays beyond the limit angle are reflected and none refracted, the phenomenon is called *total reflection*.

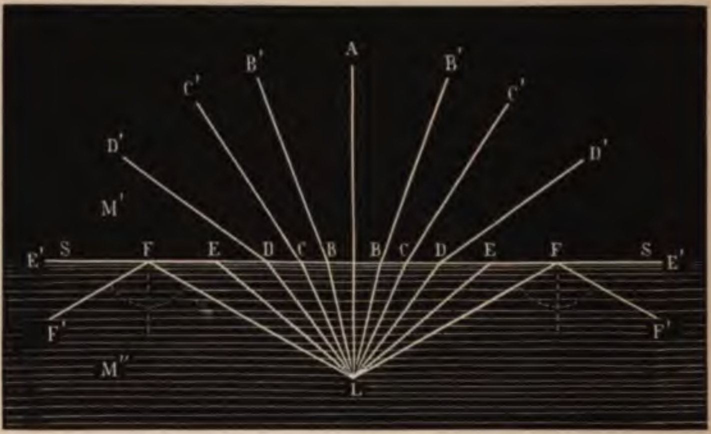

Fig. 5.

Figure 5 gives the explanation of this. It is constructed in exact accordance with the data for air and water.

Let M′ be the air, M″ the water, S S the surface, and L a luminous point in the water. The ray L A emerges from the water without being refracted. The other rays emerge from it at constantly increasing angles. L E continues its course along the surface toward

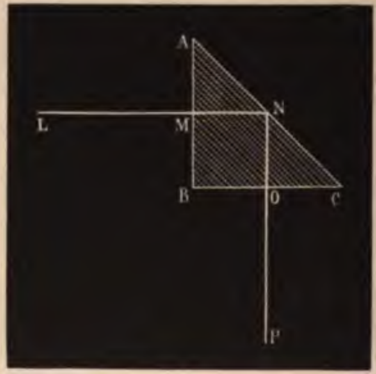

Fig. 6.

E′. A L E is the limit angle. L F does not emerge from the water, but is reflected toward F′. In a word, all rays which meet the surface at an angle greater than A L E are no longer refracted, but are reflected by the surface. From L F on, all rays are reflected in this way.

The limit angle of crown glass is 40° 49′; that of flint glass 37° 36′.

A familiar example of total reflection is afforded by the rectangular prism.

The luminous ray L M (Fig. 6) falls upon the surface of the prism normally, and passes through it without any deviation or reflection; but at N it meets the hypotenuse surface at an angle of 45°—that is to say, greater than the limit angle from glass to air. Hence it undergoes total reflection toward O, where it emerges from the prism, still without deviation or reflection, being directed perpendicularly to the surface B C. Thus it is that all the light from L is reflected by the surface A C toward P.

## Refraction by a Spherical Surface.

Let us suppose now that the rays given off from a luminous point L do not fall upon a plane surface but upon a spherical one, S H S (Fig. 7), which separates the medium M′ from a more refractive medium M″. This surface is convex on the side of the point L, and its centre of curvature is C.

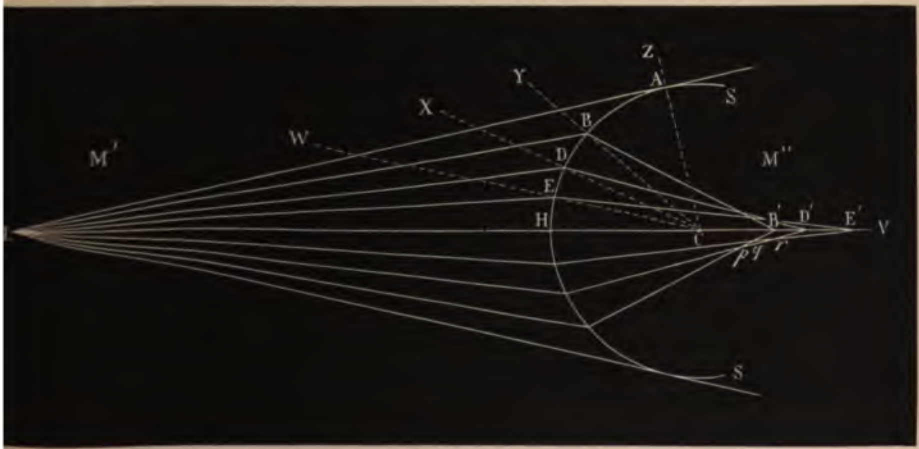

Fig. 7.

In this case the different rays strike the surface at very different angles. The spherical surface may indeed be considered as composed of an infinite number of plane surfaces inclined to each other, each being at right angles to the radius of the sphere which corresponds to it.

Take, for instance, the ray L A. It simply passes along the surface, tangent to and without penetrating it, being perpendicular to the radius C A of the sphere, and hence parallel to the surface at the place where it comes in contact with it.

Another will fall upon the surface at B, at an angle L B Y; it will be deviated. The corresponding angle of refraction will be C B B′. Hence the ray will continue its course in the direction B′.

Another ray will strike the surface at D, at an angle L D X, less than L B Y. It will still be deviated, but less so than L B, and will be directed toward D′.

Another ray, L E, still less inclined to the surface, will be refracted toward E′. The ray L H, which is directed toward the centre (C) of the surface, is perpendicular to the latter and passes through it, without deviation, toward V. It coincides with the axis of the sphere.

The same phenomenon which we have observed above this ray is also observable below it, and in every plane passed through the line L V.

Causing the surface to revolve about L V as an axis, all rays that meet it at the same angle describe, upon this surface, circles having H as a common centre; and all rays which traverse the same circle are united at one and the same point of the axis.[1] Thus the rays, which meet the surface at an angle L B Y, describe a circle, of which B is one point, and are united at B′.

All those which, in whatever plane, meet the surface at an angle L E W, are included in a circle of the surface, of which E is one point, and are united at E′. Hence all the rays which have passed through at the same distance from its summit will be united at the same point of the axis; but this point will be nearer the surface in proportion as the rays have passed through it further from its apex. The rays nearest the apex are united farthest behind the surface.

A homocentric pencil of rays—*i.e.*, one coming from a single point—will not therefore remain homocentric, that is to say, it will no longer be directed toward a single point after having passed through a spherical surface, but will form a *focal line*.

---

[1] To be quite exact, we should add, on condition that the light be *monochromatic*. Indeed, if the light be composed of different colours (as white light, for instance), it will be decomposed by refraction in such a way that rays passing through the same circle of the spherical surface will be more refracted in proportion as they belong to a part of the solar spectrum nearer the violet extremity, or as their wave-length is less. This is what is called *chromatic aberration*.

The rays having passed through the surface at very different distances from the summit, cross each other in being directed toward their respective focal points $p$, $q$, $r$ (Fig. 7). The points, where they cross each other, are distinguishable by their greater *luminous intensity*, and their union forms what is termed a *caustic*. This may be easily observed in a massive glass cylinder, or, again, by reflection, in the interior of a lamp chimney or of a drinking glass, for in reflection the same appearances are produced as by refraction.

In order that the homocentric rays may remain homocentric, the surface must have such a form that the angles of incidence shall be everywhere the same. This condition is fulfilled in ellipsoids and hyperboloid surfaces better than in spherical ones; but even then the luminous point must have a determined, fixed position.[1]

In this case all the rays given off from the same luminous (monochromatic) point will be united at a single focus, which is the *image* of that point. Indeed, the union of rays given off from a luminous point is called an image.

In the sphere, each circle of its surface forms an image of the point, and all the images are situated on the axis, forming together the focal line. In the paraboloid of revolution all the circles form their images at the same point. Thus the focal line is reduced to a point.

We seldom have to deal with parabaloid surfaces, however. We, especially, find, almost everywhere, spherical surfaces called upon to produce images. It is, however, possible to obtain an image almost reduced to a point, if in producing it one does not use a very extended surface, but only a part of it near the axis. Referring to Fig. 7, it will in fact be seen that the smaller the angle of opening of the surface is, and the more the peripheral portions of the surface are excluded, the more restricted is the focal line. In this way the rays given off from a point may be regarded as united in a single point behind the surface, and the image of a point is itself a point.

It is with this hypothesis, that the luminous rays pass through refractive surfaces very near their axes, that we are now about to take up the consideration of the laws of *spherical refraction*.

## PRINCIPAL AND CONJUGATE FOCI.

Let L′ (Fig. 8) be the luminous point and S H S a spherical surface, but in this case reduced to a smaller portion than we have drawn in the figure.

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[1] Wundt's *Physique médicale*, translated by Monoyer, p. 291.

We ask, where the rays emanating from the point L′ will be united; in other words, where will the image of L′ be formed?

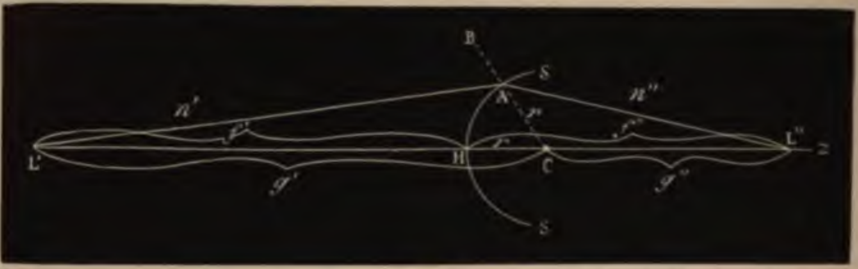


FIG. 8.

Let us draw a line L′ Z through the centre, C, of the surface, *i.e.*, its axis. We may already say that the image must be situated on this line, since it is one of the rays of the pencil given off from L′, which undergoes no deviation, because it strikes the surface at right angles. It remains only to determine in which direction a given ray L A is deviated; and where the refracted ray meets the axis, all rays emanating from L′ will be focused, and the image of L′ will be at this point.

To find the direction followed by the ray L′ A, let us erect at A the perpendicular A B, which is simply the prolongation of the ray C A. L′ A B is then the angle of incidence. Suppose A L″ to be the refracted ray; C A L″ will then be the angle we are seeking. Let $n'$ be the index of refraction of the first, and $n''$ that of the second medium. Conformably with the formula 1 *b*, we write—

$$n' \times \sin L'AB = n'' \times \sin CAL''.$$

Now the sines of the angles of a triangle are to each other as the opposite sides. We have then, in the triangle L′ A C—

$$\frac{\sin L'AC}{\sin AL'C} = \frac{L'C}{r},$$

if we designate by $r$ the radius A C of the surface; or, since the sine of the angle L′ A C is equal to the sine of the adjacent angle L′ A B—

$$\frac{\sin L'AB}{\sin AL'C} = \frac{L'C}{r}.$$

In the triangle C L″ A we have—

$$\frac{\sin CAL''}{\sin AL''C} = \frac{CL''}{r}.$$

On dividing these two equations, the first by the second, we obtain—

$$\frac{\sin L'AB}{\sin CAL''} \times \frac{\sin AL''C}{\sin AL'C} = \frac{L'C}{CL''}.$$

According to our equation 1 *a*, we may write—

$$\frac{\sin \mathrm{L'AB}}{\sin \mathrm{CAL''}} = \frac{n''}{n'}.$$

In the triangle L′ A L″ we have—

$$\frac{\sin \mathrm{AL''C}}{\sin \mathrm{AL'C}} = \frac{\mathrm{L'A}}{\mathrm{AL''}}.$$

Hence we write—

$$\frac{n'' \mathrm{L'A}}{n' \mathrm{AL''}} = \frac{\mathrm{L'C}}{\mathrm{CL''}}.$$

We have said that we take into consideration only such rays as fall upon the surface very near its summit H. In this case we may admit that L′ A = L′ H and A L″ = H L″. Our formula then becomes—

$$\frac{n'' \mathrm{L'H}}{n' \mathrm{HL''}} = \frac{\mathrm{L'C}}{\mathrm{CL''}}.$$

Let us now, for simplicity's sake, designate—

H C by $r$ = radius of curvature of the refractive surface;

H L′ by $f'$ = distance of the luminous point from the refractive surface;

H L″ by $f''$ = distance of the image of the point from the surface;

C L′ by $g'$ = distance of the luminous point from the centre of curvature;

C L″ by $g''$ = distance of the image from the centre of curvature.

In other words, we designate by $f$ the distances of the object and the image from the *surface*, by $g$ the distances from the *centre of curvature*, and, since the surface and its centre are separated by the distance $r$, we have—

$$f' = g' - r$$
$$f'' = g'' + r$$
$$g' = f' + r$$
$$g'' = f'' - r.$$

Substituting these letters, our equation becomes—

$$\frac{n'' f'}{n' f''} = \frac{g'}{g''} \quad \ldots\ldots\ldots \quad 2,$$

or, replacing their equivalents everywhere by $f$ and $r$—

$$\frac{n'' f'}{n' f''} = \frac{f' + r}{f'' - r} \quad \ldots\ldots\ldots \quad 2\,a.$$

Introducing the values $g'$ and $g''$, we have—

$$\frac{n''(g' - r)}{n'(g'' + r)} = \frac{g'}{g''} \quad \ldots\ldots\ldots \quad 3\,a.$$

This equation may be transformed in the following manner:

$$n''f'(f''-r)=n'f''(f'+r)$$

$$n''f'f''-n''f'r=n'f''f'+n'f''r$$

$$f''(n''f'-n'f'-n'r)=n''f'r$$

$$f''=\frac{n''f'r}{n''f'-n'f'-n'r}$$

and

$$\frac{1}{f'}=\frac{1}{r}-\frac{n'}{n''r}-\frac{n'}{n''f'},$$

$$\frac{n'}{n''f'}+\frac{1}{f''}=\frac{1}{r}-\frac{n'}{n''r}.$$

Multiplying all terms by $n''$:

$$\frac{n'}{f'}+\frac{n''}{f''}=\frac{n''-n'}{r} \quad . \; . \; . \; . \; . \; . \; . \; . \quad 2\,b.$$

By the same transformation is obtained, from formula 3 *a*, the following one:

$$\frac{n''}{g'}+\frac{n'}{g''}=\frac{n''-n'}{r} \quad . \; . \; . \; . \; . \; . \; . \; . \quad 3\,b.$$

Under these forms the preceding formulæ are most readily remembered.

If the first medium be the air, its index of refraction $n'$ becomes equal to 1, and, substituting simply $n$ for $n''$, the formula 2 *b* becomes—

$$\frac{1}{f'}+\frac{n'}{f''}=\frac{n-1}{r} \quad . \; . \; . \; . \; . \; . \; . \; . \quad 2\,c,$$

whence

$$r=\frac{f'f''(n-1)}{f'n+f''}.$$

The formula 3 *b* is likewise simplified for air,—*i.e.*, for $n = 1$ it becomes—

$$\frac{n}{g'}+\frac{1}{g''}=\frac{n-1}{r} \quad . \; . \; . \; . \; . \; . \; . \; . \quad 3\,c.$$

It will be seen that the luminous point and its image are intimately related to each other.

We call the luminous point and its image (L′ and L″ in our example) *conjugate foci*, and the distances $f'$ and $f''$, which separate these two points from the refracting surface, *conjugate focal distances*. One is generally called the *first*, or anterior, conjugate focal distance; the other, the *second*, or posterior, conjugate focal distance.

When the luminous point is at infinity (Fig. 9), it emits rays which are parallel to each other. Then $f'$ becomes $\infty$, and the formula 2 *b* becomes—

$$\frac{n''}{f''}=\frac{n''-n'}{r}.$$

This is an especially important case. We give the name *second principal focus* to the principal posterior focus, because it is situated behind the refracting surface, at the spot where the rays, which were parallel before passing through the refracting surface, are united; and we call the distance, which separates this focus from the surface, the *second principal focal distance.* We shall designate the second focus by $\phi''$ and the principal posterior focal distance (H $\phi''$, Fig. 9) by F″.

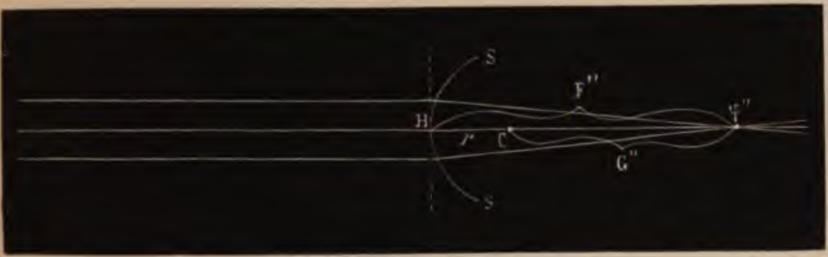


FIG. 9.

Deducing the value of F″ from the preceding formula, we obtain—

$$F'' = \frac{n'' r}{n'' - n'}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4\,a.$$

When the first medium is air, whose index of refraction equals 1, the formula becomes simpler. Let us again call the index of refraction, of the second medium, $n$, and we shall have—

$$F'' = \frac{n r}{n - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4\,b.$$

For parallel rays, $g'$ (Fig. 8) likewise becomes infinite.

Designating by G″ the distance C $\phi''$ (Fig. 9) from the centre of curvature to the second principal focus, we obtain, from the formula 3 *b*, the expression—

$$G'' = \frac{n' r}{n'' - n'}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 5\,a.$$

When the first medium is air, this formula is transformed into—

$$G'' = \frac{r}{n - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 5\,b.$$

The well-known law of reciprocity, according to which rays, coming from a point B and being directed toward a point A, follow the same path as those coming from A and directed toward B, shows us that, if there be a luminous focus at $\phi''$, *i.e.*, at the second principal focus, rays emanating from it are parallel to each other, after having passed out of the second medium through the surface S H S.

If, on the contrary, these be parallel in the second medium (Fig. 10), they should be focused somewhere in front of the surface. Then $f''$, of the formula 2 *b*, becomes infinite, and we obtain—

$$\frac{n'}{f'} = \frac{n'' - n'}{r}.$$

In this particular case the point of union, of rays which were parallel in the second medium, is called the *first principal focus* or principal anterior focus. We shall designate it by $\phi'$. The distance $\phi'$ H (Fig. 10), which separates it from the refractive surface, will then be the *first principal focal distance.*

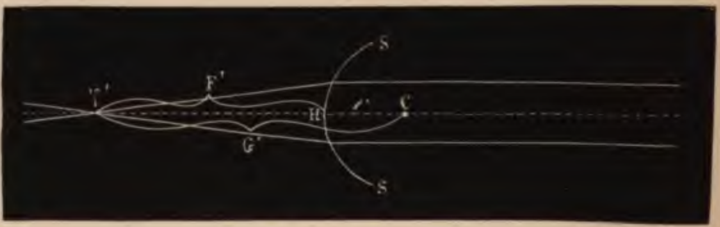


Fig. 10.

We designate it by F′ and deduce for it, from the preceding formula, the expression—

$$F' = \frac{n' r}{n'' - n'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4\ c.$$

When the first medium is air, this formula is simplified, and reads—

$$F' = \frac{r}{n - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4\ d.$$

In case the luminous point be situated at an infinite distance in the second medium, $g''$ likewise becomes equal to ∞, and from the formula 3 *b* results, for the distance C $\phi''$, which separates the first focus from the centre of curvature, the formula—

$$G' = \frac{n'' r}{n'' - n'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 5\ c.$$

and for $n' = 1$—

$$G' = \frac{n\, r}{n - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 5\ d.$$

If the light come from the more refractive medium, and pass through the surface S H S, whose concavity is toward the light, the first focal distance is equal to F″, the second to F′.

Here again we have, from mere inspection of Figures 9 and 10—

$$\begin{aligned} F'' &= G'' + r \\ F' &= G' - r \\ G'' &= F'' - r \\ G' &= F' + r \end{aligned}$$

But, farther than that, formula 4 *a*, for F″, is identical with formula 5 *c* for G′:

$$\frac{n'' r}{n'' - n'}$$

and the formula 5 *a* for G″ is identical with the formula 4 *c* for F′:

$$\frac{n' r}{n'' - n'}$$

Hence we have—

$$\begin{aligned} F'' &= G' \\ F' &= G''. \end{aligned}$$

Whence it follows that

$$F'' = G'' + r = F' + r.$$

Hence

$$F'' - F' = r.$$

Therefore the difference between G′ and G″ is likewise equal to *r*.

But there exists still another relation between F′ and F″, a relation resulting from the formulæ 4 *a* and 4 *c*:

$$\frac{F''}{F'} = \frac{\dfrac{n'' r}{n'' - n'}}{\dfrac{n' r}{n'' - n'}} = \frac{n''}{n'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 6\,a.$$

That is to say, that the focal distances are to each other as the indices of refraction of the media to which they correspond.

When $n' = 1$, *i.e.*, for air, we obtain—

$$\frac{F''}{F'} = n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 6\,b.$$

EXAMPLE.

Suppose we have a convex surface whose radius of curvature is 5 millimetres long. This surface separates air from water, the index of refraction *n*, of the latter, being $\frac{4}{3}$. We say—$\frac{F''}{F'} = \frac{4}{3}$; *i.e*, F″ is 4 when F′ is 3. The difference between the two is *r*, equal in the present case to 5 millimetres. Now, in order to obtain two numbers which shall be to each other as 4 to 3, and whose difference shall be 5, it is simply necessary to multiply 4 and 3 by 5. Hence F″ will be 4 × 5 = 20 millimetres, and F′ 3 × 5 = 15 millimetres long.

Let the curvature of the surface be the same, but medium different; glass, for instance, having an index of refraction equal to $\frac{3}{2}$; then $F''$ will be equal to $3 \times 5 = 15$ millimetres, and $F'$ will equal $2 \times 5 = 10$ millimetres. That is to say, the ratio between the two focal distances is equal to the index of refraction of the refractive medium, and their real lengths are multiples of the radius of curvature.

The same proportion exists necessarily between $G'$ and $G''$:

$$\frac{G'}{G''} = \frac{n''}{n'}$$

Dividing the equations 2 *b* and 3 *b* by $\frac{n''-n'}{r}$, and substituting the values of $F'$, $G''$, $F''$ and $G'$, these formulæ become—

$$\left.\begin{aligned} \frac{F'}{f'} + \frac{F''}{f''} = 1 \\ \frac{G'}{g'} + \frac{G''}{g''} = 1 \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad 7.$$

We have thus eliminated the expressions $n'$, $n''$ and $r$, which greatly simplifies the calculations. Provided we know the first and second focal distances, we can easily find the spot where is formed the image of a point situated at a distance $f'$ in front of, or $f''$ behind the refracting surface, or at $g'$ in front of, or $g''$ to the rear of its centre of curvature. We deduce, in fact, from the formulæ 7, for these different lengths, the following formulæ:

$$\left.\begin{aligned} f'' &= \frac{F''f'}{f'-F'} \\ f' &= \frac{F'f''}{f''-F''} \\ g'' &= \frac{G''g'}{g'-G'} \\ g' &= \frac{G'g''}{g''-G''} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \quad 8\,a.$$

Let us call the distance from the object to the first focus $(f'-F') = l'$, the distance from the image to the second focus $(f''-F'') = l''$.

In the formula $f' = \frac{F'f''}{f''-F''}$, let us subtract $F'$ from each member of the equation, and we shall obtain—

$$f' - F' = \frac{F'f''}{f''-F''} - F'$$

$$(f'-F')\,(f''-F'') = F'\,F''$$

$$l'\,l'' = F'\,F'' \quad . \; . \; . \; . \; . \; . \; . \; . \quad 7\,c.$$

The *plus* sign is given to $f'$ and $g'$ as well as to $f''$ and $g''$, as long as the object and the image are on opposite sides of the surface.

If, on the contrary, the image and object be both on the same side of the surface, $f'$ and $g'$, or $f''$ and $g''$ take the *minus* or negative sign. This may happen when the luminous point is removed from the surface beyond infinity, *i.e.*, when the rays coming from this point are neither divergent nor parallel, but *convergent*. This is not an impossible case. Although no luminous point ever emits convergent rays, still the rays coming from it may be rendered convergent, either by causing them to pass through a convex lens, or by reflecting them from a concave mirror. The object L′ is then situated at the point where the convergent rays would be focused if prolonged, *i.e.*, behind the surface, on the same side of it as the image (Fig. 11).

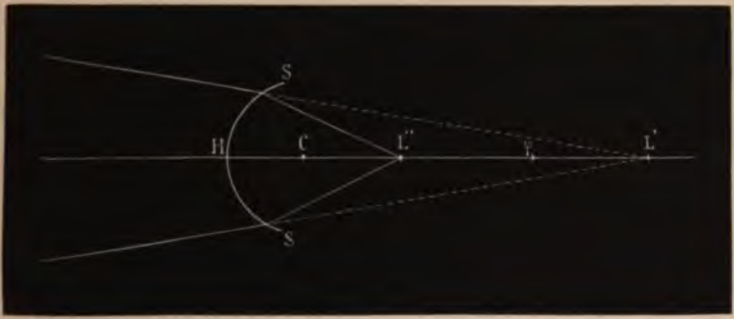


FIG. 11.

In order to find the place of the point L″, we shall have to introduce, with the minus sign, the value of $f'$ in the formula, for $f''$.

Let us take an example:

Let $F'' = 20$ millimetres
$F' = 15$ „

and suppose that the rays which strike the surface converge toward a point L′ situated 250 millimetres behind the surface; $f'$ is then negative and equals $-250$ millimetres:

$$f'' = \frac{-20 \times 250}{-250 - 15} = \frac{5000}{265} = 18{\cdot}8 \text{ millimetres.}$$

L″ is therefore situated 18·8 millimetres behind the surface, *i.e.*, nearer to it than is the principal focus. This is conceivable: parallel rays are united at the focus, in $\phi''$, and rays which are already convergent before reaching the surface must be united nearer the surface than the parallel ones, and all the nearer according as they are more convergent.

Let us invert the case, and make $L''$ the object, situated in the second medium. It follows from the law of reciprocity that, in this case, rays emanating from $L''$ diverge as if they came from a point $L'$ situated behind the surface. $L'$ is, then, the image of $L''$, but this image is virtual, *i.e.*, it has no real existence, since the rays emanating from $L'$ are nowhere focused, and we have obtained $L'$ only by supposing these rays to be prolonged backward. We shall see, however, that these divergent rays do not always pass on to become lost beyond infinity. Let us suppose, for instance, an eye which looks toward this convex surface and which is capable of focusing divergent rays upon its retina. It would collect the rays coming from $L''$, and this point would appear to it as if situated at $L'$.

Supposing now the incident rays to be less and less convergent, $L'$ and $L''$ will recede more and more from the surface, $L'$ more rapidly than $L''$, until $L'$ shall have reached infinity, when the incident rays are parallel and are focused at $\phi''$. The image $L''$ is then at the principal focus, 20 millimetres behind the surface, in our example.

When the luminous point $L'$, the object, is within infinity, *i.e.*, at a finite distance *in front* of the surface, $L''$ should be beyond $\phi''$, behind the surface; hence $f'$ and $f''$ are positive (Fig. 8).

Let us take an example. $L'$ being 250 millimetres in front of the surface, and $F''$ and $F'$, equal respectively to 20 and 15 millimetres, as in the preceding case, we shall have—

$$f'' = \frac{20 \times 250}{250 - 15} = 21{\cdot}2 \text{ millimetres.}$$

Hence $L''$ is situated 21·2 millimetres behind the surface. If there be a luminous point at this distance, in the second medium, it will produce its image 250 millimetres in front of the surface.

The nearer the object $L'$ approaches to the surface, the farther its image $L''$ recedes from it. At length, when $L'$ has reached the anterior focus ($\phi'$), the rays are no longer united, but are, as we have seen, parallel to each other after having passed through the surface (compare Fig. 14).

If the luminous point be brought still nearer the surface, and be situated between it and the anterior focus, the rays must diverge after their refraction. Supposing that the first focal distance remain 15 millimetres, and that $L'$ be 10 millimetres in front of the surface, we shall obtain for $f''$ a negative value:

$$\frac{20 \times 10}{10 - 15} = -40 \text{ millimetres,}$$

which indicates that $L''$ is situated on the same side of the surface as $L'$. In other words, that the rays given off from $L'$ diverge, after their passage through the surface, as if they came from a point situated 40 millimetres in front of it (compare Fig. 15).

When L′ is on the surface itself, the image and object of the point coincide.

## IMAGE FORMED BY A SINGLE REFRACTING SURFACE.

We have seen that a luminous point L′, situated at a distance *f*′ in front of a spherical surface, on its axis, *i.e.*, upon the line which passes through this point and the centre of curvature of the surface, forms its image L″ at the distance *f*″ from the latter.

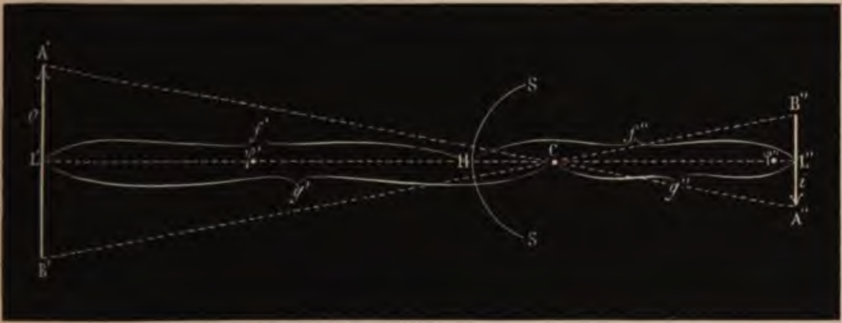


FIG. 12.

Another luminous point A′ (Fig. 12), situated at the same distance from the surface, ought to act in exactly the same way as L′. It will likewise form its image upon the axis passing through this point and the centre of curvature C of the surface, and at the same distance behind the latter as the rays emanating from L′—since L′ and A′ are equidistant from the surface.[1]

Hence we have only to draw a straight line from A′ through C, and to mark upon it the length *f*″, from the point where this line cuts the surface (or *g*″ from C), in order to find the point A″, which will be the image of A′.

The name *optic axis* or *principal axis* is generally given to the axis which, like L′ L″ in our example, is perpendicular to the refracting surface, when the latter is regarded as plane. The axes corresponding to points not situated upon the principal axis are then called *secondary axes.* These secondary axes have the same significance, for points not situated upon the principal axis, that the latter has for the points which it connects.

---

[1] We suppose the portion of the surface which produces the image of A′ B′ to be small enough to be considered parallel to A′ B′.

Since the same thing takes place in the case of B′ as occurs in the case of A′, the image of the former will be formed at B″.

The same thing is repeated, too, for all points situated between A′ and B′; all form their images between A″ and B″ upon the axes corresponding to them. All the points united form, on the one hand, the linear object A′ B′, and, on the other, its linear image B″ A″.

All lines situated in a plane perpendicular to the axis L′ L″, and, passing through L′, will thus form their images in a plane perpendicular to the axis, and passing through the point L″.

Hence a plane object, perpendicular to the optic axis, and situated at the distance $f'$ in front of the surface, forms its image at the distance $f''$ behind the surface. This image is *real* when it is situated on the opposite side of the surface, because it is then formed by the union of rays emanating from every point of the object.

We have, indeed, drawn but one ray for each luminous point, inasmuch as, $f''$ being already known from the calculation, a single ray was sufficient; but in reality each luminous point gives off an infinite number—an entire cone—of rays, which are all united at the point which corresponds to them.—(See Fig. 13 for the three points A′, L′ and B′.)

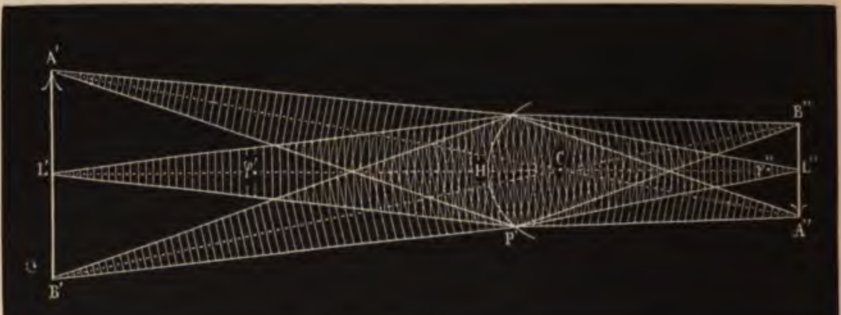


FIG. 13.

The image is *inverted* relatively to the object. What is above in the object, is below in the image; what is at the right in the object, is at the left in the image, and *vice versâ*. It is, moreover, geometrically *similar* to the object. A′ B′ and A″ B″ are in fact corresponding sides of two similar triangles, whose angles are equal. Hence, in order to find the size of the image, we write—

$$\frac{A'' B''}{A' B'} = \frac{L'' C}{L' C} = \frac{g''}{g'}.$$

Letting $o$ represent the size of the object, and $i$ that of the image, we have—

hence

$$\left.\begin{aligned} \frac{i}{o} &= \frac{g''}{g'} \\ i &= \frac{o\,g''}{g'} \end{aligned}\right\} \quad \text{. . . . . . . . . . 9 } a.$$

or, substituting for $g''$ and $g'$ their values as found in formula 8 $a$—

$$\left.\begin{aligned} i &= \frac{o \quad G''}{g' - G''} \\ i &= \frac{o\,(g'' - G'')}{G'} \end{aligned}\right\} \quad \text{. . . . . . . 9 } b.$$

Or, again, since

$$g'' = f'' - r$$

and

$$g' = f' + r,$$

$$i = \frac{o\,(f'' - r)}{f' + r}.$$

Replacing $r$ by $F'' - F'$ and substituting for $f'$ and $f''$ their values as given in formula 8 $a$, we obtain—

$$i = \frac{o\,(f'' - F'')}{F''}$$

$$i = \frac{o\,F'}{f' - F'}.$$

Finally, substituting $l''$ for $f'' - F''$, and $l'$ for $f' - F'$,

$$\left.\begin{aligned} i &= \frac{o\,l''}{F''} \\ i &= \frac{o\,F'}{l'} \end{aligned}\right\} \quad \text{. . . . . . . . . . 9 } c.$$

Hence, in order to find the size of the image formed by a spherical refracting surface, we ask ourselves, first, where its image is formed, *i.e.*, we calculate $f''$ or $g''$. It then remains only to draw the rays of direction, corresponding to the extremities of the object, through the centre of curvature. The points at which a perpendicular to the axis, passing through the point L′, intersects these two lines, will indicate the extremities of the image, and its size will be given by one of the simple formulæ above.

Let us take an example:

Let $o$, the size of the object, $= 12$ millimetres;
$f'$, its distance from the surface, $= 1000$ „
$r$, the radius of curvature, $= 5$ „
$F'' = 20^{mm} = G'$;
$F' = 15^{mm} = G''$.

According to the formula 8 $a$, we obtain for $f''$—

$$f'' = \frac{20 \times 1000}{1000 - 15} = \frac{20{,}000}{985} = 20{\cdot}3 \text{ millimetres,}$$

and for the size of the image $i$:

$$i = \frac{o\,(f'' - r)}{f' + r} = \frac{12\,(20{\cdot}3 - 5)}{1000 + 5} = 0{\cdot}18 \text{ millimetres.}$$

or, again,

$$i = \frac{o\,\mathrm{F}'}{f' - \mathrm{F}'} = \frac{12 \times 15}{1000 - 15} = 0{\cdot}18 \text{ millimetres.}$$

The same result is obtained if all the distances be reckoned from the centre of curvature, *i.e.*, with $g$ and G.

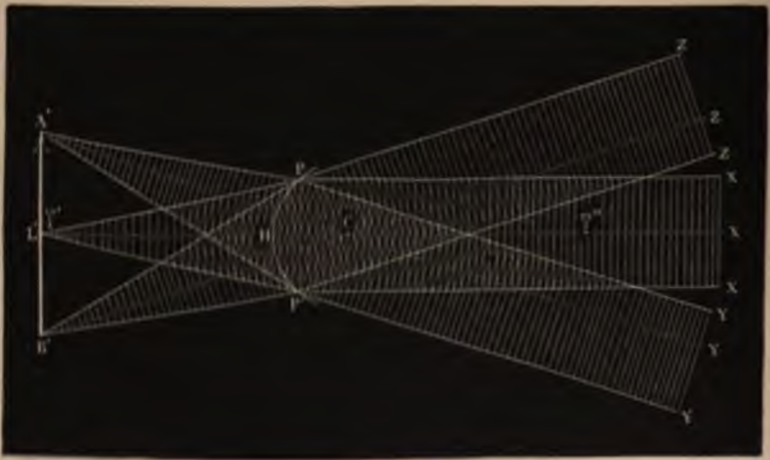


Fig. 14.

If the object approach the surface, the image recedes from it, as will be readily understood upon reference to the formula 9 $a$; $i$ increases in direct proportion with $g''$, and in inverse proportion with $g'$.

When the object is at a distance, F″ in front of the surface, the same thing occurs again for each point of the object as for the point L′,—that is to say, the rays given off from each of them are mutually parallel, and follow the direction of the ray which passes through the centre C (*Fig.* 14).

Rays coming from A′ are transmitted in the direction A′ C Y; those emanating from B′, in the direction B′ C Z.

Hence no image of the object is anywhere formed. The latter is thus in a plane which is characterised by the fact that all points belonging to it emit rays which, after having passed through the spherical surface, are parallel to each other and to the ray passing through the centre of curvature. This plane is called the ***first focal plane*** or *anterior focal plane.*

If the object approach still nearer the surface, the rays given off from each of its component points are still divergent, after having passed through the surface, but less so than they were before. Rays coming from A′ are dispersed in the directions Y Y Y, those from B′ toward Z Z Z (Fig. 15).

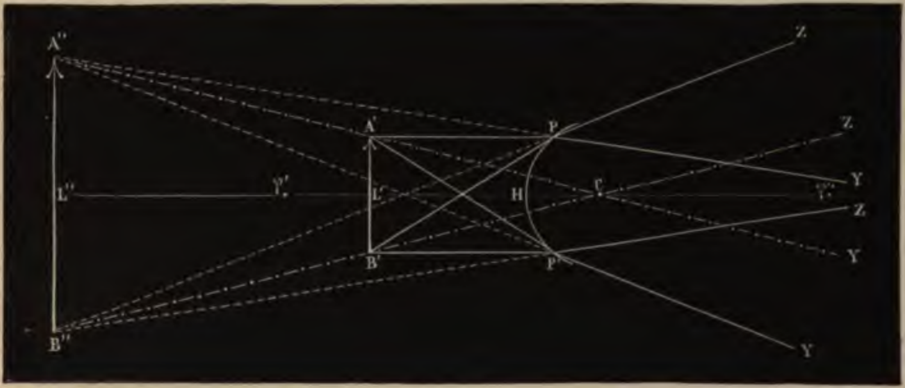


Fig. 15.

Hence they are nowhere focused, but they may be supposed to be prolonged backward, and, at the point of meeting of their prolongations is situated the *virtual* image of the object—the image of A′ at A″ (Fig. 15), that of B′ at B″. They are at distances which may be calculated from the formulae for $f''$ and $g''$, and in which $f''$ and $g''$ will have the negative sign ($-$).

It will be noticed that this virtual image has the same direction as its object, and is greater than the latter. It increases in size, and recedes from the refracting surface in proportion as the object approaches this surface.

When the object is on the other side of the surface, beyond the principal focus $\phi''$, we have the same conditions that have been heretofore considered (Figs. 12 and 13). But what was the image, in that case, has now become the object, and *vice versâ.* An object of the size of B″A″ forms, at a distance $f'$ from the surface, in the first medium, a real image of the size of B′A′, inverted relatively to the direction of the object.

When the object is in a plane passed through the second principal focus $\phi''$, the rays given off from each of its component points are, after having passed out of the medium M′, parallel to the ray drawn from each point through the centre of curvature. They are nowhere focused, unless they be concentrated by means of a convex lens or other collective system. For instance, an eye which is able to focus parallel rays upon its retina will see the object situated behind the surface, and in its true position, *i.e.*, not inverted.

When the object is between the second principal focus and the surface, the rays coming from it are divergent after having passed through the latter (Fig. 16).

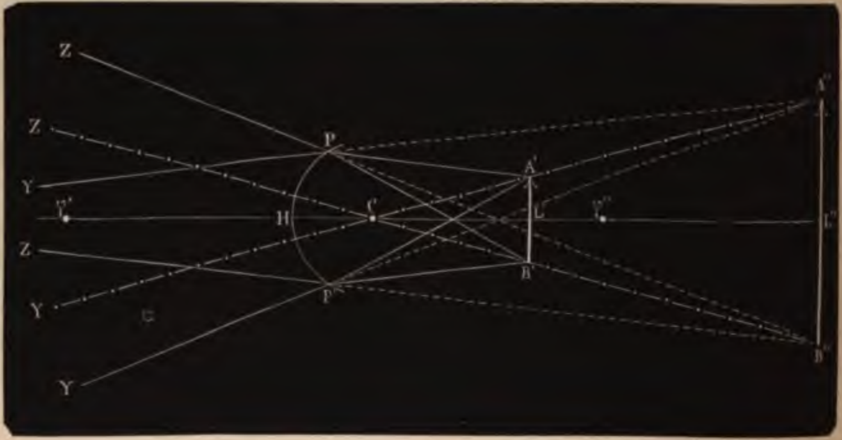


FIG. 16.

Rays emanating from A′ pass off in the direction Y Y Y, those coming from B′ go toward Z Z Z. But to an eye that could focus divergent rays, those given off by A′ would seem to come from a point A″ situated on the backward prolongation of the ray of direction A′ C, where all the prolongations of rays coming from A′ meet. Those given off from B′ would appear to arise from B″, and those coming from points between A′ and B′ would correspond to points between A″ and B″. In short, the observer would see the object A′ B′ enlarged (A″ B″) in its true position, and situated to the rear of its real location.

Inverting the case, it may be said that convergent rays which, without the refracting surface, would have formed an image A″ B″, are rendered more convergent by the surface through which they pass, and are focused by it nearer, into an image A′ B′, which is smaller than it would have been without this refraction.

We encounter all these possibilities in practice; and it is for this reason that we lay stress upon these different cases.

## CARDINAL POINTS.

In studying the course of light through a spherical surface separating two media of different indices of refraction, we have met with certain points which play a preponderant *rôle* in dioptrics. They are all situated on the optic axis.

1. The point of intersection of the surface with the optic axis. This is called the *principal point;* we give it the sign H.

We term the plane passed through the principal point, perpendicularly to the optic axis, the *principal plane.*

2. The *first principal focus* ($\phi'$), characterised by the fact that all rays emanating from it are parallel to each other and to the axis, after having passed through the refracting surface; or, again, the point at which are focused all the rays which were parallel to the axis, in the interior of the second medium.

The plane passed through the first principal focus, perpendicularly to the axis, is called the *first principal focal plane.* It contains all the points whose rays are, after having passed through the refracting surface, parallel to the ray drawn through the centre of curvature; or the plane upon which are focused the rays which, behind the surface, were parallel to the various secondary axes.

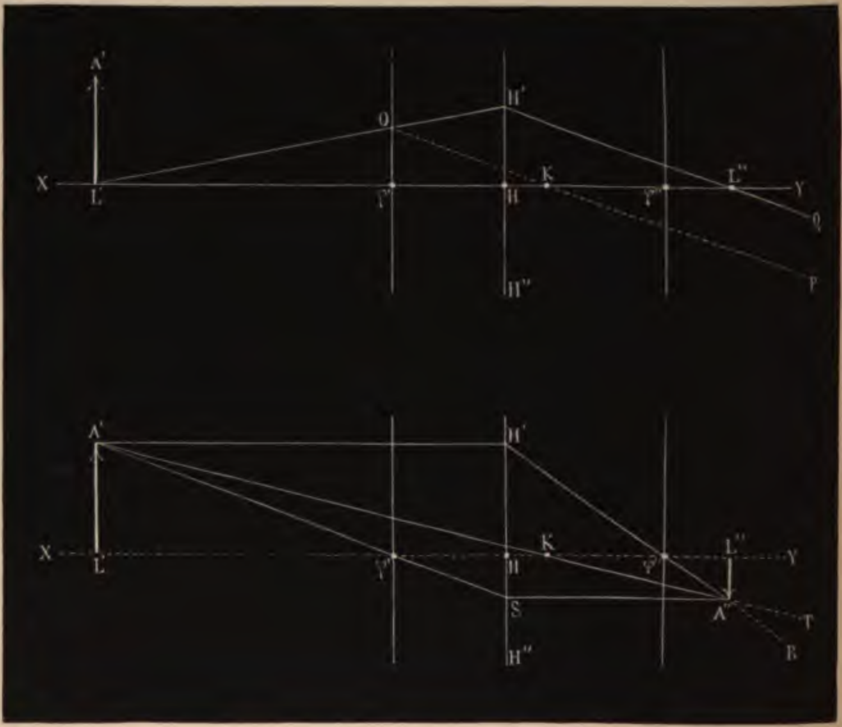
3. The *second principal focus* ($\phi''$), at which are focused the rays which, before entering the second medium, were parallel to the axis; or, again, it is the point which emits rays that, after having passed through the refracting surface, are parallel to the axis.

The plane passed through the second principal focus is the *second principal focal plane.* All of its constituent points emit rays which are parallel to the several secondary axes which correspond to them; or, what amounts to the same, all rays which, before entering the second medium, were parallel to one of the axes, are focused upon the second principal focal plane.

4. Finally, we have the *centre of curvature of the surface.* All rays directed toward it pass through the refracting surface without being deflected. This point is specially important in determinations of the size of the image, since all straight lines, which go from the various points of the object to corresponding points of the image, must pass through it. This is called the *nodal point,* and we shall designate it by K.

By means of these data we may, without calculation, find the place where the image of an object is formed, as well as the size of this image.

Let L′ A′ (Fig. 17) represent the object, $\phi'$ the first focus, $\phi''$ the second focus, H′ H H″ the refracting surface or the principal plane, and K the nodal point. The image of the point L′ must be formed somewhere on the axis X Y. If we have merely the direction of another ray emanating from L′, the image of L′ will be given by the intersection of this line with the axis.



Figs. 17 and 18.

Let us take a ray, L′ H′, which passes through the focal plane at O, and through the principal plane at H′. Supposing that the point O itself be luminous, the rays emitted by it will be parallel to each other after having passed through the surface, and, moreover, parallel to the ray of direction O K P drawn through the nodal point K. The ray O H′ may evidently be regarded as coming likewise from O. Hence it will be parallel to O P after having passed through the surface, and will be directed toward Q. The ray H′ Q meets the principal axis at L″. Hence L″ is the image of L′.

Something analogous to this will occur for the rays given off from the point A′. But, in order not to render the figure perplexing, we have drawn another one (Fig. 18) for the construction of the image of the point A′.

The ray A′K passes without deviation through the surface, because it is directed toward K. It indicates consequently the line A′KT on which the image of A′ is situated. Another ray emanating from A′, A′H′, is parallel to the axis XY. All rays which are parallel to this axis pass through the second principal focus $\phi''$. Hence $H'\phi''R$ is the direction that the ray A′H′ will take after its passage through the surface. Continuing its course, it meets the ray A′T at A″. It is here, then, that the image of A′ is formed.

The same point may be found by still another way, by means of the ray A′S (Fig. 18), which passes through the first principal focus $\phi'$. All rays emanating from $\phi'$ are parallel to the axis XY, after having passed through the surface. Hence $\phi'S$ (or A′S) will continue its course parallel to the principal axis, and will also meet A′T at the point A″.

A″L″ is therefore the image of A′L′.

The same thing would evidently have been produced if we had chosen an object extending below the axis; its image would be produced on the other side of the axis above L″A″.

## PASSAGE OF LIGHT THROUGH A SYSTEM MADE UP OF SEVERAL REFRACTING SURFACES.

When, after having passed through one surface, the light encounters a second, then a third, a fourth, &c., all of which separate media having different indices of refraction, the problem relative to the place where the image is formed, and its size, becomes much more complicated.

It is easy to conceive that the image formed by the first surface becomes the object for the second surface, and that this is true whether the image be actually formed, or whether the rays be intercepted by the second surface, before their reunion. The image for the second becomes the object for the third surface, and so forth.

Performing successively the calculations for all the refractive surfaces and media, we obtain a final image. This image, formed by the last surface, is the image of the object after the passage of its rays through the entire system.

These calculations might become extremely complicated, especially in an apparatus like the eye, where we have to do with a considerable number of refracting surfaces. As illustrating this, it suffices to recall the surfaces of the different layers of the crystalline.

Very fortunately, these calculations have been considerably reduced by the eminent mathematician GAUSS.[1] He has found, for the course of light through a centred system of whatever number of surfaces, comparatively simple and highly demonstrative formulæ. They have since been completed by MŒBIUS.[2]

These *savans* have demonstrated that there exists for every dioptric system, composed of whatever number of centred spherical surfaces, *i.e.*, whose centres are all situated on the same axis, three pairs of cardinal points, likewise situated on the axis and comparable with those that we have found for a single refracting surface; and two pairs of planes passed through four of these points perpendicularly to the axis.

The six cardinal points are—the *two principal foci*, the *two principal points*, and the *two nodal points*.

The FIRST PRINCIPAL FOCUS is the point at which the incident rays must cross each other in order that the rays which have passed through the system, *i.e.*, the emergent rays, may be parallel to the principal axis.

The SECOND PRINCIPAL FOCUS is the point where the emergent rays cross each other when the incident rays have been parallel to the principal axis.

The PRINCIPAL POINTS are characterised by the following property: When an incident ray, prolonged if necessary, passes through the first principal point, the corresponding emergent ray, or its prolongation, passes through the second principal point, but the incident is not parallel to the emergent ray. The *principal points* are the images of one another.

The NODAL POINTS are two points of the principal axis, so situated that every ray which, before being refracted, is directed toward the first of them, seems, after its refraction, to come from the second one, and takes a direction parallel to that which it had at first. These two parallel rays are called *lines of direction*, and act, in the combined system, the same part as the line passing through the nodal point of a single refracting surface.

The *second nodal point* is the image of the *first*, and *vice versâ*.

The distance between the two *principal points* is equal to that which separates the two *nodal points*.

The two pairs of *planes* are—the two FOCAL PLANES, passed through the principal foci, and the two PRINCIPAL PLANES, passed through the principal points, perpendicular to the principal axis.

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[1] Gauss, *Dioptrische Untersuchungen in Verhandlungen der k. Gesellschaft der Wissenschaften zu Göttingen*, Bd. i., 1840.

[2] Mœbius, *Kurze Darstellung der Haupteigenschaften eines Systems von Linsengläsern* (*Crelles Journal*, Bd. v., p. 113).

The FOCAL PLANES have the same property as those of a single refracting surface. The rays given off from a point of the first focal plane are, after refraction, parallel to each other and to the rays of direction.

The rays that were parallel before refraction are focused in a point of the second focal plane. This point is likewise indicated by the rays of direction.

The PRINCIPAL PLANES are defined as follows: If a parallel to the principal axis be drawn through the point at which an incident ray, or its prolongation, pierces the first principal plane, the point where this line pierces the second principal plane is in the course of the corresponding emergent ray or its prolongation. In other words, the directions of any incident ray, and of the corresponding emergent ray, pierce the first and second principal planes in two points situated *on the same side of*, and at the *same distance from, the principal axis of the system.*

The *second principal plane* is the optical image of the first, and *vice versa.* These are the only two conjugate images which have the same size and direction. The two principal planes of the composite system correspond to the single principal plane of the simple system (having but one refracting surface).

The FIRST PRINCIPAL FOCAL DISTANCE is the interval which separates the *first principal focus* from the *first principal point.*

The SECOND PRINCIPAL FOCAL DISTANCE is the interval which separates the *second principal point* from the *second principal focus.*

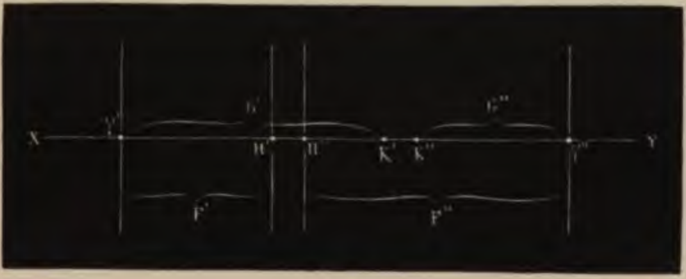

FIG. 19.

Let X Y (Fig. 19) be the principal axis;
φ' and φ'' the first and second foci;
H' and H'' the first and second principal points;
K' and K'' the first and second nodal points.
φ' H' is the first principal focal distance = F';
φ'' H'' is the second principal focal distance = F''.

*The distance* (G′) *from the first focus to the first nodal point is equal to the second principal focal distance, and the distance* (G″) *from the second nodal point to the second focus is equal to the first principal focal distance:*

$$\phi' K' = \phi'' H'' = G' = F''$$
$$\phi' H' = \phi'' K'' = F' = G'',$$

if, as for the single refracting surface, we designate by F′ and F″ the distances from the foci to the principal points, and by G′ and G″ those from the foci to the nodal point.

From this it follows that the respective distance, of each principal point and the nodal point of the same kind, is equal to the difference of the two focal distances:

$$H' K' = H'' K'' = F'' - F',$$

and that, moreover, the distance between the two principal points is the same as that between the two nodal points:

$$H' H'' = K' K''.$$

Finally, the two principal focal distances are to each other as the indices of refraction of the first and last media:

$$\frac{F'}{n'} = \frac{F''}{n''''} \text{ or } \frac{F'}{F''} = \frac{n'}{n''''},$$

if $n''''$ be the index of refraction of the last,—of the fourth medium, for instance.

If, then, the last medium be of the same nature as the first, and we have $n' = n''''$ (as in most optical instruments, but not in the eye), the two principal focal distances are equal, and the principal points coincide with the nodal points.

As soon as we know the cardinal points of a dioptric system, it is easy to calculate or construct, for any object, the image of such object as formed by the system in question. Hence, when one has to deal with a dioptric system, it is essential at the outset to ascertain the location of its cardinal points.

## TWO SURFACES SEPARATING TWO REFRACTIVE MEDIA.

### LENSES.

Let us first take the simplest case—that of *two surfaces* separating one medium from another. This is the case with lenses in air or the crystalline of the eye, supposed to be homogeneous, since the aqueous humor and the vitreous body have, as we shall see, the same index of refraction.

Light coming from the first medium passes through the first surface of the lens, is transmitted by the second medium, and leaves the latter, passing through the second surface to re-enter the first medium.

The line, on which are situated the centres of curvature of the two surfaces, is called the principal axis of the lens. This is the principal optic axis.

Considering again, as for the single surface, only a part of the lens very near the axis (3° to 5°), it is found that when the incident pencil is homocentric, the emergent pencil is equally so.

The luminous point and the point at which the emergent rays meet are therefore conjugate foci. The latter may still be real or virtual.

Two groups of lenses are distinguished (Fig. 20).

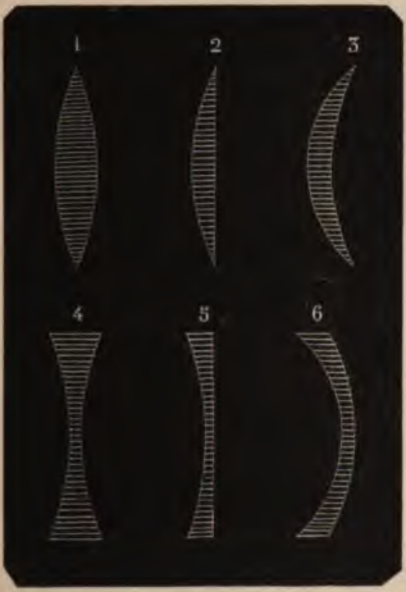

FIG. 20.

I. Lenses with sharp edges (thicker at the centre), *convergent* or *positive* lenses.

II. Lenses with blunt edges (thinner at the centre), *divergent* or *negative* lenses.

The first group comprises—

1. The *biconvex* lens (1);
2. The *plano-convex* lens (2);
3. The *convergent meniscus* (3).

The second group comprises—

4. The *biconcave* lens (4);
5. The *plano-concave* lens (5);
6. The *divergent meniscus* (6).

### Biconvex Lens.

Let us take a biconvex lens, surrounded by a single **medium, and** seek to determine its cardinal points, taking as a guide the definition which we have just given.

We shall call—

| | |
|---|---|
| The index of refraction of the ambient medium, | $n'$ |
| The index of refraction of the lens, | $n''$ |
| The radius of curvature of the first surface S′, | $r'$ |
| The radius of curvature of the second surface S″, | $r''$ |
| The first focal distance of the first surface S′ $\phi'$ | $f_1'$ |
| The second focal distance of the first surface S′ $\phi''$, | $f_1''$ |
| The first focal distance of the second surface S″ $\pi'$ | $f_2'$ |
| The second focal distance of the second surface, S″ $\pi''$, | $f_2''$ |
| The thickness of the lens, | $e$ |

(Compare Fig. 21).

The first surface S′ gives us the following formulæ:[1]—

For the first focal distance, $f_1' = \frac{n' r'}{n'' - n'}$;

For the second focal distance, $f_1'' = \frac{n'' r'}{n'' - n'}$.

The second surface S″ gives—

For the first focal distance, $f_2' = \frac{n'' r''}{n'' - n''}$;

For the second focal distance, $f_2'' = \frac{n' r''}{n'' - n'}$.

Between the second focal distance of the first surface, and the first focal distance of the second surface exists, therefore, the relation—

$$\frac{f_1''}{f_2'} = \frac{n'' r'}{n'' - n'} : \frac{n'' r''}{n'' - n'} = \frac{r'}{r''},$$

that is to say, that these two distances are to each other as the radii of curvature of their respective surfaces.

### *Principal Planes and Points of the Biconvex Lens.*

According to Gauss' definition, all rays which before refraction were directed toward a point in the first principal plane, seem, after refraction, to come from a point, in the second principal plane, which is situated on the same side of the axis as the first, and equally distant from the axis. This is the case if the two planes are the images of each other, the two images being equal in size, and the first having relation to the course of the rays in the first medium, the second to the course of rays in the last medium.

---

[1] See formulæ 4*a* and 4*c*.

In order to ascertain the location of the two principal planes, it may be assumed that both of them are images of a common luminous plane, seen from either side of the refractive system.

Let us seek, then, for the biconvex lens, the plane of which each of the two surfaces furnishes an image of the same size, or which, seen from the two surfaces of the lens, appears of the same size. Let O P (Fig. 21) be the section of this plane.

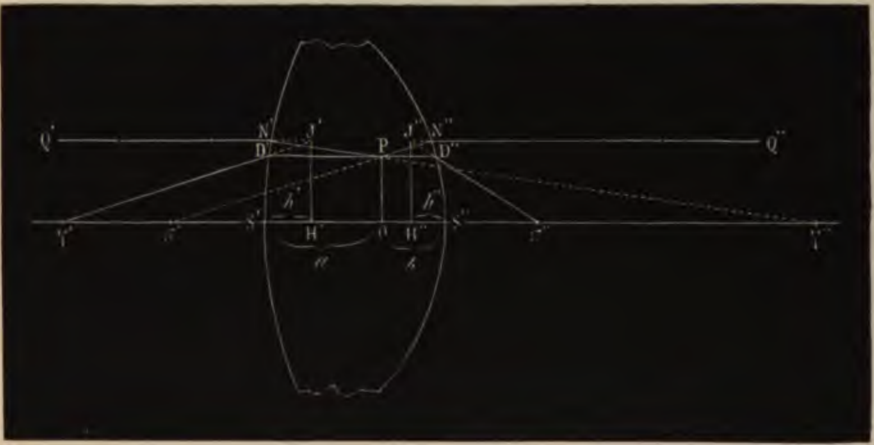

Fig. 21.

In order to find the image of it, which the surface S′ furnishes, let us draw a ray, P D′, parallel to the axis. It will be refracted toward the first focus $\phi'$ of this surface.

Let a line be drawn from the second focus $\phi''$ of this surface, through P, to the first surface at N′, and N′ Q′, parallel to the axis, will be the direction taken by the ray P N′ after refraction. The (virtual) image of P furnished by the surface S′, is evidently situated where the two rays, given off from P, meet each other. This is at J′, $\phi'$ D′ and Q′ N′ being prolonged backward. Hence J′ H′ is the virtual image of O P furnished by the first surface S′.

For the image of the second surface we proceed in a similar way. P D″ $\pi''$ is the ray which, in the interior of the lens, was parallel to the axis, and which is refracted toward the second focus $\pi''$ of the second surface. P N″ Q″ is the direction of a ray emanating from P, and appearing to come from the first focus $\pi'$ of this surface. The two rays, prolonged backwards, meet at J″. J″ H″ is therefore the virtual image of P O furnished by the second surface.

Now, if P O, as we have supposed, has been well chosen, J′ H′ and J″ H″ are images situated in the principal planes, and must be of equal size. J′ H′ being equal to J″ H″, the lines Q′ J′ and J″ Q″ must form but a single line, Q′ Q″, parallel to the axis. To find the location of O, we make use of the similar triangles $\phi''$ S′ N′ and $\phi''$ O P, whence we derive, designating O S′ by $a$—

$$\frac{S'N'}{OP} = \frac{S'\phi''}{O\phi''} = \frac{f_1''}{f_1''-a}.$$

From the similarity of the triangles $\pi'$ S″ N″ and $\pi'$ O P, it results that—

$$\frac{S''N''}{OP} = \frac{\pi' S''}{\pi' O} = \frac{f_2'}{f_2'-b}$$

representing O S″ by $b$.

S′ N′ and S″ N″ being equal to J′ H′ = J″ H″, we have, according to our previous supposition—

$$\frac{f_1''}{f_1''-a} = \frac{f_2'}{f_2'-b}$$

whence

$$\frac{a}{b} = \frac{f_1''}{f_2'} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad 10\,a.$$

That is to say, that in order to find in a lens the point O, whose images are two principal points, the thickness of the lens must be divided into two parts, of which one is to the other as the second focal distance of the first surface is to the first focal distance of the second surface; or, again, as the radius of the first is to the radius of the second, since we have seen that—

$$\frac{f_1''}{f_2'} = \frac{r'}{r''}.$$

When the two surfaces have radii of equal length, O is at the middle of the lens; when they are of different curvature, O is nearer that one of the surfaces which has the shorter radius.

Again we may write—

$$\left.\begin{array}{l} \dfrac{a}{f_1''} = \dfrac{b}{f_2'} \\[2ex] \dfrac{b}{f_2'} = \dfrac{a}{f_1''} \end{array}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad 10\,b.$$

Expressing $a$ and $b$ by the thickness ($S'S''=e$) of the lens, we obtain—

$$\left.\begin{aligned} a&=\frac{e f_1''}{f_1''+f_2'},\\ b&=\frac{e f_2'}{f_1''+f_2'}\end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \quad 11.$$

The situation of the point O, and consequently that of the plane O P, are thus determined. Let us now seek that of the points $H'$ $J'$ and $H''$ $J''$ relatively to the two surfaces of the lens; in other words, the values $S'H'=h'$ and $S''H''=h''$.

For this purpose we make use of the similar triangles $\phi'$ $J'$ $H'$ and $\phi'$ $D'$ $S'$, whence we derive

$$\frac{J'H'}{D'S'}=\frac{\phi' H'}{\phi' S'}.$$

Now

$$J'H'=N'S'$$
$$D'S'=OP$$
$$\phi' H'=f_1'+h'.$$
$$\phi' S'=f_1'.$$

Hence we may write—

$$\frac{J'H'}{OP}=\frac{f_1'+h'}{f_1'}.$$

From the similar triangles $\phi''$ $S'$ $N'$ and $\phi''$ O P we derive in a similar way—

$$\frac{S'N'}{OP}=\frac{J''H''}{OP}=\frac{f_1''}{f_1''-a},$$

and $J'$ $H'$ being equal to $J''$ $H''$, we have—

$$\frac{f_1'+h'}{f_1'}=\frac{f_1''}{f_1''-a}.$$

Substituting for $a$ its value derived from equation 11, we obtain—

$$\frac{f_1'+h'}{f_1'}=\frac{f_1''}{f_1''-\dfrac{e f_1''}{f_1''+f_2'}}$$

and

$$h'=\frac{e f_1'}{f_1''+f_2'-e} \quad . \; . \; . \; . \; . \; . \; . \; . \quad 12\,a.$$

From the similarity of the triangles $\pi''$ $H''$ $J''$ and $\pi''$ $S''$ $D''$, we conclude that

$$\frac{J''H''}{OP}=\frac{H''\pi''}{S''\pi''}=\frac{f_2''+h''}{f_2''}.$$

J″ H″ being equal to S″ N″, we may write—

$$\frac{f_2'' + h''}{f_2''} = \frac{f_2'}{f_2' - b} = \frac{f_2'}{f_2' - \frac{e f_2'}{f_1'' + f_2'}}$$

Substituting for $b$ its value taken from formula 11, we obtain, for the distance of the second principal point, or plane, from the second surface, the formula—

$$h'' = \frac{e f_2''}{f_1'' + f_2' - e} \quad . \; . \; . \; . \; . \; . \quad 12\,b.$$

or, expressed by the indices of refraction and the radii of the two surfaces—

$$\left.\begin{aligned} h' &= \frac{e\, n'\, r'}{n''\, r' + n''\, r'' - e\,(n'' - n')} \\ h'' &= \frac{e\, n'\, r''}{n''\, r' + n''\, r'' - e\,(n'' - n')} \end{aligned}\right\} \quad . \; . \; . \quad \mathbf{13}\,a \text{ and } b.$$

There exists, then, between $h'$ and $h''$, the same relation as between $r'$ and $r''$.

Hence the principal planes, united, play the same part as one single refracting surface.

The distances for the optical centre ($a$ and $b$) and for the principal points ($h'$ and $h''$) are reckoned—$a$ and $h'$ positively to the right and negatively to the left of the first surface, $b$ and $h''$ positively to the left and negatively to the right of the second surface.

The positive sign which results for them, from their formula of the biconvex lens, proves that these points are all situated in the interior of the lens.

To find the point O and the principal planes by construction, one must follow the course opposite to that which we have taken in our deductions,—*i.e.*, one must commence by drawing the line Q′ Q″ parallel to the axis, and then, from their intersections with the surfaces, the lines N′ $\phi''$ and N″ $\pi'$ which give the point P. Finally D′ D″ is drawn, through P, parallel with the axis. D′ $\phi'$ and D″ $\pi''$, prolonged backward, will give the points J′ and J″ of the principal planes.

### *Principal Focal Distances of the Biconvex Lens.*

The *second principal focus* of the lens, Φ″ (Fig. 22), is the point at which are focused all the rays which, before entering the lens, were parallel to the axis. Let Q′ Y′ be one of these rays. The surface S′ alone would refract it toward its second focus $\phi''$. Hence Y′ Y″ is the path taken by the ray in the interior of the lens. Y″ is a point in the course of the rays after refraction.

On the other hand we know that, when a ray meets the first principal plane at J′, it continues its course, after refraction, as if it came from the point J′ of the second principal plane, J′ J″ being parallel to the axis. Hence J″ Y″ Φ″ is the final direction of the ray Q′ Y′, and Φ″ the *second focus*, of the lens, geometrically determined.

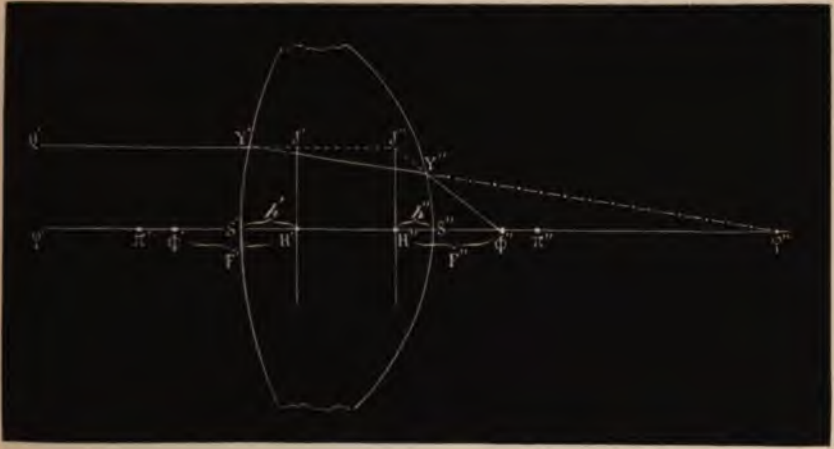

FIG. 22.

The *first principal focus* (Φ′) of the lens is found by means of a similar construction. We have only to repeat the same construction, taking a parallel ray coming from the opposite side, refracted at the surface S″, toward $\phi'$. Its prolongation would meet the second principal plane at J″, &c., as previously for Q′ Y′ J′.

F′ = H′ Φ′ is the *first focal distance of the lens;* H″ Φ″ = F″ is its *second focal distance.*

The similar triangles, $\phi''$ S′ Y′ and $\phi''$ S″ Y″, give—

$$\frac{S'\,\phi''}{S''\,\phi''} = \frac{S'\,Y'}{S''\,Y''},$$

the triangles Φ″ H″ J″ and Φ″ S″ Y″—

$$\frac{H''\,\Phi''}{S''\,\Phi''} = \frac{H''\,J''}{S''\,Y''},$$

and, since S′ Y′ = H″ J″,

$$\frac{F''}{S''\,\Phi''} = \frac{f_1''}{S''\,\phi''}.$$

If we designate H″ S″ by $h''$,

$$S''\Phi'' = F'' - h''$$

$$S''\phi'' = S'\phi'' - S'S'' = f_1'' - e,$$

hence

$$\frac{F''}{F'' - h''} = \frac{f_1''}{f_1'' - e}.$$

Substituting for $h''$, in this formula, its value in 12 *b*, we obtain for the second focal distance—

$$F'' = \frac{f_1'' f_2''}{f_1'' + f_2' - e} \quad . \; . \; . \; . \; . \quad 14\,a.$$

By an analogous construction and similar calculations the expression of the first focal distance of the lens is obtained:

$$F' = \frac{f_1' f_2'}{f_1'' + f_2' - e} \quad . \; . \; . \; . \; . \quad 14\,b.$$

Now, from the values that we have indicated for the focal distances of the two surfaces, it results that

$$f_1'' f_2'' = f_1' f_2';$$

hence

$$F'' = F'.$$

That is to say, that *the two principal focal distances of a lens, placed in a single medium, are equal, whatever be the radii of curvature of the surfaces.* It should, however, be remembered that they are not reckoned from the same point, and that the principal point of the more convex surface is nearer to this surface than that of the less convex surface is to the latter. For thin lenses this difference may, however, be disregarded.

### *Optical Centre of the Biconvex Lens.*

Let us draw through the centres of curvature, C′ and C″ (Fig. 23), of the two surfaces of the lens, two parallel rays, C′ J′ and C″ J″. The planes tangent to the refractive surfaces at J′ and J″ are parallel, since they are perpendicular to the parallel rays C′ J′ and C″ J″. Hence, if the ray C′ J′ meet the first surface at such an angle that it follows J′ J″ in entering, the corresponding emergent ray J″ U will be parallel to the incident ray, for the ray will thus have passed through a refractive medium limited by parallel surfaces.

Connecting J′ and J″, we find upon the optic axis the point O, which is called the *optical centre.*

The position of the point O results from the similarity of the triangles C′ J′ S′ and C″ J″ S″, and of the triangles O S′ J′ and O S″ J″:

$$\frac{C'S'}{C''S''} = \frac{S'J'}{S''J''}$$

$$\frac{S'J'}{S''J''} = \frac{OS'}{OS''}$$

$$\frac{OS'}{OS''} = \frac{r'}{r''}.$$

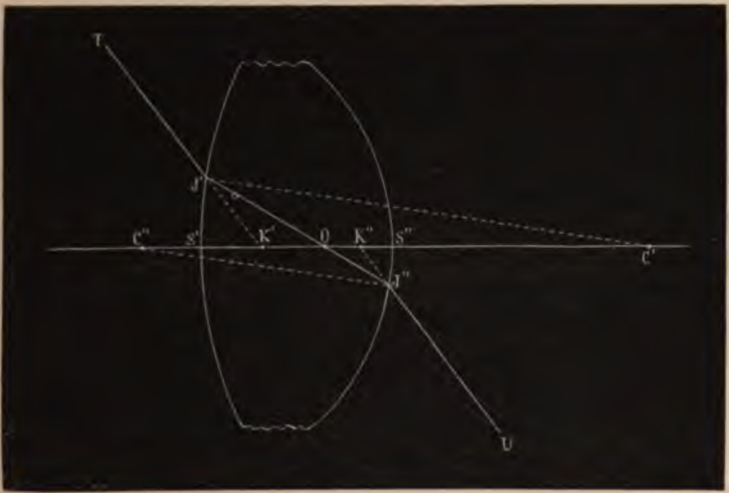

FIG. 23.

In a lens surrounded by a single medium, the radius of the first surface is to that of the second as the second focal distance of the first is to the first focal distance of the second. Hence we may write—

$$\frac{OS'}{OS''} = \frac{r'}{r''} = \frac{f_1''}{f_2'}$$

This is equivalent to saying that, in order to find the point O, the thickness of the lens must be divided into two parts, which will be to each other as the radii of the corresponding surfaces. Hence O is midway between the two surfaces of the lens, when they are of equal curvature, and nearer the more convex surface when they differ in curvature.

*Every incident ray, refracted by the first surface in such a way as to pass through the optical centre, emerges from the system in a direction parallel to its primitive one.*

### *Nodal Points of the Biconvex Lens.*

Comparing the above equation, which gives us the situation of the point O, with the equation 10*a*, page 38, we see that the two are identical.

Hence the optical centre is at the same time the object of which the two principal points would be the images formed by the two surfaces of the lens.

Let us again consider this point O, in Fig. 23, as a luminous point, and we shall find the corresponding images in still another way. We have only to draw the line J′ O J″ and regard it as composed of two rays, O J′ and O J″, emanating from the point O, and meeting the respective surfaces at J′ and J″. If, taking into account the curvature of the surface and the index of refraction, we construct the emergent ray J′ T, its prolongation backward from the surface S′ gives us, at the point K′ where it meets the optic axis, the image of the optical centre furnished by this surface. In the same way the emergent ray J″ U, prolonged backward from the surface S″, will give, at the point K″, where it meets the axis, the optical image furnished by the second surface. Hence the points K′ and K″ are still the virtual images of O, inasmuch as both belong to rays given off from O, the first to the rays O S′ and O J′, the second to the rays O S″ and O J″.

Moreover K′ is also the image of K″.

On the other hand, we have seen that the incident ray T J′, which was directed toward K′, leaves the lens in a direction parallel to its primitive one, and as if it came from K″. The points K′ and K″ correspond, therefore, to the definition we have given of *nodal points*. They are, indeed, the nodal points of the lens.

Inasmuch as they are the images of O, and the points H′ and H″ are likewise the images of O, it is proved that, *for the lens surrounded by a single medium, the first nodal point coincides with the first principal point, and the second nodal with the second principal point.*

### *Conjugate Foci of the Biconvex Lens.*

Being thus acquainted with the principal points and planes of the lens, it is easy to find the *conjugate foci*, as well as the relation between the *object* and *its image* formed by the lens.

Let H′ and H″ (Fig. 24) be the principal or nodal points of the lens; $\phi'$ and $\phi''$ its principal foci, and L′ a luminous point situated on the axis.

To find its conjugate focus, let us draw a ray L′ J′, which cuts the focal plane at A and the first principal plane at J′. Draw J′ J″ parallel to the axis; J″ is the point from which the ray, after refraction, appears to arise.

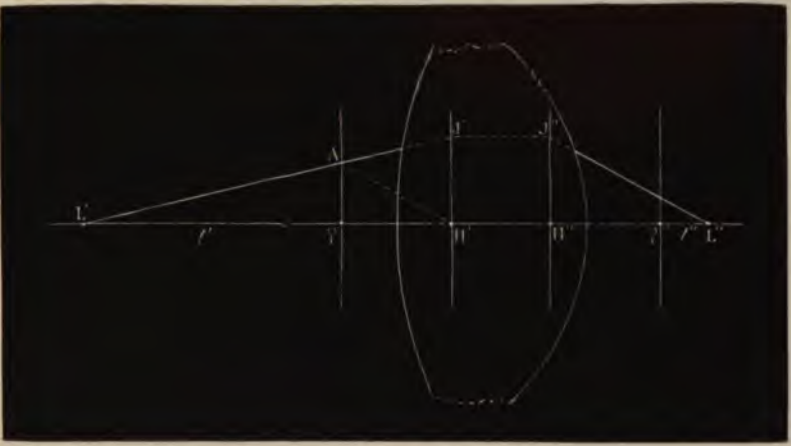

FIG. 24.

Suppose A to be a luminous point. One of the rays, emanating from it, is directed toward the first nodal point, H′. It would pass out of the system, remaining parallel to this direction, but appearing to come from H″.

The point A being situated in the focal plane, all rays emanating from it should, after refraction, be parallel to each other and to A H′. A J′ may be considered as one of these rays. Hence J″ L″, parallel to A H′, is the final direction of the ray, and L″, on the axis, the image of L′.

The triangles L′ H′ J′ and H″ $\phi''$ B; L″ $\phi''$ B and $\phi'$ H′ A are similar.

Let us call the focal distances (equal to each other)—

$$
\begin{aligned}
H' \phi' &= H'' \phi'' = F \\
L' H' &= a \\
L'' H'' &= b \\
L' \phi' &= a - F = l' \\
L'' \phi'' &= b - F = l'',
\end{aligned}
$$

and we may say—

$$\frac{l'}{F} = \frac{A\,\phi'}{B\,\phi''}$$

and

$$\frac{F}{l''} = \frac{A\,\phi'}{B\,\phi''};$$

hence

$$\frac{l'}{F} = \frac{F}{l''}$$

$$l' \, l'' = F^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 15\,a.$$

or, again, substituting $a - F$ for $l'$ and $b - F$ for $l''$

$$\frac{1}{F} = \frac{1}{a} + \frac{1}{b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 15\,b.$$

### *Size of the Image furnished by the Biconvex Lens.*

Let A′ L′ (Fig. 25) be an object situated in a plane perpendicular to the axis. Its image will likewise be situated in a plane perpendicular to the axis.

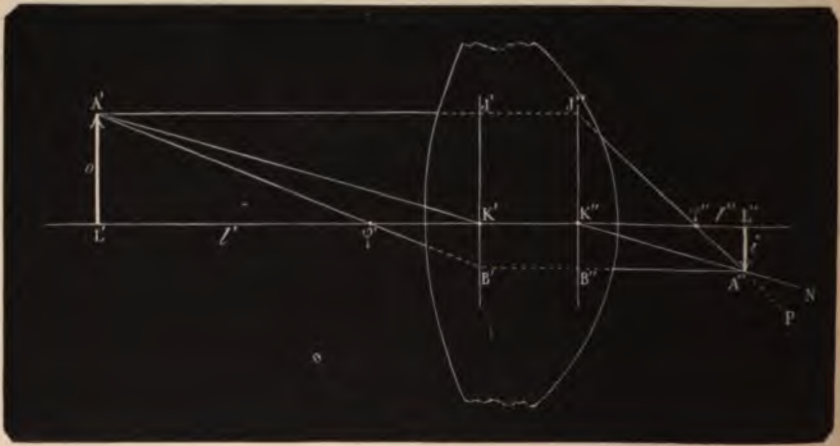

FIG. 25.

In order to find the point where the rays emanating from A′ are focused, let us draw the line of direction A′ K′. The image should be situated on the line of direction K″ N, parallel to A′ K′.

Another ray, given off from A, parallel to the axis, pierces the first principal plane at J′, the second at J″, and emerges following J″ $\phi''$ P. The point A″, in which the two rays intersect each other, is therefore the image of A′. Erect the perpendicular, and A″ L″ will be the image of A′ L′.

Again, the course taken by a third ray coming from A′, that which passes through the first focus $\phi'$, may easily be found. It meets the first principal plane at B′, the second at B″, and passes out of the lens, in a direction parallel to the axis, toward A″.

As to the size of the image, it is easily seen, on making

$$A' L' = o,$$
$$A'' L'' = i,$$
$$L' K' = a,$$
$$L'' K'' = b,$$

that

$$\left.\begin{aligned} \frac{i}{o} &= \frac{b}{a} \\ i &= \frac{o\,b}{a} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 9\,a'.$$

Let us consider that

$$i = B' K',$$
$$o = J'' K''$$
$$K' \phi' = K'' \phi'' = F,$$
$$L' \phi' = l',$$
$$L'' \phi'' = l''.$$

It results from the similar triangles $\phi' A' L'$ and $\phi' B' K'$; $\phi'' J'' K''$ and $\phi'' A'' L''$ that $l' l'' = F^2$, as above, and that

$$\left.\begin{aligned} \frac{i}{o} &= \frac{F}{l'} = \frac{l''}{F} \\ i &= \frac{o\,F}{l'} = \frac{o\,l''}{F} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 9\,c'.$$

As long as the image and object are on *opposite* sides of the lens, they are both positive,—*i.e.*, *real* and *inverted* relatively to each other, as is easily explained by Fig. 25.

Moreover, it results from the above formulæ that *the nearer the object is to the convex lens, the further the image recedes from the lens, and the more it increases in size.*

*If the object be at infinity, the image is formed in the second focal plane, and is infinitely small relatively to the object.*

*When the object is at a distance from the lens double the focal distance, the image is formed at the same distance* (2 F) *on the other side of the lens,* because $a = b = 2\,F$, and *it is of the same size as the object.*

*When the object is in the first focal plane, the image is formed at infinity, and is infinitely great, relatively to the object.*

*When the object is between the focus and the lens,* $l'$ becomes *negative,* and the formula 15 $b$ gives for $a$ likewise a *negative* value. *Hence the image is on the same side of the lens as the object. It is virtual, has the same direction as, and is larger than, the object. It increases in size in proportion as the object is brought nearer the lens.*

In other words, as long as the object is to the left of $\phi'$, the rays emanating from it are rendered convergent by the lens, and form therefore a real image to the right of the latter.

When it is in the principal plane, all rays coming from any point of the object are rendered parallel to the corresponding ray of direction.

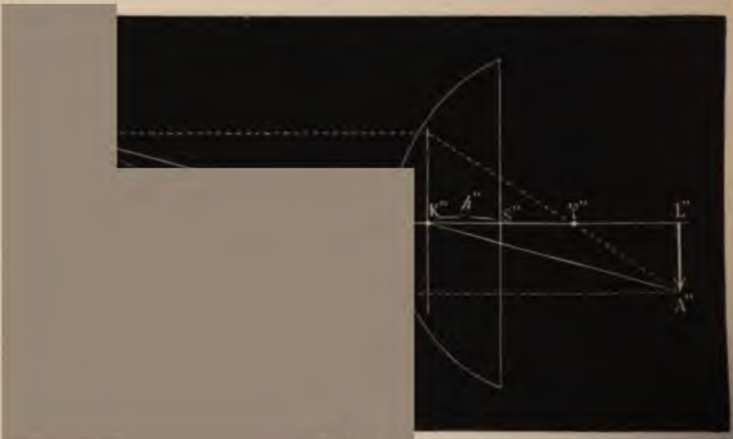

FIG. 26.

Between the focus and the lens, the object emits rays which are still divergent after having passed through the lens, and the image is obtained only by supposing the rays to be prolonged backward. Hence the image has the same direction as the object; but it is not real, it is *virtual*.

## PLANO-CONVEX LENS.

We shall pass more rapidly over the plano-spherical lenses and the menisci. They are of less practical importance than the bi-spherical lenses.

It suffices, moreover, to point out the differences between them and the latter in order to find readily the cardinal points, the conjugate foci, and the images which correspond to them.

The *optical centre*, the *first principal point* and the *first nodal point* all occupy one and the same position at the summit (Fig. 26) of the convex surface of the lens.

The *second principal point* and *second nodal point* coincide at K'', situated in the interior of the lens and determined by the equation

$$S'' K'' = h'' = \frac{e}{n}$$

in which $e$ equals the thickness and $n$ the index of refraction of the lens.

The focal distances are positive and have for common value—

$$F = f_1' = \text{focal distance of the convex surface} = \frac{r}{n-1}$$ [1]

The conjugate foci, as well as the relations of position and size between an object and its image, are determined by the same formulæ as are used in the case of the biconvex lens.

The construction is indicated by Fig. 26, and needs no explanation.

### THE CONVERGENT MENISCUS.

In the convergent meniscus the radius of the convex surface is shorter than that of the concave surface.

The *principal foci* of the *first* surface S′ are *real*, and its focal distances, $f_1'$ and $f_1''$, are *positive*.

The *principal foci* of the *second* surface S″ are *virtual*, and its focal distances, $f_2'$ and $f_2''$, *negative*.

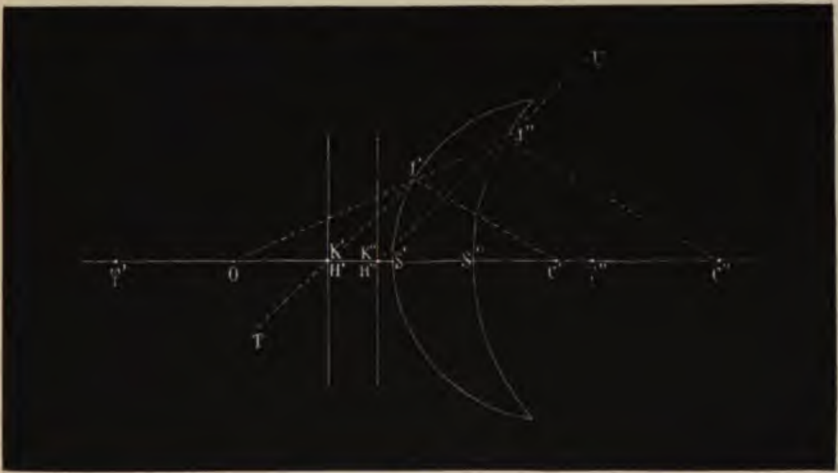

FIG. 27.

The construction of the *optical centre*, as well as that of the *principal points*, which coincide with the *nodal points*, is indicated by Fig. 27.

It is analogous to Fig. 23. The radii, C′ J′ and C″ J″, of the two surfaces, are chosen parallel to each other. The optical centre O is found where the connecting line J′ J″ cuts the axis of the meniscus.

Considering J′ J″ as a luminous ray passing through the meniscus, we construct the incident ray T J′ and the emergent ray J″ U, which corresponds to it, according to the principle discussed on pages 42-43. They are parallel because the portions of the surfaces S′ and S″, through which they pass, are parallel. The ray T J′ marks, upon the axis, the point K′ or H′,—*i.e.*, the first nodal (or principal) point; the prolongation of the ray J″ U marks the second nodal (or principal) point K″ or H″.

---

[1] Gavarret, *loc. cit.*, p. 91.

It results from the construction (Fig. 27) that the *optical centre* and the *nodal points* are *not in the interior* of the meniscus, but at a certain distance from it, on the side of its convexity.

The distances $S'O = a$ and $S'H' = h'$ are negative.

The distances $S''O = b$ and $S''H'' = h''$ are positive.

The principal focal distances (F) are equal: $\phi' H' = \phi'' H''$.

In ascertaining all these values the same formulæ are used as for the biconvex lens.

The relations of distance, position, and size, between the object and image, are likewise determined by the same formulæ and by the same construction as in the case of the biconvex lens.[1]

## THE BICONCAVE LENS.

### Cardinal Points of the Biconcave Lens.

The *cardinal points of the biconcave lens* are determined in the same way as those of the biconvex lens. It must, however, be borne in mind that, for this lens, the *foci* of the first surface S′, as well as those of the second S″, are *virtual*.

Hence *all the focal distances are negative*.

The *optical centre* is again found by taking two parallel radii of curvature, and connecting the points in which they meet the surfaces. The point, at which this line cuts the axis, is the optical centre. Hence it is situated in the interior of the biconcave lens.

Maintaining the same considerations as for the biconvex lens, we find, for the situation of the *optical centre* of the biconcave lens, the same formulæ:

$$\left.\begin{aligned} a &= \frac{e f_1''}{f_1'' + f_2'} \\ b &= \frac{e f_2'}{f_1'' + f_2'} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad 11\,b$$

and for the *principal points*, which again coincide with the *nodal points*—

$$\left.\begin{aligned} h' &= \frac{e f_1'}{f_1'' + f_2' - e} \\ h'' &= \frac{e f_2''}{f_1'' + f_2' - e} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \quad 12\,b$$

---

[1] Gavarret, *loc. cit.*, p. 99.

and for the *focal distances*, which are again the same as for the convex lens—

$$F = \frac{f_1' f_2'}{f_1'' + f_2' - e} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 14\,b.$$

Remembering that in these formulæ $f_1'$, $f_1''$, $f_2'$, and $f_2''$ are negative, it will be seen that the formulæ 11 *b* and 12 *b* give *positive* values; that is to say, that, agreeably to our supposition, *the optical centre, as well as the principal* (or *nodal*) *points are situated in the interior of the lens.*

The formula 14 *b*, on the contrary, gives a *negative* value for F. This signifies that, although the principal focal distances of the lens are equal, the focus situated on the side from which the light comes must be regarded as the *second* focus, and the focus on the opposite side as the *first*.

It seems unnecessary to give ampler explanations concerning the determination of the cardinal points of the biconcave lens. It would be but a repetition of the calculations and constructions of the biconvex lens.

### *Conjugate Foci of the Biconcave Lens.*

Let $\phi'$ and $\phi''$ (Fig. 28) be the foci, H′ and H″ the principal points of the biconcave lens, and L′ a luminous point on its axis.

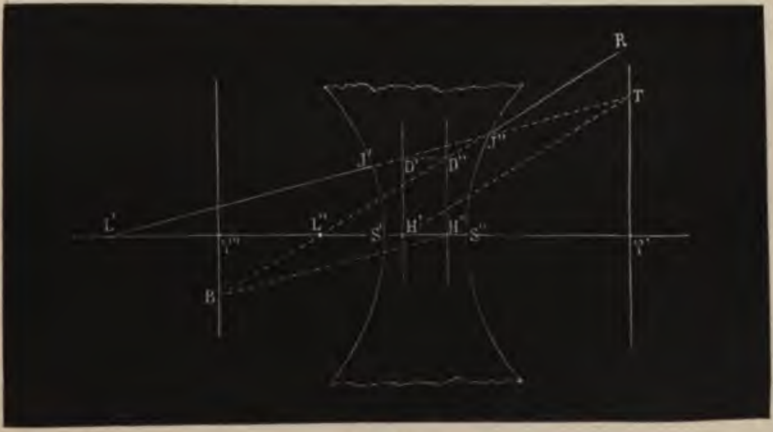

Fig. 28.

A ray given off from L′ passes through the first principal plane at D′, and through the first focal plane at T. We draw the line D′ D″ parallel to the axis, and we know that the ray leaves the lens in the direction D″ R, parallel to H′ T.

Prolonged backward, it crosses the axis at L″, which is the image of L′. Hence L′ and L″ are conjugate foci.

To find the relation between the distances of the conjugate foci, prolong R L″ to the second focal plane, which it meets at B. B H″ is parallel to L′ T.

The similar triangles L′ T $\phi'$ and H″ B $\phi''$, H′ T $\phi'$, and L″ B $\phi''$ give—

$$\frac{L'\phi'}{T\phi'}=\frac{F}{B\phi''},$$

$$\frac{F}{T\phi'}=\frac{L''\phi''}{B\phi''}.$$

Let us call $L'\phi'=l'$ and $L''\phi''=l''$. Dividing the first equation by the second, we obtain—

$$l'\,l''=F^2, \quad \ldots\ldots\ldots\ldots \quad 16\,b,$$

or, introducing the distances, from the conjugate foci to the principal points,—

$$L'\,H'=l'-F=a$$

and $L''\,H''=F-l''=b$;

$$\left.\begin{array}{l}\dfrac{1}{F}=\dfrac{1}{b}-\dfrac{1}{a}\\[2ex] \text{or}\quad \dfrac{1}{a}+\dfrac{1}{b}=-\dfrac{1}{F}\end{array}\right\} \quad \ldots\ldots \quad 15\,b.$$

### *Images furnished by the Biconcave Lens.*

Let A′ (Fig. 29) be a point not situated on the principal axis and belonging to the object A′ L′.

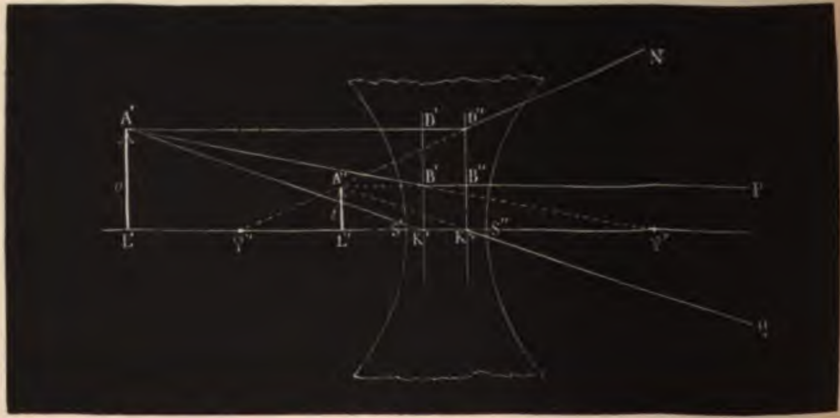

FIG. 29.

The ray A′ D′ D″, parallel to the axis, is directed toward N, when it passes out of the lens, as if it came from the second focus $\phi''$. The ray A′ B′, directed toward the first focus $\phi'$, leaves the system in a direction parallel to the axis, and passes on toward P from the point B′, where it meets the first principal plane. A third ray A′ K′ continues its course, parallelly to this direction, as if it came from K″. The three rays, prolonged backward, meet at A″, where, accordingly, the *virtual image* of the point A′ is formed.

Letting fall the perpendicular A″ L″, we obtain the virtual image A″L″ corresponding to the object A′ L′.

For the size $i$ of the image, relatively to the size $o$ of the object, we have—

$$\left.\begin{aligned} \frac{i}{o} &= \frac{F}{L'\phi'} = \frac{l''}{F} \\ i &= \frac{o\,F}{l'} = \frac{o\,l''}{F} \end{aligned}\right\} \quad . \; . \; . \; . \; . \quad 18\,b.$$

or

$$\frac{i}{o} = \frac{b}{a}$$

$$i = \frac{o\,b}{a} \quad . \; . \; . \; . \; . \; . \; . \; . \quad 17\,b.$$

## The Plano-Concave Lens.

The *optical centre*, the *first nodal point* and the *first principal point*, coincide with the summit of the *concave surface* of the lens.

The second nodal point is situated in the interior of the lens, at a distance L″ K″, equal to $\frac{e}{n}$, from the second surface.

The *first principal plane* is identical with the concave surface; the *second principal plane* passes through the second nodal point K″.

The *principal foci are virtual.* The *focal distances* are *negative*, *equal to each other*, and have the value $F = f_1' = -\frac{r}{n-1}$, *i.e.*, the value of the first focal distance of the concave surface.

For the *conjugate foci* and the *images* the same formulæ are applicable as for the biconcave lens.[1]

## The Divergent Meniscus.

One surface of the divergent meniscus is convex, the other concave.

The concave surface has the shorter radius of the two. The focal distances of the *first* surface, $f_1'$ and $f_1''$, are *positive*, those of the *second*, $f_2'$ and $f_2''$, are *negative*.

The construction of the *optical centre* and of the *nodal points* (which are coincident with the principal points) is indicated by Fig. 30.

First we draw the parallel rays C′ J′ and C″ J″.

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[1] Gavarret, *loc. cit.*, p. 117.

J′ J″, prolonged to its intersection with the axis, gives the optical centre O.

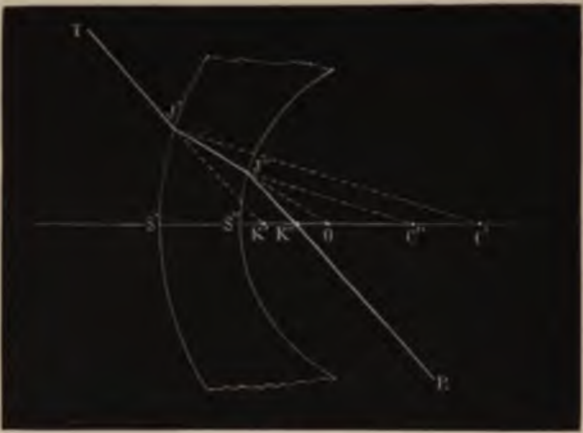

Fig. 30.

The incident ray T J′, prolonged, gives the *first nodal point* K′, and J″ R, parallel to T K′, gives the *second nodal point* K″.

None of these three points is in the interior of the meniscus, and all are situated to that side of it toward which its concave surface is turned. They are determined by the same formulæ as those of the biconvex and biconcave lenses (11, 12, 13 and 14). However, taking into account the signs of $f_1'$, $f_1''$, $f_2'$ and $f_2''$, the expression for—

S′ O $= a$, becomes positive ;
S″ O $= b$, negative ;
S′ K′ $= h'$, positive ;
S″ K″ $= h''$, negative;

and the principal foci, F, become negative.

The *conjugate foci*, as well as the relations of position and size of an object and of *its image*, are determined by the same formulæ as in the case of the biconcave lens.[1]

## INFINITELY THIN LENSES.

Infinitely thin lenses have for us a peculiar importance. Spectacle glasses may generally be considered as lenses of this kind. Hence we may properly devote a special chapter to a consideration of them.

The formulæ for such lenses are much simpler than those for the lenses that we have heretofore studied.

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[1] Gavarret, *loc. cit.*, p. 119.

In the first place, the thickness of the glass being disregarded, the summits S′ and S″ of the two surfaces, the principal and nodal points, and the optical centre, are all situated at the point where the principal axis passes through the lens. It is from this one point that all distances and lengths are reckoned. In fact the expression $e$, for the thickness of the lens, does not appear in these formulæ as in those which precede.

Moreover, the infinitely thin lenses under consideration are surrounded by atmospheric air; hence the index of refraction of the first medium is 1, and we have to consider only the index of refraction $n$ of the substance of which the lens is composed.

The lens has an anterior and a posterior focus; but these are equidistant from the united surfaces. Their common value is—

$$F = \frac{f_1' f_2'}{f_1'' + f_2''} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 14\ c.$$

Substituting for these values the index of refraction and the radii of curvature of the two surfaces, and taking into account the signs of $r'$ and $r''$, we obtain—

For the CONVERGENT LENSES:

Biconvex lens: $F = \dfrac{r'\,r''}{(n-1)\,(r'+r'')}$;

Plano-convex lens: $F = \dfrac{r}{n-1}$;

Convergent meniscus: $F = \dfrac{r'\,r''}{(n-1)\,(r''-r')}$.

For the DIVERGENT LENSES:

Biconcave lens: $F = -\dfrac{r'\,r''}{(n-1)\,(r'+r'')}$;

Plano-concave lens: $F = -\dfrac{r'}{n-1}$;

Divergent meniscus: $F = -\dfrac{r'\,r''}{(n-1)\,(r'-r'')}$.

In most cases the two surfaces, of the biconvex and biconcave lenses, have the same curvature. Hence $r' = r''$ in the preceding formulæ for bispherical lenses. If we write $r$ simply, the formulæ for the *biconvex* lens becomes—

$$F = \frac{r}{2\,(n-1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 20\ a.$$

and that for the *biconcave* lens—

$$F = -\frac{r}{2\,(n-1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 20\ b.$$

The formulæ for the *conjugate foci*, which we have indicated above as serving to determine the place where the image of a point is formed, are the same, for infinitely thin, as for thick lenses. The distances $a$ and $b$, as well as F, are, however, reckoned directly from the centre of the lens, where the principal and nodal points and the optical centre all coincide in one point. We mention them again, here, only for the sake of completeness.

Let $a$ be the distance of a luminous point from the lens, and $b$ that of its image; we have then, for the convex lens—

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{F},$$

whence

$$a=\frac{b\,F}{a+b}$$

$$F=\frac{a\,b}{a+b};$$

and for the *concave lens*—

$$\frac{1}{a}+\frac{1}{b}=-\frac{1}{F},$$

whence

$$a=\frac{b\,F}{b+F}$$

$$-F=\frac{a\,b}{a+b}.$$

Letting $l'$ be the distance from the luminous point to the anterior focus, and $l''$ the distance from the image to the posterior focus, we have again—

$$l'\,l''=F^2.$$

These formulæ are generally deduced in another way, and since the bispherical lenses, of equal surfaces, infinitely thin and surrounded by air, play so important a part in ophthalmology, it seems desirable to give here the deduction of their formulæ in the form they assume when, from the outset, the thickness of the lens is disregarded.

### Deduction of the Formulæ for the Conjugate Foci of an Infinitely Thin Convex Lens.

It is well known that a prism generally deviates luminous rays according to their angles of incidence. But, for prisms of slight angles, this difference of deviation may be disregarded, and it may be admitted that the deviation produced by such a prism is equal for all incidental rays, at whatever inclination they meet the prism.

Now biconvex lenses, whose thickness may be disregarded, may be considered as prisms of small angles.

Let A B (Fig. 31) be a lens of this kind:

The ray L′ A undergoes the same deviation as the ray P A, which is parallel with the axis.

The latter is deviated toward the focus $\phi$. Hence the incident and the refracted ray form an angle P A $\phi$.

L′ A L″, the angle of deviation of the ray L′ A, must be equal to the angle P A $\phi$. The direction of the refracted ray A L″ is therefore found by adding, above A $\phi$, an angle $\phi$ A L″ equal to the angle P A L′.

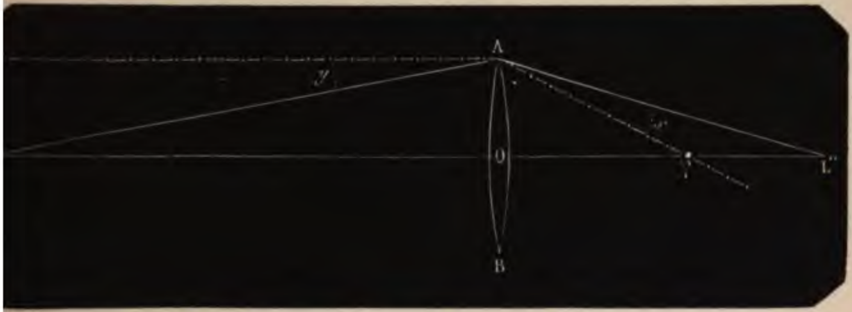

FIG. 31.

Let A B (Fig. 32) be a lens whose foci are $\phi'$ and $\phi''$. A ray P A, parallel to the axis, will be refracted toward the focus $\phi''$. In what lirection is a ray, which arises from the point L′, deflected?

Construct the angle $\phi''$ A L″ equal to the angle L′ A P, and mark upon the line P A G the distances A C and A D = A O = the semi-liameter of the lens.

From the points C and D let fall the perpendiculars C H and D N.

The angles L′ A P and $\phi''$ A L″ are equal and very small.

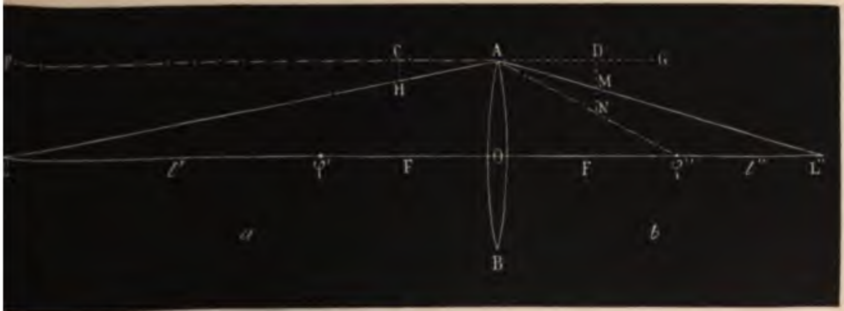

FIG. 32.

Moreover, the angle G A L″ is so small that M N is almost per-endicular to A L″. Hence we may admit that

$$C\,H = M\,N$$

nd

$$D\,N = D\,M + M\,N = D\,M + C\,H \quad . \; . \; . \; . \; . \quad x.$$

The triangles $\phi'' A O$ and $A D N$ are similar, because their sides are parallel. Hence—

$$\frac{\phi'' O}{A O} = \frac{A D}{D N}.$$

Making A O equal to 1, and the focal distance of the lens equal to F, we obtain—

$$\frac{F}{1} = \frac{1}{D N},$$

hence

$$D N = \frac{1}{F}.$$

The triangles $L'' A O$ and $A D M$ are likewise similar, because their sides are parallel. Hence—

$$\frac{L'' O}{A O} = \frac{A D}{D M}.$$

Let us call the second conjugate focal distance $L'' O = a$, and we shall have—

$$\frac{a}{1} = \frac{1}{D M}.$$

Hence

$$D M = \frac{1}{a}.$$

From the similarity of the triangles, $A C H$ and $A L' O$, it results that—

$$\frac{L' O}{1} = \frac{1}{C H}$$

and, designating $L' O$ (the first conjugate focal distance) by $b$—

$$\frac{b}{1} = \frac{1}{C H}$$

$$C H = \frac{1}{b}.$$

Substituting for D N, D M, and C H their values in formula $x$, we obtain—

$$\left.\begin{aligned} \frac{1}{F} &= \frac{1}{a} + \frac{1}{b} \\ \frac{1}{b} &= \frac{1}{F} - \frac{1}{a} \end{aligned}\right\} \quad \ldots\ldots\ldots \quad 15\,c.$$

Let us call $L' \phi' = l'$, and $L'' \phi'' = l''$, and write—

$$a = l' + F$$

and

$$b = l'' + F.$$

Substituting these values in formula 15 *c*, we obtain—

$$\left.\begin{aligned} l' l'' &= F^2 \\ \text{or} \qquad l'' &= \frac{F^2}{l'} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad 15\,d.$$

When the two conjugate foci are equidistant from the lens, $a$ becomes equal to $b$.

Hence

$$\frac{1}{F} = \frac{2}{a}$$

$$a = 2\,F.$$

Let us suppose a luminous point to be situated at the anterior focus of the first surface of the lens. It will then be at a distance $F'$ from the lens.

A ray, from this luminous point, passes through the lens in a direction parallel to the axis, and reaches the second surface, which presents its concavity to the ray. Here it undergoes the same deviation as in passing from the air, through the first surface, into the substance of the lens—only in an inverse direction. It is directed toward a point, on the axis, which is likewise situated at a distance $F'$ from the lens. Hence we find ourselves dealing with precisely the same case as that already cited, in which the two conjugate foci are equally distant from the lens, and we have

$$F' = 2\,F.$$

Now F is, according to formula 4 *d*, equal to $\frac{r}{n-1}$, whence we deduce the focal distance of the lens:

$$F = \frac{r}{2\,(n-1)},$$

an expression which we have already found (p. 55) in formula 20 *a*.

If a biconvex lens be made of a kind of glass whose index of refraction is $n = 1{\cdot}5$, we have—

$$F = \frac{r}{2\,(1{\cdot}5 - 1)}$$

$$F = r$$

That is to say, that in this case the focal distance is equal to the radius of curvature of one of the surfaces of the biconvex lens.

For $n = 1{\cdot}792$ (the heaviest flint glass), we obtain $F = 0{\cdot}631\,r$.

For $n = 1{\cdot}664$ (flint glass of medium density), $F = 0{\cdot}753\,r$.

For $n = 1{\cdot}562$ (as in rock crystal), $F = 0{\cdot}889\,r$.

For $n = 1{\cdot}53$, the average index of refraction of our spectacle glasses, $F = 0{\cdot}943\,r$.

For $n = 1{\cdot}528$ (Saint-Gobain glass), $F = 0{\cdot}946\,r$.

For $n = 1{\cdot}516$, the index of refraction of crown glass of the least density, $F = 0{\cdot}969\,r$.

## RELATIONS OF POSITION AND SIZE BETWEEN THE OBJECT AND ITS IMAGE FORMED BY AN INFINITELY THIN CONVEX LENS.

Let A′ (Fig. 33) be a point not situated upon the principal axis of the lens. In order to find its image, it is necessary only to draw, from A′, the ray of direction, which passes, without deviation, through the centre (O) of the lens, this centre representing both nodal points united. Another ray A′ B, parallel with the axis, passes, after refraction, through the focus $\phi''$. The point A″, where these two rays intersect, is the image of A.′

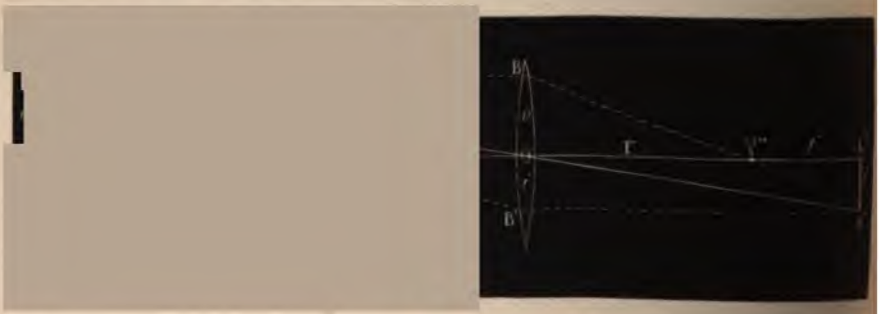

FIG. 33.

Let A′ L′ be an object; we have only to erect, upon the axis, the perpendicular A″ L″, and A″ L″ is the image of A′ L′.

If the question be to determine, by calculation, the size (*i*) of the image relatively to that (*o*) of the object, we shall first deduce, by means of formula 5 *c*, the distance O L″ = *b*, at which the image is formed.

This having been ascertained, we have simply to draw the ray of direction, and to let fall a vertical line from L″. The point of intersection, of these two lines, gives the intersection point A″ of the image. From the similar triangles L″ A″ O and L′ A′ O we then deduce—

$$\frac{i}{o} = \frac{L''O}{L'O} = \frac{b}{a},$$

$$i = \frac{o\,b}{a}.$$

Considering that the triangles A′ L′ $\phi'$ and B′ O $\phi'$ are similar, and that $A'L' = o$; $L'\phi' = l'$; $OB' = L''A'' = i$ and $\phi'O = F$, we obtain—

$$i = \frac{o\,F}{l'}.$$

Here again, as well as in the case of a single refraction surface, the distances, from the object and the image to the dioptric system, are *positive* so long as they are on *opposite* sides of the system.

When both are on the *same* side, either one or the other is *negative*.

The same is true with regard to the object and the image. The *image* is *real* and *inverted*, relatively to the object, as long as it is on the *opposite* side of the lens, because it is formed by the meeting of convergent rays. It is *virtual*, and has the *same direction* as the object when they are on the *same side* of the dioptric system. In this case the image is formed only by the meeting of the supposed prolongations of rays, which in reality are divergent.

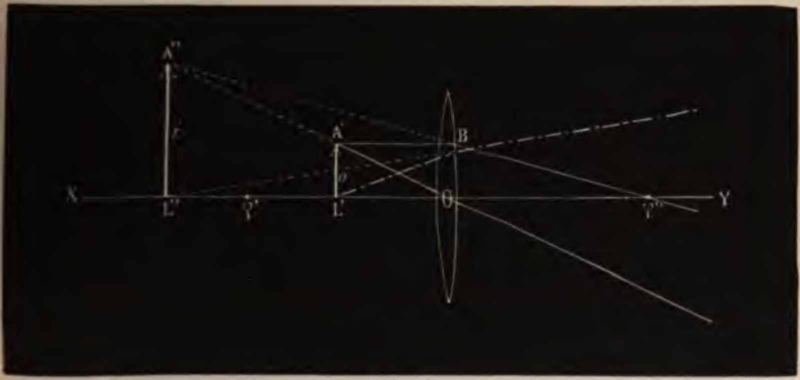

FIG. 34.

Fig. 34 shows the construction of the image L″ A″ of an object L′ A′, situated between the focus $\phi'$ and the lens.

After all that we have said, this figure scarcely requires explanation.

## THE INFINITELY THIN BICONCAVE LENS.

We have already said that, for the biconcave lens, the same formulæ are applicable as for the biconvex lens. F, however, is negative.

Thus we have—

$$-F = \frac{r}{2(n-1)}$$

$$l' l'' = F^2 \text{ (in which } l' \text{ is negative)}$$

and

$$\left.\begin{aligned} \frac{1}{a} + \frac{1}{b} &= -\frac{1}{F} \\ \frac{1}{b} &= -\left(\frac{1}{F} + \frac{1}{a}\right) \end{aligned}\right\} \quad \dots\dots\dots \quad 15\,e.$$

or

$$-\frac{1}{b}=\frac{1}{F}+\frac{1}{a}$$

$$\left.\begin{aligned} i &= -\frac{ob}{a} \\ i &= -\frac{oF}{l'} \end{aligned}\right\} \quad \dots\dots \quad 17\,c.$$

The foci of the concave lens being negative, the parallel rays are *divergent* after having passed through the lens, as if they came from the second focus $\phi''$, which is situated on the same side of the lens as the origin of the incident rays.

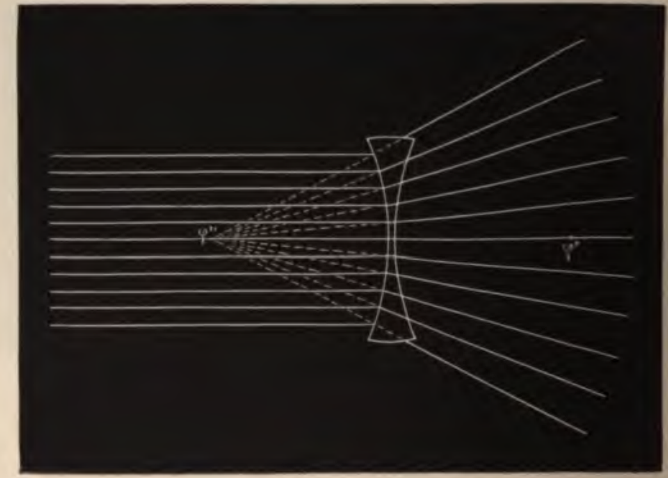

FIG. 35.

It results, from formula 15*e*, that $b$ always has a negative value, *i.e.*, the image is always situated on the *same side* of the lens as the object.

Moreover $a$ and $b$ increase and decrease together as the image and object approach the lens or recede from it.

Let the negative focal distance $\phi'$ O of the lens (Fig. 36) be equal to $F=25$ millimetres. Let the distance from a luminous point L′ to the lens, L′ O, equal $a=60$ millimetres. We seek the distance of the conjugate focus L″ $O=b$, and we write—

$$\frac{1}{b} = -\left(\frac{1}{F}+\frac{1}{a}\right),$$

$$\frac{1}{b} = -\left(\frac{1}{25}+\frac{1}{60}\right) = -\frac{1}{17{\cdot}6}.$$

Hence *b* equals 17·6 millimetres, and the minus sign indicates that this length is on the same side of the lens with *a*. L″ is the point from which all the rays given off from L′ seem to come, when they pass out of the lens.

The same thing that happens in the case of L′, takes place as well in the case of a point not situated upon the optic axis, the point A′ for instance.

The rays emanating from it diverge, on passing out of the lens, as if they came from a point A″, situated at a distance *b* in front of the lens, and upon the ray of direction A′ O N, which passes through the centre of the lens without undergoing any deflection.

If A′ L′ be an object, A″ L″ is its image. Being situated on the same side as the object, the image is therefore virtual, and has the same direction as the object.

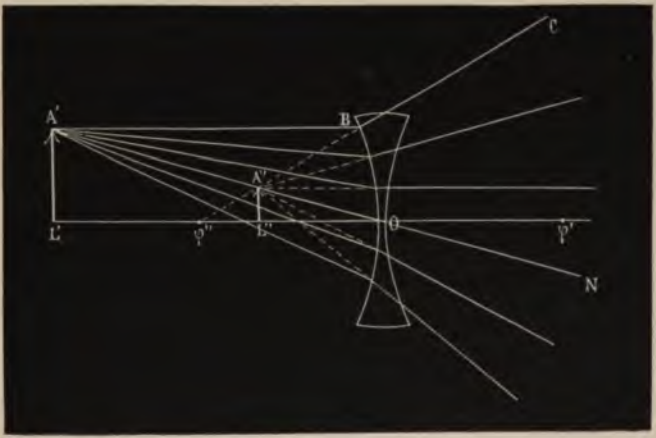

Fig. 36.

As to the size of the object, it results from formula 17*c*, in which *a* is always greater than *b*, and *l*′ greater than F, that the virtual image is always smaller than its object. This is, moreover, an *a priori* conception: the concave lens rendering rays coming from any point more divergent, as if they came from a nearer object, the virtual image ought always to appear smaller than its object.

Fig. 36 indicates the construction of the image for an infinitely thin concave lens. The ray A′ O N passes through the lens without deviation, because it passes through the optical centre.

The ray A′ B, parallel with the axis, passes out of the lens in a direction as if it came from the second focus, and is directed toward

C. Hence rays given off from the point A′ are divergent. Prolonging them backward, they meet at A″, which is therefore the *virtual* image of A′. Letting fall upon the axis the perpendicular A″ L″, we obtain A″ L″, the *virtual* image of A′ L′.

## THE COMBINATION OF INFINITELY THIN LENSES.

The more a lens refracts light, the stronger it is said to be. Now the greater the deviation imparted to parallel rays by such a lens, the nearer is its focus to the lens, and the shorter is its focal distance. Hence *the refractive power and the focal distance of a lens are inversely proportional to each other*. Let us call the refractive power $d$ and the focal distance F; we may then say—

$$d = \frac{1}{F}$$

or

$$F = \frac{1}{d}$$

If a positive or negative lens be applied to another having the same sign, the strength of both is evidently combined. If the first have a focal distance F′ and the second one of F″, we shall have—

$$\frac{1}{F'} + \frac{1}{F''} = \frac{1}{X}$$

in which $\frac{1}{X}$ represents the strength of the lens resulting from the combination of both, X being its focal distance.

Thus, by combining a convex lens, having a focal distance of 333 millimetres, with another whose focal distance is 250 millimetres we obtain—

$$\frac{1}{333} + \frac{1}{250} = \frac{1}{144}$$

That is to say, that both together act as a lens having a focal distance of 144 millimetres.

Adding to this a third lens, with a focal distance F‴, we have evidently—

$$\frac{1}{F'} + \frac{1}{F''} + \frac{1}{F'''} = \frac{1}{Y}.$$

For instance—

$$\frac{1}{333} + \frac{1}{250} + \frac{1}{166} = \frac{1}{77}.$$

The three lenses together have the same power as would be possessed by a single one having a focal distance of 77 millimetres.

If to a convex lens we add a concave one, the two, having opposite effects, partially neutralise each other, and, instead of adding their refractive powers, we have to subtract the one from the other.

Suppose a convex or positive lens, having a focal distance of F′ millimetres, to be applied to a concave or negative lens whose focal distance equals F″ millimetres; we say—

$$\frac{1}{F'} - \frac{1}{F''} = \frac{1}{X}.$$

Thus a convex glass of 333 millimetres focal distance, combined with a concave one having a focal distance of 500 millimetres, will give—

$$\frac{1}{333} - \frac{1}{500} = + \frac{1}{997}.$$

That is, a convex glass whose focal distance is 997 millimetres, and consequently weaker than that of 333 millimetres.

If the concave lens be stronger—if it have, for instance, a focal distance of 83 millimetres, we obtain—

$$\frac{1}{333} - \frac{1}{83} = - \frac{1}{111}.$$

The resultant lens is negative. It acts as a concave glass, having a focal distance of 111 millimetres.

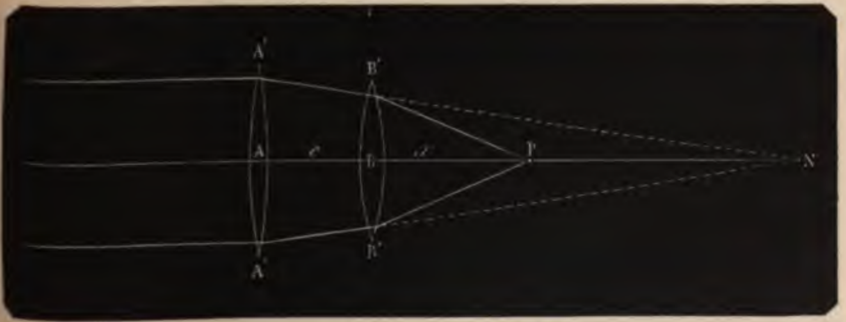

FIG. 37.

When the glasses are not in immediate contact with each other, but are a certain distance apart, the resultant lens is evidently not the same as in the preceding case. Their separation must be taken into account.

Let us take two lenses, A′ A′ and B′ B′ (Fig. 37).

The former has a focal distance A N = F′; the second, a focal distance $F''$. The parallel rays coming from the left would be refracted by the lens $A'A'$ alone, toward its focus N. At a distance $AB=e$, behind this lens, they strike the lens $B'B'$.

Up to this they still converged toward N; this point is, however, no longer separated from B by the distance $F'$, but by $F'-e$.

The convergence of the rays is not expressed by $\frac{1}{F'}$, but by $\frac{1}{F'-e}$. $A'A'$ being placed at a certain distance, $e$, from $B'B'$, it is as if we had added, to the lens $B'B'$, another one having a refractive power $\frac{1}{F'-e}$, and the combination of the two gives, consequently, the sum—

$$\frac{1}{F'-e}+\frac{1}{F''}=\frac{1}{X}, \quad \text{. . . . . . . . .} \quad 21.$$

Let us again suppose $F'$ equal to 333 millimetres, and $F''$ to 250 millimetres; but, instead of bringing the two lenses in contact with each other, let us separate them by a distance, $e$, of 133 millimetres, we shall then have, for the resultant lens—

$$\frac{1}{X}=\frac{1}{333-133}+\frac{1}{250}=\frac{1}{200}+\frac{1}{250}=\frac{1}{111};$$

that is to say, the two lenses combined, but separated by a distance of 133 millimetres, have the same action as a single lens with a focal distance of 111 millimetres.

Something analogous to this is produced when a convex is combined with a concave lens.

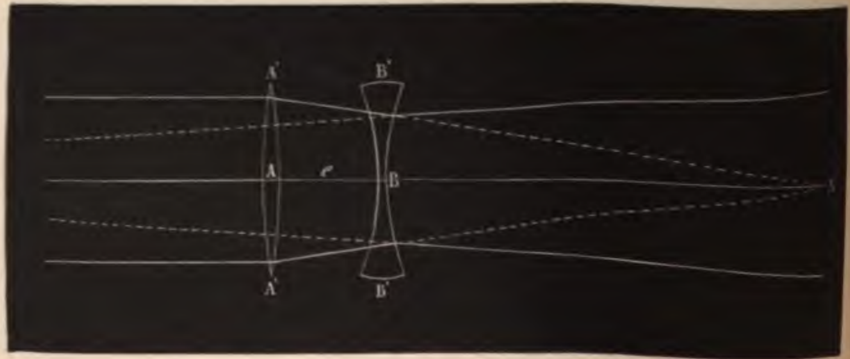

FIG. 38.

Let $A'A'$ (Fig. 38) be a convex lens having a focal distance of $+F'$, and $B'B'$ a concave lens with a focal distance of $-F''$.

They are separated by the space $e$.

The parallel rays are refracted, by the lens A′ A′, toward the focus N. Their convergence at A′ A′ equals $\frac{1}{F'}$; but, on reaching the lens B′ B′, this convergence is only $\frac{1}{F'-e}$.

Hence we have, so to say, combined a convex lens $\frac{1}{F'-e}$ with a concave one $\frac{1}{F''}$, and the result is—

$$\frac{1}{F'-e}-\frac{1}{F''}=\frac{1}{X}.$$

Letting

$$F' = 333^{\text{mm.}}$$
$$F'' = 83^{\text{mm.}}$$
$$e = 133^{\text{mm.}}$$

we shall have—

$$\frac{1}{333-133}-\frac{1}{83}=-\frac{1}{142};$$

that is to say, the combination of the two lenses is equivalent to a single concave lens, having a negative focal distance of 142 millimetres. In other words, the rays, parallel before reaching the lens A′ A′, take a direction, on leaving the lens B′ B′, as if they came from a point situated 142 millimetres behind it.

It will at once be seen that, with two lenses of this kind, the direction of the rays coming from a system, thus formed, may be varied at will; it is only necessary to vary the reciprocal distance of the two lenses.

This principle has found a broad application in the Galilean telescope, which is composed of a powerful concave ocular and a weak convex objective.

Many optometers are likewise based upon the variation to which the direction of luminous rays is subjected by the greater or less separation of two lenses of opposite signs.

Let us, for instance, combine a convex lens, of $F'=100$ millimetres of focal distance, with a concave lens having a focal distance of $-F''=50$ millimetres. When they are brought into contact, we have a concave lens of

$$\frac{1}{100}-\frac{1}{50}=-\frac{1}{100}.$$

When they are separated by $e=50$ millimetres, we have

$$\frac{1}{100-50}-\frac{1}{50}=\frac{1}{50}-\frac{1}{50}=0,$$

*i.e.*, they neutralise each other.

When they are 80 millimetres apart, we have—

$$\frac{1}{100-80}-\frac{1}{50}=\frac{1}{20}-\frac{1}{50}=+\frac{1}{33},$$

and so forth for the rays which come from the side of the convex glass.

Thus the parallel rays have at first been rendered divergent; then, on separating the glasses, we have gradually diminished the divergence until it became entirely annulled at the instant when the lenses neutralised each other. Continuing to withdraw the convex lens, the rays become more and more convergent.

Giving the desired values to the several terms of the formula 21—

$$\frac{1}{F' - e} - \frac{1}{F''} = \frac{1}{X},$$

all possible kinds of Galilean telescopes and optometers may be produced. The division, of the instrument, which indicates the separation, of the two glasses, requisite to produce a given direction of the divergent rays, is obtained by calculating $e$ by means of this formula 21.

When the rays come from the side of the concave lens, the formula is necessarily changed. Thus, in the preceding example ($+F' = 100$; $-F'' = 50$; $e = 80$) we should have obtained—

$$\frac{1}{100} - \frac{1}{50 + 80} = + \frac{1}{433}$$

## NUMBERING SPECTACLE GLASSES—THE MEASURE OF REFRACTION IN PRACTICE—THE DIOPTRY.

Spectacle glasses are, as we have said, lenses which may be considered as infinitely thin, *i.e.*, the thickness of which may be disregarded. Having already discussed their action, let us now treat briefly of the principle according to which they are designated—the *numbering* of spectacle glasses.

Experience, extending over centuries, has taught which lenses we need oftenest as spectacle glasses. These are the ones ordinarily contained in our trial cases. It is of very exceptional occurrence that an eye demands a number stronger than is to be found in our cases, and it is never necessary to go below the weakest lens. These glasses are manufactured in large quantities, and are so chosen as to be easy to designate.

### *The Old System of Numbering Spectacle Glasses.*

The numbers of glasses, according to the old system, indicate, *in inches, the radius of curvature of the surface*, or surfaces, of the lens.

Generally, but not always, the lenses we use are *bispherical.*

This way of numbering lenses is very convenient for the manufacturer, because, without taking its strength into consideration, he simply notes upon the lens the number of the "tool" in which it was made. These tools become worn, and are not always carefully inspected and regulated. But, even if the number thus given were exact, it would teach us nothing respecting the action of the lens. However, it is the power of refraction of a lens we want to know, and not its radius. The strength of the lens is, as we have seen, the converse of its focal distance, and this focal distance depends, not alone upon the radius of curvature, but also upon the substance of which the lens is composed. We have given, p. 55, the formula (20) which indicates the relations between the focal distance (F), the radius of curvature ($r$), and the index of refraction ($n$) of a bispherical lens:

$$F = \frac{r}{2(n-1)}$$

Hence the radius of curvature is not a rational criterion for the designation of lenses. If, notwithstanding this, the old system of numbering was long maintained, it was only because the radius of curvature and the focal distance of a bispherical lens were regarded as equal. This is true only when the index of refraction, of the material of which it is composed, is 1·5. But the index of refraction of the glass most commonly used is not 1·5; it is always greater.

EXAMPLES.[1]

| SUBSTANCE. | $n$ | $r$ Radius of Curvature. | F Focal Distance. |
|---|---|---|---|
| Flint glass, . . . . . . | 1·6 | 1000 mm. | 833 mm. |
| Rock crystal, . . . . . | 1·56 | ,, | 893 ,, |
| Ordinary glass for spectacles, . . | 1·53 | ,, | 943 ,, |
| Crown glass, . . . . . . | 1·5 | ,, | 1000 ,, |

We may admit 1·53 as the average index. Hence the focal distance of the lens No. 36 is not 36″, but only 34″. The lens is stronger than it is supposed to be.—(Compare columns I. and II., VII. and VIII. of our table, p. 76.)

Besides this inaccuracy, we encounter here the great inconvenience that the inch is not an invariable or universal unit of measurement, but varies considerably in different countries.

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[1] Müller und Pouillet, *Lehrbuch der Physik*, iii., p. 538; Wundt, *Physique Médicale*, translated by Monoyer.

The English inch = 25·40 millimetres.
The Prussian „ = 26·15 „
The Austrian „ = 26·34 „
The Paris „ = 27·07 „ &c.

It would be easy to cite 20 or 30 more inches of different length.

This is a source of notable errors, especially when we are dealing with problems in physiological optics.

But the old system has still another disadvantage. In practice we have much more to do with refractive powers than with focal distances. These two properties are inversely proportional. A lens is stronger in proportion as its focal distance is shorter; the greater the focal distance, the weaker is the lens. A lens of 4″ focal distance (F) has a refractive power ($d$) of $\frac{1}{4''}$, since

$$d = \frac{1}{\mathrm{F}}.$$

In the same way, the focal distance is the inverse of the refractive power:

$$\mathrm{F} = \frac{1}{d}.$$

The lenses usually employed in ophthalmological practice vary from 2″ to 80″ in focal distance. Hence their refractive powers vary from $\frac{1}{2}$ to $\frac{1}{80}$. Accordingly the simplest addition or subtraction of lenses to or from each other, to or from the refraction of the eye, becomes an addition or subtraction of fractions, which is a rather complicated calculation for practical use.

For instance, let us take an eye whose *punctum remotum* is situated at a distance of 42″: Its range of accommodation may equal $\frac{1}{14}$, corresponding to the age. What glass does it need in order to see at 8″? We say that the eye in question has a myopia of $\frac{1}{42}$. Adding to this the increase in refraction, of which the eye, by virtue of its accommodation, is susceptible, and it represents $\frac{1}{42} + \frac{1}{14} = \frac{1}{10 \cdot 5}$. In order to see at 8″ it needs a refractive power of $\frac{1}{8}$. Hence the eye in our example demands $\frac{1}{8} - \frac{1}{10 \cdot 5} = \frac{1}{33 \cdot 6}$ of refractive power in order to see at 8″,—*i.e.*, a convex glass having a focal distance of 33″·6.

However simple this calculation may be, it would be still too complicated to be carried out mentally in the short time of a consultation, where we have so many other questions to answer. But if we content ourselves with seeking empirically the glass that adapts the eye to the given distance without taking into account whether or not it corresponds to the one called for by the patient's age, we lose much time, and many of the most significant alterations of accommodation escape our notice.

We meet with such cases, and even more complicated ones, every day, in practice.

All these inconveniences have been done away with by "*the new system of numbering spectacle glasses.*" The principle of the new system of numbering spectacle glasses is *the introduction of the* METRE *in measuring the focal distances, and the designation of the lens by its* REFRACTIVE POWER.

First of all, we no longer concern ourselves with the radius of curvature, but with the refractive power of the lens. We require the optician to furnish us lenses of certain powers or focal distances. It matters little to us whether they have a greater or less radius, or a higher or lower index of refraction, provided they have the required strength.

Moreover, we do not measure the focal distance in inches, but with THE METRE.[1] The majority of lenses that we use having focal distances of less than a metre, this focal distance (F) becomes a *fraction* of the unit, and its converse, the refractive power ($d$), a *whole number*. Thus a lens having a focal distance of half a metre ($F = \frac{1^{\text{m.}}}{2}$) has a refractive power of $\frac{1}{\frac{1}{2}} = 2$; ($d = \frac{1}{F}$).

A lens of $\frac{1^{\text{m.}}}{3}$ focal distance has $\frac{1}{\frac{1}{3}} = 3$ units of refractive power.

These whole numbers 2, 3, 4, &c., which are the converses of the focal distances (fractions of a metre), represent refractive powers, and are the NUMBERS of the new system. It is found, indeed, that in dividing the metre successively by the fractions $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, &c., up to $\frac{1}{20}$, values of focal distances are obtained which meet, satisfactorily, the exigencies of practice. They give, for the refractive powers, a regular series of whole numbers—1, 2, 3, 4, 5, &c., up to 20.

Indeed, just as the focal distances are fractions of the metre, so the numbers which express the refractive powers are multiples of the lens, which has a focal distance of one metre,

$$F = 1^{\text{m.}}$$
$$d = \frac{1}{1^{\text{m.}}} = 1.$$

This lens represents the *unit of the new system*, the unit of measure of refraction. NAGEL has given it the name "Meterlinse" (*metre-lens*) (*Ml*). This name is well chosen, since it embodies the definition of the object it serves to designate, and renders further explanation of it almost superfluous. It has, however, seemed too long, especially in French (*lentille métrique*), and, following Monoyer's example, the word dioptry (D) has been substituted for it.

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[1] We beg the reader to note that we measure with THE METRE, and not with the centimetre or millimetre.

The number 4 is, then, a lens of 4 dioptries (four metre-lenses), a lens four times as strong as that which has a focal distance of 1 metre; its focal distance being four times smaller: $\frac{1^{\text{m.}}}{4}$

In practice we sometimes need lenses weaker than the metre-lens *i.e.*, those having focal distances greater than a metre. The numbers of such lenses are fractions: thus, lenses having focal distances of $\frac{4}{3}$ (1·333), 2 and 4 metres, have been added to the series. The numbers of these lenses are $\frac{3}{4} = 0{\cdot}75$, $\frac{1}{2} = 0{\cdot}5$, and $\frac{1}{4} = 0{\cdot}25$ dioptry.

Quarter-dioptries have also been interpolated between the numbers up to 2·5, and demi-dioptries from 2·5 up to 5·5, in order to obtain smaller intervals. In the higher numbers, on the other hand, an interval of one dioptry has appeared too small, even a slight variation in the distance between the lens and the eye serving to produce an effect stronger than one dioptry. For this reason the number 19 has been suppressed.

In this way the series, found in column V. of the table, has been arranged.

The numbers of the new system represent, as we have said, refractive powers. In *combining these lenses*, we are no longer forced to calculate with fractions, but may use the whole numbers, represented by the numbers of the lenses. Thus the numbers (convex or concave) 3, 4, and 5 combined, give the number $13 = 3 + 4 + 5$. The lens $+3$ with the $-12$ gives $+3 - 12 = -9$.

*To find the focal distance corresponding to a number of the new system*, one has only to remember that it is the inverse of the refractive power, *i.e.*, of the number, and that the numbers are multiples of a lens having a focal distance of 1 metre. Thus the number 9 has a focal distance of $\frac{1^{\text{m.}}}{9}$; if centimetres are desired, this value will be written $\frac{100^{\text{cm.}}}{9} = 11^{\text{cm.}}$; if millimetres, $\frac{1000^{\text{mm.}}}{9} = 111^{\text{mm.}}$, and so forth.

Again, *the refractive power* is always the inverse of the focal distance. A lens having a focal distance of 25 millimetres always has $\frac{1}{25}$ of refractive power, but in order to assimilate the refractive powers to the numbers of the lenses and express them in dioptries, the focal distance should be given in terms of the metre, and not in centimetres or millimetres.

In our example it will be asked — what part of a metre is

$25^{\text{mm.}}$ ?—$\frac{25^{\text{mm.}}}{1000^{\text{mm.}}}=\frac{1^{\text{m.}}}{40}$, hence $\frac{1}{\frac{1}{40}}=40$ dioptries. If the focal distance be 25 centimetres, we have $25^{\text{cm.}}=\frac{25}{100}=\frac{1^{\text{m.}}}{4}$. Hence $\frac{1}{\frac{1}{4}}=4$ dioptries.

If the distance be expressed in metres, $4^{\text{m.}}$ for instance, we shall have at once $\frac{1}{4}$ dioptry or 0·25, if the latter form be considered preferable.

Simpler still, *the number of dioptries may be found by means of a fraction whose numerator is one metre, and whose denominator is the focal distance in question.*

Thus, for 40 centimetres, we say—

$$\frac{1}{0{\cdot}40}=\frac{100}{40}=2{\cdot}5 \text{ dioptries.}$$

For 17″ (English) we say:

$$17'' \text{ (English)} = 431 \text{ millimetres ;}$$

Hence

$$\frac{1}{0{\cdot}431} \text{ or } \frac{1000}{431} = 2{\cdot}31 \text{ dioptries.}$$

In this way it is easy to express any given focal distance by its equivalent in dioptries.

This is what we do, not only in the case of lenses, but also to express the refraction of the eye, its range of accommodation, &c. &c.

It may be of interest to inquire what form the classical formula, $\frac{1}{F}=\frac{1}{A}+\frac{1}{B}$, assumes when expressed in dioptries. It is a formula which is constantly used, since it indicates the relations which exist between the distance (A) from the object, and that (B) from the image to the lens, and the focal distance (F) of this lens. We use it whenever we wish to know where the image of an object is formed.

The values F, A, and B, in the formula, are linear values. We have only to express them in vulgar fractions of a metre, and the formula will become applicable to dioptries.

Let us take, for example, a convex lens having a focal distance of 25 centimetres. We wish to know where the image of an object, situated 40 centimetres from it, will be formed. According to the classical formula, we say—

$$\frac{1}{25}=\frac{1}{40}+\frac{1}{B}$$

$$\frac{1}{B}=\frac{1}{25}-\frac{1}{40}=\frac{1}{66}$$

$$B = 66 \text{ centimetres.}$$

or

$$25 \text{ cm.} = \frac{1}{4} \text{ metre} = 4 \text{ D.}$$

$$40 \text{ ,,} = \frac{1}{2{\cdot}5} \text{ ,,} = 2{\cdot}5 \text{ D.}$$

Hence

$4 = 2{\cdot}5 + b$ (in which case $b$ equals a number of dioptries)

$b = 4 - 2{\cdot}5 = 1{\cdot}5$ D, the focal distance of which is $\frac{100^{\text{cm.}}}{1{\cdot}5} = 66$ cm.

In a general way, in the system of dioptries, the formula

$$\frac{1}{F} = \frac{1}{A} + \frac{1}{B}$$

becomes

$$\frac{1^{\text{m}}}{F} = \frac{1^{\text{m}}}{A} + \frac{1^{\text{m}}}{B}$$

or, if we designate by $f$, $a$, and $b$ the number of dioptries corresponding to a given length—

$$f = a + b.$$

This second method of calculating is not always simpler than the first, but the dioptric system having once been adopted, the question will take a different form; one will not generally speak of a lens of such and such a focal distance, but of a certain refractive power, and it will be asked where the image of a point, situated 40 centimetres from a lens of 4 dioptries, may be formed?

The problem, stated in these terms, may be solved in two ways. We may calculate the focal distance corresponding to the number of dioptries of our lens, and apply the formula in its classical form; or we may calculate the number of dioptries corresponding to the distance of the object $\left(\frac{100^{\text{cm.}}}{40} = 2{\cdot}5 \text{ D}\right)$, and apply the modified formula. In this way we obtain a number of dioptries whose focal distance will be the distance of the image from the lens. For each case, as it arises, one may choose the simpler method.

*Relation between the Old and New Systems of numbering Lenses.*

The unit of the new system—the dioptry—has a focal distance of one metre. One metre equals 39·3708 English inches. Hence—

1 dioptry is equivalent to $\frac{1}{39{\cdot}37''}$

2 dioptries are ,, $\frac{2}{39{\cdot}37''}$

3 ,, ,, $\frac{3}{39{\cdot}37''}$, &c.

Inversely, a lens of 6″ focal distance, having $\frac{1}{6''}$ of refractive power, represents a number of dioptries equal to the number of times $\frac{1}{39{\cdot}37}$ is contained in $\frac{1}{6}$; $\dfrac{\frac{1}{6}}{\frac{1}{39{\cdot}37}} = \frac{39{\cdot}37}{6} = 6{\cdot}56d.$

Thus, in order to obtain the number of dioptries corresponding to a focal distance expressed in inches, we have to divide 39·37 by that focal distance.

This is the method of procedure when it is a question of expressing, in dioptries, the strength of a lens whose *focal distance* is given in inches.

The NUMBERS of spectacle glasses, according to the old system, do not give the focal distances of the lenses, but simply their radii of curvature. Hence the equivalent of the dioptry would not bear the NUMBER 39·3708; but, instead, for an index of refraction of 1·53, the number 41·73.

Therefore, in order to find the NUMBER of the old system corresponding to a given number of dioptries, 41·73 (which, for simplicity's sake, may be called 42) must be divided by that number.

Six dioptries correspond to the number 7 $\left(i.e., \frac{42}{6}\right)$ of the old system; more exactly $\frac{41 \cdot 73}{6} = 6 \cdot 95$.

Inversely, being given the number of dioptries, we divide 42 (exactly, 41·73) by that number. For instance, 20 dioptries $= \frac{42}{20} =$ number 2·1 of the old system.

We give, in columns I., II., V., and VI. of our table, a synopsis of the numbers and focal distances according to the old and new systems. Supposing the index of refraction to be 1·53, that of the glass generally used in the manufacture of spectacle glasses, and Table I. corresponds to the English, and Table II. to the French inch. We have separated them in order to avoid confusion.

Everybody agrees that it was indispensable to abandon the system of numbering which was based upon the radius of curvature of the lenses.

Nor can any serious objection be made to the introduction of the metric system. The metre is the only uniform measure, and it possesses the very great advantages of the decimal system, and agrees directly with the optical constants of the eye, which are generally expressed in metric terms.

There are, however, a few of our colleagues who disapprove of the principal of numbering according to the *refractive power*, and of the introduction of the dioptry. They would prefer to write the *focal distance* as the number on the lens.

To these we reply that this distance is virtually to be found expressed in metres in the numbers of the new system, since it is the inverse of this number. Number 5, for instance, has a focal distance of $\frac{1}{5}$ of a metre. If it were written in centimetres or millimetres, the unit of the new system would not be the metre, but the centimetre or millimetre.

The fractions, corresponding to the refractive powers, would then become scarcely manageable, and such calculations as we are obliged to perform every day in practice would be just as much more complicated in the new than in the old system, as the centimetre or millimetre is less than the inch.

## TABLE I.

| OLD SYSTEM. | | | | NEW SYSTEM. | | | |
|---|---|---|---|---|---|---|---|
| I. No. of the Lens, Old System. | II. F. Focal Distance in Engl. inches for $n=1·53$. | III. F. Focal Distance in Millimetres. | IV. D. Equivalent in Dioptries. | V. No. of the Lens, New System. | VI. F. Focal Distance in Millimetres. | VII. F. Focal Distance in Engl. inches. | VIII. No. corresponding of the Old System for $n=1·53$. |
| 72 | 67·9 | 1724 | 0·58 | 0·25 | 4000 | 157·48 | 166·94 |
| 60 | 56·6 | 1437 | 0·695 | 0·5 | 2000 | 78·74 | 83·46 |
| 48 | 45·3 | 1150 | 0·87 | 0·75 | 1333 | 52·5 | 55·63 |
| 42 | 39·6 | 1005 | 0·99 | 1 | 1000 | 39·37 | 41·73 |
| 36 | 34 | 863 | 1·16 | 1·25 | 800 | 31·5 | 33·39 |
| 30 | 28·3 | 718 | 1·39 | 1·5 | 666 | 26·22 | 27·79 |
| 24 | 22·6 | 574 | 1·74 | 1·75 | 571 | 22·48 | 23·83 |
| 20 | 18·8 | 477 | 2·09 | 2 | 500 | 19·69 | 20·87 |
| 18 | 17 | 431 | 2·31 | 2·25 | 444 | 17·48 | 18·53 |
| 16 | 15 | 381 | 2·6 | 2·5 | 400 | 15·75 | 16·69 |
| 15 | 14·1 | 358 | 2·79 | 3 | 333 | 13·17 | 13·9 |
| 14 | 13·2 | 335 | 2·98 | 3·5 | 286 | 11·26 | 11·94 |
| 13 | 12·2 | 312 | 3·20 | 4 | 250 | 9·84 | 10·43 |
| 12 | 11·3 | 287 | 3·48 | 4·5 | 222 | 8·74 | 9·26 |
| 11 | 10·3 | 261 | 3·82 | 5 | 200 | 7·87 | 8·35 |
| 10 | 9·4 | 239 | 4·18 | 5·5 | 182 | 7·16 | 7·6 |
| 9 | 8·5 | 216 | 4·63 | 6 | 166 | 6·54 | 6·93 |
| 8 | 7·5 | 190 | 5·25 | 7 | 143 | 5·63 | 5·97 |
| 7 | 6·6 | 167 | 5·96 | 8 | 125 | 4·92 | 5·22 |
| 6½ | 6·13 | 155 | 6·42 | 9 | 111 | 4·37 | 4·63 |
| 6 | 5·6 | 142 | 7·0 | 10 | 100 | 3·94 | 4·17 |
| 5½ | 5·2 | 132 | 7·57 | 11 | 91 | 3·58 | 3·8 |
| 5 | 4·7 | 119 | 8·4 | 12 | 83 | 3·27 | 3·46 |
| 4½ | 4·2 | 106 | 9·4 | 13 | 77 | 3·03 | 3·21 |
| 4 | 3·8 | 96 | 10·4 | 14 | 71 | 2·8 | 2·96 |
| 3½ | 3·3 | 84 | 11·9 | 15 | 67 | 2·64 | 2·8 |
| 3¼ | 3·1 | 79 | 12·7 | 16 | 62 | 2·44 | 2·59 |
| 3 | 2·8 | 71 | 14·0 | 17 | 59 | 2·32 | 2·46 |
| 2¾ | 2·6 | 66 | 15·1 | 18 | 55 | 2·17 | 2·29 |
| 2½ | 2·36 | 60 | 16·7 | 20 | 50 | 1·97 | 2·09 |
| 2¼ | 2·1 | 53 | 18·7 | | | | |
| 2 | 1·88 | 48 | 20·94 | | | | |

When one becomes thoroughly conversant with the system of dioptries, it is easy to answer all the objections that have been made to it. It has, moreover, itself refuted them in the most effective way, by the services it has rendered to ophthalmological practice.

## TABLE II.

| OLD SYSTEM. | | | | NEW SYSTEM. | | | |
|---|---|---|---|---|---|---|---|
| I. No. of the Old System. | II. F. Focal Distance in Paris inches for $n=1{\cdot}53$. | III. F. Focal Distance in Millimetres. | IV. D. Equivalent in Dioptries. | V. No. of the New System. | VI. F. Focal Distance in Millimetres. | VII. F. Focal Distance in Paris inches. | VIII. No. corresponding of the Old System for $n=1{\cdot}53$. |
| 72 | 67·9 | 1837 | 0·54 | 0·25 | 4000 | 148 | 156 |
| 60 | 56·6 | 1523 | 0·65 | 0·5 | 2000 | 74 | 78 |
| 48 | 45·3 | 1225 | 0·81 | 0·75 | 1333 | 49 | 52 |
| 42 | 39·6 | 1072 | 0·93 | 1 | 1000 | 37 | 39·2 |
| 36 | 34 | 920 | 1·08 | 1·25 | 800 | 29·6 | 31·2 |
| 30 | 28·3 | 766 | 1·30 | 1·5 | 666 | 24·6 | 26·1 |
| 24 | 22·6 | 612 | 1·63 | 1·75 | 571 | 21 | 22·3 |
| 20 | 18·8 | 509 | 1·96 | 2 | 500 | 18·5 | 19·5 |
| 18 | 17 | 460 | 2·17 | 2·25 | 444 | 16·4 | 17·4 |
| 16 | 15 | 406 | 2·46 | 2·5 | 400 | 14·8 | 15·6 |
| 15 | 14·1 | 383 | 2·61 | 3 | 333 | 12·3 | 13·0 |
| 14 | 13·2 | 357 | 2·8 | 3·5 | 286 | 10·5 | 11·1 |
| 13 | 12·3 | 332 | 3·0 | 4 | 250 | 9·23 | 9·78 |
| 12 | 11·3 | 306 | 3·26 | 4·5 | 222 | 8·22 | 8·7 |
| 11 | 10·3 | 280 | 3·56 | 5 | 200 | 7·4 | 7·8 |
| 10 | 9·4 | 254 | 3·9 | 5·5 | 182 | 6·71 | 7·1 |
| 9 | 8·5 | 230 | 4·35 | 6 | 166 | 6·15 | 6·5 |
| 8 | 7·5 | 203 | 4·9 | 7 | 143 | 5·29 | 5·59 |
| 7 | 6·6 | 178 | 5·6 | 8 | 125 | 4·6 | 4·89 |
| 6½ | 6·13 | 166 | 6·02 | 9 | 111 | 4·1 | 4·35 |
| 6 | 5·6 | 152 | 6·52 | 10 | 100 | 3·7 | 3·91 |
| 5½ | 5·2 | 140 | 7·12 | 11 | 91 | 3·37 | 3·56 |
| 5 | 4·7 | 127 | 7·83 | 12 | 83 | 3·07 | 3·26 |
| 4½ | 4·2 | 115 | 8·70 | 13 | 77 | 2·84 | 3·01 |
| 4 | 3·8 | 102 | 9·72 | 14 | 71 | 2·63 | 2·8 |
| 3½ | 3·3 | 89 | 11·2 | 15 | 67 | 2·47 | 2·60 |
| 3¼ | 3·1 | 83 | 12·0 | 16 | 62 | 2·3 | 2·44 |
| 3 | 2·8 | 76 | 13·0 | 17 | 59 | 2·18 | 2·30 |
| 2¾ | 2·6 | 70 | 14·4 | 18 | 55 | 2·03 | 2·17 |
| 2½ | 2·36 | 64 | 15·7 | 20 | 50 | 1·85 | 1·95 |
| 2¼ | 2·1 | 57 | 17·4 | | | | |
| 2 | 1·88 | 51 | 19·6 | | | | |

As will be seen, the old system extends from $\frac{1}{67{\cdot}9}$ to $\frac{1}{1{\cdot}88}$. It comprises, therefore, a variation of refraction of $\frac{1}{1{\cdot}88} = \frac{1}{67{\cdot}9} = \frac{1}{1{\cdot}93}$; or, more simply stated, of $20{\cdot}94 - 0{\cdot}58 = 20{\cdot}36$ dioptries, with 32 numbers. The new system includes $20 - 0{\cdot}25 = 19{\cdot}75$ dioptries, with 30 numbers.

This arrangement of the new system, which is based on experience gained in the long-continued use of the old system, has shown itself to be adapted to the requirements of practice, and we should not care to alter it in any way, unless it were, perhaps, to add a number 6·5, the interval between 6 and 7 dioptries having sometimes seemed too great. This number is nearly equivalent to the English 6½ or the French 6 of the old system. Hence there is a "tool" for it already in existence, and it could easily be interpolated in our series of trial glasses.

It greatly simplifies the questions of refraction, accommodation and optical adaptation of the eyes, and, since the introduction of the *metre angle*, even those of their direction.

This is the opinion of all those who, like us, have employed for many years the old and the new system of calculation, and have thus become fully acquainted with the relative merits of each.

Having received the dioptry rather coldly at first, we now render it full justice, and warmly recommend it to all our colleagues who are still burdened with the old system.

## THE COMBINATION OF THREE SPHERICAL REFRACTIVE SURFACES.

### Dioptric System of the Eye.

Let us now resume the course of our discussion at the point where we digressed to consider infinitely thin lenses—that is, at the end of the preceding chapter, page 54.

We have followed the course of light through *a single refracting surface*, and subsequently through *two*. Let us now take the case of three surfaces. This brings us to the principal problem in these discussions, THE COURSE OF LIGHT THROUGH THE DIOPTRIC SYSTEM OF THE EYE. This system may indeed be considered as composed of three refractive surfaces.

We have a first refractive surface, $S^{\circ}$, convex: the CORNEA, separating the *air*, whose index of refraction is $n = 1$, from a more refractive medium, the *aqueous humor*, whose index of refraction is $n'$.

A *second* convex surface $S'$ (the anterior surface of the crystalline) separates the medium $n'$ from $n''$, the latter being more refractive than $n'$.

Finally, a *third* surface $S''$ (the posterior surface of the crystalline), concave, forms the limit between the medium $n''$ and a new medium $n'$, the VITREOUS BODY. The aqueous humor and the vitreous body have, indeed, the same index of refraction.

We wish to determine the CARDINAL POINTS of this compound system.

We can decompose it into two: 1. the *cornea*; and 2. the *crystalline* surrounded by the medium $n'$.

We shall have then to determine: first, the cardinal points of the cornea, by means of the formulæ laid down for a single surface; then the cardinal points of the crystalline, by means of the formulæ above given for the biconvex lens surrounded by a single medium.

Finally, by combining the cardinal points of both systems, we shall obtain those of the entire system—the *eye*.

The radii of curvature of these surfaces, as well as their relative distances, and the indices of refraction of the refractive media, have been ascertained by mensuration and calculation.

We give here these values, conformably to the latest investigations of Helmholtz, published by Reich,[1] in which a few errors in calculation have been corrected by Stammeshaus.[2]

Index of refraction of the air, . . . . . . $n = 1$
Index of refraction of the aqueous humor and vitreous body, $n' = 1{\cdot}3365$
Total index of refraction of the crystalline, . . $n'' = 1{\cdot}4371$
Radius of curvature of the cornea, . . . . . $r = 7{\cdot}829^{\text{mm.}}$
Radius of the anterior surface of the crystalline, . $r' = 10^{\text{mm.}}$
Radius of the posterior surface of the crystalline, . $r'' = 6^{\text{mm.}}$
Distance from the anterior surface of the cornea to the anterior surface of the crystalline (Fig. 39), . . $A\,A' = 3{\cdot}6^{\text{mm.}}$
Distance from the anterior surface of the cornea to the posterior surface of the crystalline, . . . $A\,A'' = 7{\cdot}2^{\text{mm.}}$
Hence, thickness of the crystalline, . . . . $\epsilon = 3{\cdot}6^{\text{mm}}$

It need scarcely be said that these are all average values.

The radii $r$ and $r'$ are positive: the radius $r''$ is negative.

Let us commence by determining the foci of the various refracting surfaces. The formulæ 4*a*, 4*b*, 4*c*, and 4*d* will help us in doing this.

I. SURFACE OF THE CORNEA:

First focal distance $$f_0' = \frac{r}{n'-1} = \frac{7{\cdot}829}{1{\cdot}3365-1} = 23{\cdot}266^{\text{mm.}}$$

Second focal distance $$f_0'' = \frac{n'r}{n'-1} = \frac{1{\cdot}3365 \times 7{\cdot}829}{1{\cdot}3365-1} = 31{\cdot}095^{\text{mm.}}$$

II. ANTERIOR SURFACE OF THE CRYSTALLINE:

First focal distance $$f_1' = \frac{n'r'}{n''-n'} = \frac{1{\cdot}3365 \times 10}{1{\cdot}4371-1{\cdot}3365} = 132{\cdot}853^{\text{mm.}}$$

Second focal distance $$f_1'' = \frac{n''r'}{n''-n'} = \frac{1{\cdot}4371 \times 10}{1{\cdot}4371-1{\cdot}3365} = 142{\cdot}853^{\text{mm.}}$$

III. POSTERIOR SURFACE OF THE CRYSTALLINE:

First focal distance $$f_2' = \frac{n''r''}{n'-n''} = \frac{1{\cdot}4371 \times 6}{1{\cdot}3365-1{\cdot}4371} = 85{\cdot}7117^{\text{mm.}}$$

Second focal distance $$f_2'' = \frac{n'r''}{n'-n''} = \frac{1{\cdot}3365 \times 6}{1{\cdot}3365-1{\cdot}4371} = 79{\cdot}7113^{\text{mm.}}$$

---

[1] *Arch. f. Ophth.*, xx., i, p. 107.

[2] *Dioptrik des normalen menschlichen Auges*, p. 131 *et seq.*

The *principal points of the cornea* coincide with its summit, and the *principal plane* with its anterior surface.

The *principal foci of the crystalline* are positive, because the index of refraction $n''$, of the crystalline, is higher than that of the ambient medium $n'$.

The *focal distances of the crystalline* are equal. Let us call them $\psi$, and the thickness of the crystalline $(3{\cdot}6^{\text{mm.}}) = \epsilon$.

We shall have, according to formulæ 14, p. 42—

$$\psi = \frac{f_1' f_2'}{f_1'' + f_2' - \epsilon} = \frac{132{\cdot}853 \times 85{\cdot}7117}{142{\cdot}853 + 85{\cdot}7117 - 3{\cdot}6} = 50{\cdot}6168^{\text{mm.}} \quad . \quad . \quad . \quad 1.$$

*For the principal points of the crystalline*, which coincide with the *nodal points* (N′ and N″, Fig. 39), we have, according to formulæ 12*a* and 12*b* (pp. 39-40), the formulæ—

$$\left.\begin{aligned} N'A' &= \frac{\epsilon f_1'}{f_1'' + f_2' - \epsilon} = \frac{3{\cdot}6 \times 132{\cdot}853}{142{\cdot}853 + 85{\cdot}7117 - 3{\cdot}6} = 2{\cdot}12597^{\text{mm.}} \\ N''A'' &= \frac{\epsilon f_2''}{f_1'' + f_2' - \epsilon} = \frac{3{\cdot}6 \times 79{\cdot}7117}{142{\cdot}853 + 85{\cdot}7117 - 3{\cdot}6} = 1{\cdot}2756^{\text{mm.}} \end{aligned}\right\} \quad . \quad 2.$$

The cardinal points of the two systems, the cornea and crystalline, are thus determined.

It will be readily understood that the principal plane of the cornea plays the same part, with reference to the principal planes of the crystalline, as the first refracting surface of a lens with reference to the second surface. We shall find the principal planes of the compound system in a similar way.

## I.—PRINCIPAL POINTS AND PLANES OF THE EYE.

Let $a'$ and $a''$ (Fig. 39) be the *foci of the first surface* $S_0$ (*cornea*), $b'$ and $b''$ the *principal foci of the crystalline*, N′ and N″ the *principal points of the crystalline*.

A ray Q J, parallel to the axis, passes through the first surface, the principal plane of the cornea, at J, and is deflected toward $a''$.

A ray coming from Q, parallel to the axis, meets the first principal plane of the crystalline at J′, and is deviated toward $b'$.

J $a''$ and J′ $b'$ intercross at S. Let fall the perpendicular S O from this point upon the axis.

The two pairs of similar triangles, $a''$ J A and $a''$ S O, $b'$ J′ N′ and $b'$ S O, give—

$$\frac{a''A}{AJ} = \frac{a''A - AO}{OS}$$

and

$$\frac{b'\,N'}{A\,J} = \frac{b'\,O}{O\,S}$$

$$\frac{f_o''}{\psi} = \frac{f_o'' - A\,O}{\psi - N'\,O}$$

$$\frac{A\,O}{N'\,O} = \frac{f_o''}{\psi}.$$

Whence

$$A\,O = \frac{f_o'' \times N'\,O}{\psi}$$

and

$$N'\,O = \frac{\psi \times A\,O}{f_o''}.$$

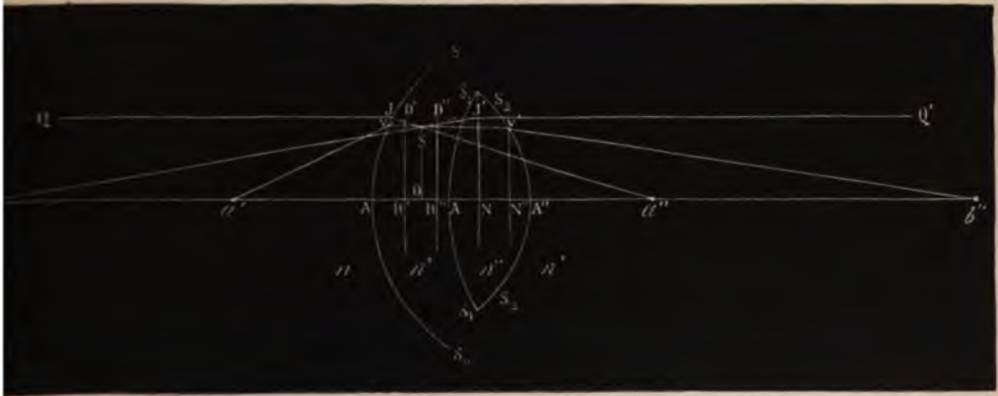

FIG. 39.

Let us consider A N′ as equal to $e$. This is a known value; it is— $e = A\,A' + A'\,N' = 3{\cdot}6 + 2{\cdot}12597 = 5{\cdot}72597$ millimetres.

We have, moreover—

$$A\,O + N'\,O = e$$

$$A\,O = e - N'\,O$$

$$N'\,O = e - A\,O.$$

From these three equations, for N′ O, we derive the following:

$$A\,O = \frac{e f_o''}{f_o'' + \psi} = \frac{5{\cdot}72597 \times 31{\cdot}0950}{31{\cdot}0950 + 50{\cdot}6168} = 2{\cdot}1789^{\text{ mm.}}$$

The two equations for A O give—

$$N'\,O = \frac{e\,\psi}{f_o'' + \psi},$$

or simply—

$$N'\,O = e - A\,O = 5{\cdot}72597 - 2{\cdot}1789 = 3{\cdot}54698^{\text{ mm.}} \qquad \Bigg\} \quad . \quad \text{I.}$$

## II.—Principal Points of the Eye.

Having found the point O, let us seek the principal points and planes of the system called *the eye*.

Let O S (Fig. 39) be an object. We have to determine the place where the images of it, furnished conjointly by the surface S° (cornea) and by the crystalline, are formed.

The ray S V, parallel with the axis, is refracted toward $a'$. Prolonged backward, it meets the line Q Q′ at D′. Hence D′ is the virtual image of S, because this is the point of intersection of the rays J Q and V $a'$, both appertaining to the point S.

The ray S V′, likewise parallel with the axis, meets the second principal plane of the crystalline at V′, and is refracted toward $b''$. Prolonged backward, it meets the line Q Q′ at D″, and thus determines the image (D″) of S, furnished by the crystalline. Hence D′ H′ and D″ H″ are the images of S O seen through the surface S° S° and through the crystalline. They are erect and of equal size.

The planes passed through H′ D′ and H″ D″ are, therefore, the PRINCIPAL PLANES OF THE DIOPTRIC SYSTEM OF THE EYE.

The triangles $a''$ J A and $a''$ S O are similar, as are also $a'$ D′ H′ and $a'$ V A.

Moreover—

$$\mathrm{J\,A} = \mathrm{H'\,D'};\ \mathrm{V\,A} = \mathrm{S\,O} \text{ and } \mathrm{A\,O} = \frac{e f_0''}{f_0'' + \psi}.$$

From these equations we derive the following:

$$\mathrm{A\,H'} = \frac{e f_0'}{f_0'' + \psi - e}$$

$$\mathrm{A\,H'} = \frac{5{\cdot}72597 \times 23{\cdot}2660}{31{\cdot}0950 + 50{\cdot}6168 - 5{\cdot}72597} = 1{\cdot}7532^{\text{ mm.}} \quad . \ . \ . \quad \text{II. } a.$$

that is to say, the *first principal point of the eye* is situated 1·7532 $^{\text{mm.}}$ behind the anterior surface of the cornea.

For H″ we make rise of the similar triangles $b'$ J′ N′ and $b'$ S O; $b''$ D″ H″ and $b''$ V′ N″. We bear in mind, too, that J′ N′ = D″ H″; V′ N″ = S O, and N′ O = $\frac{e\psi}{f_0'' + \psi}$.

It follows that—

$$\mathrm{N''\,H''} = \frac{e\psi}{f_0'' + \psi - e}$$

$$\mathrm{N''\,H''} = \frac{5{\cdot}72597 \times 50{\cdot}617}{31{\cdot}0950 + 50{\cdot}6168 - 5{\cdot}72597} = 3{\cdot}8143^{\text{ mm.}} \quad . \ . \quad \text{II. } b.$$

The position of N″ is determined by the second of these equations. N″ is situated 1·2756 millimetres in front of the posterior surface of the crystalline, hence $A\,A'' - N''\,A'' = A\,N''$, *i.e.*, $7{\cdot}2 - 1{\cdot}2756 = 5{\cdot}9244$ millimetres behind the cornea.

For the situation of the second principal point ($H''$) of the eye, we write—

$$A\,H'' = A\,N'' - H''\,N'' = 5{\cdot}9244 - 3{\cdot}8143 = 2{\cdot}1101\ ^{\text{mm.}}$$

Hence the *second principal point of the eye* is 2·1101 millimetres behind the anterior surface of the cornea.

The two principal points are, therefore, situated in front of the crystalline, in the anterior chamber; and the distance between them is—

$$H'\,H'' = A\,H'' - A\,H' = 2{\cdot}1101 - 1{\cdot}7532 = 0{\cdot}3569\ ^{\text{mm.}}$$

## III.—PRINCIPAL FOCAL DISTANCES OF THE EYE.

Let $H'$ and $H''$ (Fig. 40) be the principal points of the eye, $N'$ and $N''$ the principal points of the crystalline.

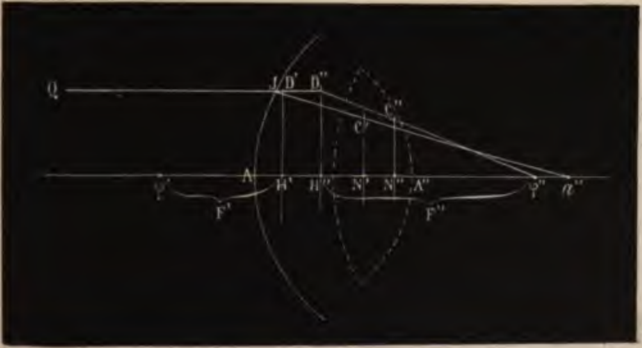

FIG. 40.

A ray coming from Q, parallel to the axis, meets the first surface at J, and is deviated toward its second focus $\alpha''$. This ray meets the first principal plane of the crystalline at $C'$. $C'\,C''$ being parallel with the axis, we have, in $C''$, one point in the path of the ray, after its refraction by the crystalline.

Another point in this path is given by $D''$ of the second principal plane of the optical system of the eye. Drawing a straight line through $D''$ and $C''$, we find the point $\phi''$ of the axis, at which all rays parallel to the axis are focused,—in short, the *second focus* of the compound system—the eye.

Hence $H''\,\phi'' = F''$ is the *second principal focal distance of the eye.*

In order to find the formula for it, let us employ the similar triangles $a''$ J A and $a''$ C″ N′; $\phi''$ D″ H″ and $\phi''$ C″ N″, and bear in mind that J A = D″ H″; C″ N″ = C′ N′; A N′ = $e$, and H″ N″ = $\frac{e\,\psi}{f_0''+\psi-e}$.

We shall obtain—

$$\frac{f_0''}{F''}=\frac{f_0''-e}{F''-\frac{e\,\psi}{f_0''+\psi-e}}.$$

whence

$$F''=\frac{f_0''\,\psi}{f_0''+\psi-e}\quad\ldots\ldots\ldots\quad \text{III. } a.$$

In figures, this equation reads—

$$F''=\frac{31{\cdot}0950\times 50{\cdot}6168}{31{\cdot}0950+50{\cdot}6168-5{\cdot}72597}=20{\cdot}7136^{\text{ mm.}}$$

This is the SECOND PRINCIPAL FOCAL DISTANCE OF THE EYE.

Since H″ is 2·1101 millimetres behind the cornea (A), the second principal focus of the eye is situated A H″ + H″ $\phi''$ = 2·1101 + 20·7136 = 22·8237 millimetres behind the cornea. This number represents the *length of the emmetropic eye.*

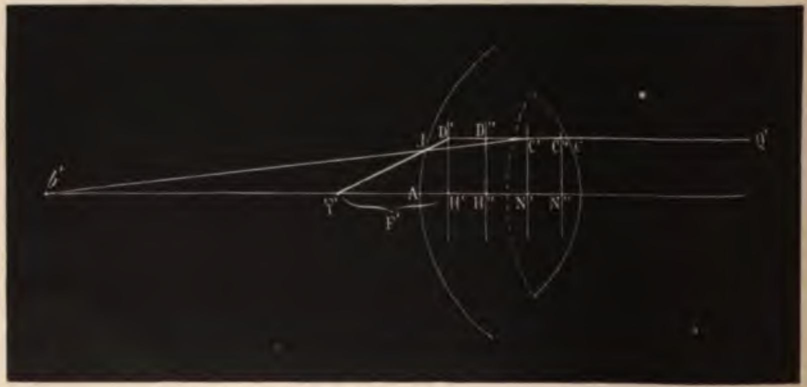

FIG. 41.

An analogous construction gives the formula for the first principal focal distance of the eye, H′ $\phi'$ = F′ (Fig. 41).

The first focus is the point at which all rays, that were parallel in the interior of the eye, are united. Let Q′ C be one of these rays. It meets the second principal plane of the crystalline at C″, the first principal plane at C′, and would intersect the axis at $b'$, the first focus of the crystalline, if it were not deviated, at J, by the cornea. The point J, at which it strikes the cornea, is one point in the path of the ray through the entire system.

Prolonging the ray Q′ C to the first principal plane, H′ D′, of the entire system, a new point, D′, of this path, is found. Let us now connect D′ and J, and, prolonging this line until it meets the axis, we find the *first focus*, $\phi'$, of the eye.

From the similar triangles $b'$ C′ N′ and $b'$ J A; $\phi'$ D′ H′ and $\phi'$ J A, it results that—

$$\frac{b'\,\mathrm{N}'}{\mathrm{C}'\,\mathrm{N}'} = \frac{b'\,\mathrm{A}}{\mathrm{J}\,\mathrm{A}}$$

$$\frac{\phi'\,\mathrm{H}'}{\mathrm{C}'\,\mathrm{N}'} = \frac{\phi'\,\mathrm{A}}{\mathrm{J}\,\mathrm{A}}.$$

By division we obtain—

$$\frac{b'\,\mathrm{N}'}{\phi'\,\mathrm{H}'} = \frac{b'\,\mathrm{A}}{\phi'\,\mathrm{A}}$$

in which

$$b'\,\mathrm{N}' = \psi$$
$$\phi'\,\mathrm{H}' = \mathrm{F}'$$
$$b'\,\mathrm{A} = \psi - e$$
$$\phi'\,\mathrm{A} = \mathrm{F}' - \mathrm{A}\,\mathrm{H}' = \mathrm{F}' - \frac{e\,f_0'}{f_0'' + \psi - e}.$$

Hence our equation becomes—

$$\frac{\psi}{\mathrm{F}'} = \frac{\psi - e}{\mathrm{F}' - \dfrac{e\,f_0'}{f_0'' + \psi - e}}$$

from which we have the first principal focal distance of the eye—

$$\mathrm{F}' = \frac{f_0'\,\psi}{f_0'' + \psi - e} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{III. } b.$$

Expressed numerically, this equation reads—

$$\mathrm{F}' = \frac{23{\cdot}266 \times 50{\cdot}6168}{31{\cdot}0950 + 50{\cdot}6168 - 5{\cdot}72597} = 15{\cdot}4983^{\text{mm.}}$$

This is the FIRST PRINCIPAL FOCAL DISTANCE OF THE EYE.

The *first principal focus of the eye* is situated $\phi'$ H′ − A H′ = 15·4983 − 1·7532 = 13·7451 millimetres in front of the cornea.

From the expressions already found for the two focal distances of the compound system (the eye), it results that—

$$\frac{\mathrm{F}''}{\mathrm{F}'} = \frac{f_0''}{f_0'} = \frac{n'}{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{III. } c.$$

This is equivalent to saying that *the ratio of the second focal distance, of the system, to the first, is equal to the ratio of the index of refraction, of the last medium, to that of the first.*

This is a general law which is not exactly rendered evident by our example, because we have supposed that the light, after having passed through the third medium ($n''$), re-enters the second ($n'$), as is the case in the eye. If, instead of having the same index of refraction as the second, the fourth medium had had an index of refraction $n'''$, our formula would have given us—

$$\frac{F''}{F'} = \frac{n'''}{n}.$$

For the dioptric system under consideration (the eye), the index of refraction of the first medium, the air, is $n = 1$.

Hence the formula

$$\frac{F''}{F'} = \frac{f_0''}{f_0'} = \frac{n'}{n}$$

is simplified, and becomes

$$\frac{F''}{F'} = \frac{f_0''}{f_0'} = n'.$$

Performing the calculation, with the help of the values which we have already found, we obtain, in fact—

$$\frac{20{\cdot}7136}{15{\cdot}4983} = \frac{31{\cdot}0950}{23{\cdot}2660} = 1{\cdot}3365.$$

This, it will be remembered, is the index of refraction of the last medium—the vitreous body.

## IV.—Nodal Points of the Eye.

Let E (Fig. 42) be a point in the first focal plane. A ray, coming from it in a direction parallel to the axis, pierces at C′ and C″ the two principal planes of the eye, and is directed toward the second principal focus $\phi''$; C″ $\phi''$ is therefore the direction finally taken by all rays emanating from E.

Now, according to the definition, rays which, before being refracted, are directed toward the first nodal point, have, after refraction, a direction parallel to their primitive one, and seem to come from the second nodal point. Hence the ray which is directed toward the first nodal point must be likewise parallel to C″ $\phi''$. This is the ray E K′, which meets the axis at K′. Therefore K′ is the *first nodal point of the eye.*

The ray E K′ meets the first principal plane at D′. One point of its ultimate course is, then, the point D″ in the second principal plane, if D′ D″ be parallel to the axis. Hence we have only to draw, from D″, a line parallel to E K′ or to C″ $\phi''$, and the point K″, at which it meets the axis, will be the *second nodal point of the eye.*

Now the triangles $\phi' K' E$ and $H'' \phi'' C''$ are equal, because $E \phi' = C'' H''$, and all their angles are equal, being the included angles formed by parallel lines; hence $\phi' K' = H'' \phi''$.

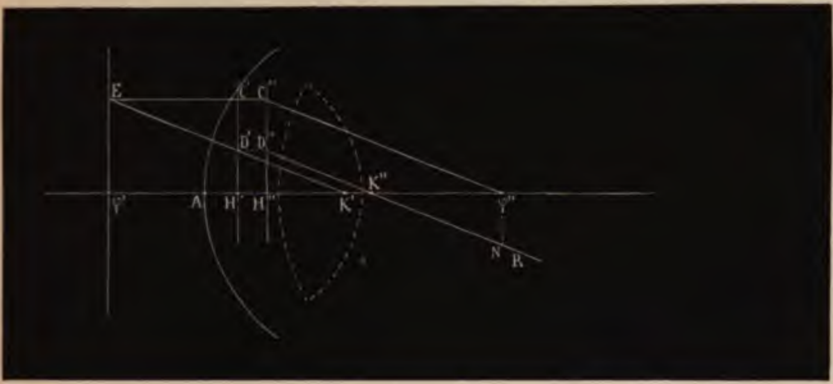

FIG. 42.

We call $\phi' K'$ equal to $G'$, and $H'' \phi''$ equal to $F''$.

We have then—

$$G' = F'' \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{IV. } a.$$

Erecting at $\phi''$ the vertical $\phi'' N$, we obtain two other equal triangles, $E C' D'$ and $K'' \phi'' N$, from which it results that $E C' = \phi' H' = K'' \phi''$ or

$$G'' = F' \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{IV. } b.$$

Finally, $K' K''$ and $D' D''$ are equal, being opposite sides of a parallelogram. And, since $D' D'' = H' H''$, we have—

$$K' K'' = H' H'' \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{IV. } c.$$

Subtracting $F'$, from each member of the equation IV. $a$, we obtain—

$$G' - F' = F'' - F'.$$

Now

$$G' - F' = H' K'$$

or

$$H' K' = F'' - F'.$$

By an analogous operation we deduce from IV. $b$—

$$F'' - G'' = F'' - F',$$

or

$$F'' - G'' = H'' K'',$$

or

$$H'' K'' = F'' - F'.$$

Hence $H' K'$, the distance between the first principal point and the first nodal point, and $H'' K''$, the distance between the second principal point and the second nodal point, have values equal to the difference between the two focal distances. It will be remembered that we have found a similar relation in the case of a single refracting surface: the principal point (summit of the surface) is separated from the nodal point (centre of curvature) by the difference ($r$) between F′ and F″ (p. 19).

Let us now express these equations numerically:—

$$H' K' = F'' - F' = 20{\cdot}7136 - 15{\cdot}4983 = 5{\cdot}2153^{\text{mm.}}$$

and

$$A K' = A H' + H' K' = 1{\cdot}7532 + 5{\cdot}2153 = 6{\cdot}9685^{\text{mm.}}$$

The FIRST NODAL POINT OF THE EYE is, therefore, situated 6·9685 millimetres behind the summit of the cornea.

The posterior surface of the crystalline being 7·2 millimetres behind the corneal surface, the first nodal point is situated in the crystalline, 0·2315 millimetres in front of its posterior surface.

For the second nodal point we shall write—

$$A K'' = A K' + K' K'' = 6{\cdot}9685 + 0{\cdot}3569 = 7{\cdot}3254^{\text{mm.}}$$

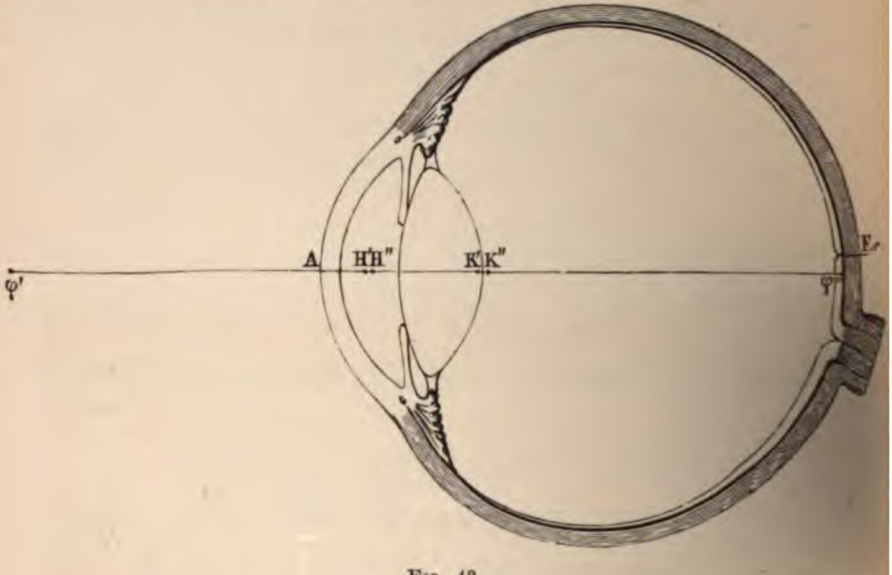

FIG. 43.

Schematic eye magnified three times. $\phi'$, anterior or first principal focus; A, anterior surface of the cornea; H′ and H″, principal points; K′ and K″, nodal points; $\phi''$, posterior or second principal focus; F c, *fovea centralis*; $\phi' \phi''$, optic axis.

Hence the SECOND NODAL POINT OF THE EYE is situated 7·3254 millimetres behind the surface of the cornea, *i.e.*, $7{\cdot}3254 - 7{\cdot}2 = 0{\cdot}1254$ millimetres behind the posterior surface of the crystalline.

We have thus found all the cardinal points of the average or schematic eye, and can add, to the anatomical, the physiological table of the optical constants of the eye.

Taking as starting-point the anterior surface of the cornea (A, Fig. 43) we find—

### *The Cardinal Points of the Eye.*

The first principal focus, $\phi'$, is situated $A\phi' = 13{\cdot}7451$ millimetres *in front* of the cornea. The other points are *behind the cornea :*

| | |
|---|---|
| The first principal point, H′ by $AH' =$ . . . | $1{\cdot}7532^{\text{mm.}}$ |
| The second principal point, H″ by $AH'' =$ . . | $2{\cdot}1101^{\text{mm.}}$ |
| The difference, $H'H'' = K'K''$ is . . . . | $0{\cdot}3569^{\text{mm.}}$ |
| The first nodal point, K′ by $AK' =$ . . . | $6{\cdot}9685^{\text{mm.}}$ |
| The second nodal point, K″ by $AK'' =$ . . | $7{\cdot}3254^{\text{mm.}}$ |
| The second principal focus, $\phi''$ by $A\phi'' =$ . . | $2.28237^{\text{mm.}}$ |

These values are shown in Figure 43, but three times as great as in nature.

## V.—Conjugate Foci of the Eye.

After having determined the cardinal points of the composite system, it still remains for us to find the formula of the conjugate foci. In other words, being given a luminous point, we endeavour to find where the image of it, furnished by the dioptric system of the eye, is formed.

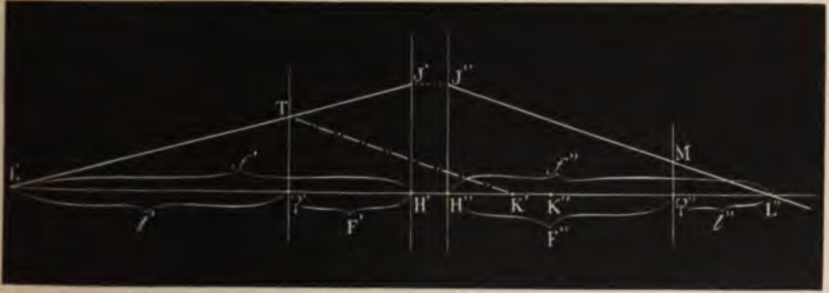

Fig. 44.

This problem is easy of solution. We have only to bear in mind the properties of the cardinal points and planes, and we may without difficulty follow the luminous rays, given off from the object-points, through the dioptric system to their destinations.

This is, moreover, as we shall see, almost a repetition of what occurs in the case of a single refractive surface, or in that of a lens. The only difference is, that we have here two principal and two nodal points. But this is also the course which light takes in passing through the most complicated centred system, made up of any number of surfaces and of different media.

Let $\phi'$ and $\phi''$ (Fig. 44) be the principal foci of the eye,
H′ and H″ the principal points,
K′ and K″ the nodal points.[1]

Let L′ be a luminous point on the axis. We wish to know where its image, L″, is formed.

We write—

$$\begin{aligned}
\phi' \, H' &= F' = G'' = \phi'' \, K'' \\
\phi'' \, H'' &= F'' = G' = \phi' \, K' \\
L' \, H' &= f' = g'' = L'' \, K'' \\
L'' \, H'' &= f'' = g' = L' \, K \\
L' \, \phi' &= l' = f' - F' \\
L'' \, \phi'' &= l'' = f'' - F''.
\end{aligned}$$

The values F′ and $f'$ are positive to the left of H′, negative to the right of it;

F″ and $f''$ are positive to the right of H″, negative to the left of it;

G′ and $g'$ are positive to the left of K′, negative to the right of it;

G″ and $g''$ are positive to the right of K″, negative to the left of it.

The point L′ being situated upon the axis, its image will also be formed upon this line. In order to ascertain the location of this image, we have only to seek the point where another ray, given off from L′, meets the axis.

Let L′ J′ be a ray piercing the first focal plane at T, and the first principal plane at J′. The point J″, of the second principal plane, from which the ray in the last medium appears to arise, is found by drawing a line, parallel with the principal axis, from J′ to the second principal plane. J″ is this point.

In order to determine the final direction of the ray, in the last medium, we draw the line T K′ toward the first nodal point, bearing in mind that, since T is situated in the first focal plane, all rays emanating from this point are parallel to each other in the last medium, and parallel to the ray of direction which passes through the first nodal point.

T J′ may be regarded as another ray coming from T. Its final direction, J″ L″, will therefore be parallel to T K′. Hence the point L″, at which this ray meets the axis, is the image of L′.

---

[1] We do not indicate in this figure either the cornea or the surfaces of the crystalline, because, their dioptric action being expressed by the cardinal points and planes, they have no significance for the construction or calculation of the course of luminous rays through the system.

In order to find by calculation the place where this image is formed, let us consider the similar triangles L″ $\phi''$ M and L″ H″ J″, L′ $\phi'$ T and L′ H′ J′ (Fig. 44). The first pair gives us—

$$\frac{f''-F''}{\phi'' M}=\frac{f''}{H'' J''};$$

the second pair—

$$\frac{f'-F'}{\phi' T}=\frac{f'}{H'' J''}$$

$$(H'' J''=H' J').$$

Dividing the first equation by the second, we obtain—

$$\frac{\phi' T\,(f''-F'')}{\phi'' M\,(f'-F')}=\frac{f''}{f'}.$$

From the similarity of the triangles, $\phi''$ M L″ and $\phi'$ T K′, it results that—

$$\frac{\phi'' M}{\phi' T}=\frac{f''-F''}{\phi' K'},$$

whence

$$\phi'' M=\frac{\phi' T\,(f''-F'')}{\phi' K'}.$$

Now, according to the equation IV. *a* (p. 87), $\phi'$ K′ = F″; hence—

$$\phi'' M=\frac{\phi' T\,(f''-F'')}{F''}.$$

Let us introduce this expression in the above formula, and $\phi'$ T, as well as $f''-F''$, will disappear upon performing the division. We shall have left—

$$\frac{F''}{f'-F'}=\frac{f''}{f'},$$

whence

$$f=\frac{f' F''}{f'-F'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{V. } a.$$

or, reckoning from the nodal points, K′ and K″, *i.e.*, expressing their values by G and $g$, we obtain—

$$g'=\frac{g'' G'}{g'-G''} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{V. } b.$$

For $f'$, we deduce the formula—

$$f'=\frac{f'' F'}{f''-F''} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{V. } c.$$

and for $g''$

$$g''=\frac{g' G''}{g'-G'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{V. } d.$$

Finally, subtracting F′ from each member of the equation given for $f'$, we obtain, furthermore—

$$\left.\begin{aligned} l'\, l'' &= F'\, F'' \\ l'' &= \frac{F'\, F''}{l'} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \quad \text{V. } e.$$

It will be observed that these formulæ are identical with those furnished by a single refracting surface (formulæ 8 *a* and 7 *c*, p. 20). The only difference exists in the absolute values of the lengths, which are not reckoned here from a single point, H or K, but from two points, H′ and H″, or K′ and K″.

## VI.—SIZE OF THE IMAGE FURNISHED BY THE EYE OR BY ANY OTHER COMPOUND SYSTEM.

In order to ascertain the size of the image corresponding to an object A′ L′, let us seek the spot where the image of the point A′ (Fig. 45) is formed.

We first draw the ray A′ J′ J″ parallel with the axis, and which, after having passed through the system, is directed toward the second principal focus $\phi''$.

The image of A′ is found at that point in which the ray J″ $\phi''$ A″ meets the perpendicular let fall from L″. This is at the point A″. Hence A″ L″ is the image of A′ L′.

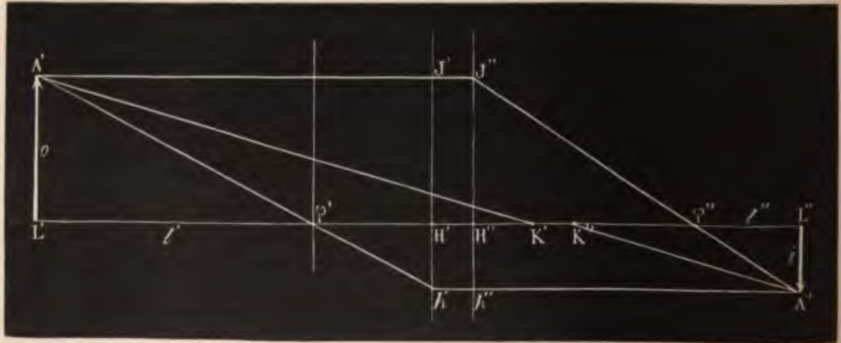

Fig. 45.

We may find the point A″ in still another way,—that is, by using the ray which, before entering the system, passes through the first focus $\phi'$. After leaving the point $h'$, in which it meets the first principal plane, it is parallel to the principal axis, and is also directed toward A″.

The simplest method of finding the image of an object, and the one oftenest employed when the place, where the image is formed, is already known, is to draw the rays of direction from the point A′ through the nodal points. The ray A′ K′, which, before entering the system, is directed toward the first nodal point, is parallel to this direction in the last medium, and continues its course as if it came from the second nodal point. Hence K″ A″ is parallel with A′ K′, and indicates, at its point of intersection with the vertical H″ A″, the image A″, of A′.

A formula, giving the size of the image, results from the similarity of the triangles A′ L′ K′ and A″ L″ K″ (Fig. 45).

Letting *i* represent the size of the image, and *o* that of the object, we obtain—

$$\left.\begin{aligned} \frac{i}{o} &= \frac{L''\,K''}{L'\,K'} = \frac{g''}{g'} \\ i &= \frac{o\,g''}{g'} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \quad \text{VI. } a.$$

The similar triangles A′ L′ $\phi'$ and H′ *h*′ $\phi'$ give us another formula for the size of the image, if we consider that H′ *h*′ = L″ A″ = *i*:

$$\left.\begin{aligned} \frac{i}{o} &= \frac{F'}{l'} \\ i &= \frac{o\,F'}{l'} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{VI. } b.$$

Again, from the similarity of the triangles L″ A″ $\phi''$ and H″ J″ $\phi''$, it results that—

$$\left.\begin{aligned} \frac{i}{o} &= \frac{l''}{F''} \\ i &= \frac{o\,l''}{F''} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{VI. } c.$$

We see that the formulæ, relating to the size of the image furnished by a large number of refractive surfaces, are still like those which we have found for a single surface: 9 *a*, 9 *b*, and 9 *c* (p. 25), or for a lens, 9 *a*′ and 9 *c*′ (p. 47).

Let us take an example:

Suppose an object, A′ L′ (Figs. 44 and 45) to be 100 millimetres from the first principal point of the eye, and three millimetres in height, where will its image be formed?

The formula V. *a* shows us that—

$$f'' = \frac{f'\,F''}{f' - F'}$$

$$f'' = \frac{100 \times 20{\cdot}7}{100 - 15{\cdot}5} \;\; 24{\cdot}49^{\text{mm.}}$$

that is to say, the image is formed 24·49 millimetres behind the second principal point. The latter being situated $1{\cdot}75+0{\cdot}365=2{\cdot}115$ millimetres behind the cornea, we may say that the image of an object situated $100-1{\cdot}75=98{\cdot}25$ in front of the cornea is formed $24{\cdot}49+2{\cdot}115$ millimetres $=26{\cdot}6$ millimetres behind the latter.

This is, by the way, the distance from the cornea to the retina of a myopic eye whose *punctum remotum* is situated 98·25 millimetres in front of the cornea. Such an eye is, therefore, $26{\cdot}6-22{\cdot}8=3{\cdot}8$ millimetres longer than an emmetropic eye.

If the question be to find the distance of the image from the second nodal point, as is necessary when calculating the size of this image, we shall employ the formula V. *d*.

In our example, $g'=f'+H'\,K'=100+5{\cdot}216=105{\cdot}216$; hence—

$$g''=\frac{105{\cdot}216\times 15{\cdot}5}{105{\cdot}216-20{\cdot}7}=19{\cdot}28^{\text{mm.}}$$

The value of $g''$ may, moreover, be also obtained from $f''-H''\,K''=24{\cdot}49-5{\cdot}21=19{\cdot}28^{\text{mm.}}$ Again, according to the formula V. *e*, we write—

$$l''=\frac{F'\,F''}{l'}$$

in which

$$l'=L'\,H'-F'=100-15{\cdot}5=84{\cdot}5^{\text{mm.}}$$

$$l''=\frac{15{\cdot}5\times 20{\cdot}7}{84{\cdot}5}=3{\cdot}79^{\text{mm.}}$$

which signifies that the image is formed 3·79 millimetres behind the second principal point of the eye.

This length $l''$ has for us a very special importance. It is the difference in length between an emmetropic and an (axially) myopic eye. The retina of the emmetropic eye is at the focus $\phi''$, while that of the myopic eye is at $L''$, *i.e.*, a distance of $l''$ beyond $\phi''$.

For the size of the image, we have, according to formula VI. *a*—

$$i=\frac{3\times 19{\cdot}28}{105{\cdot}21}=0{\cdot}549^{\text{mm.}}$$

or, according to formula VI. *b*—

$$i=\frac{3\times 15{\cdot}5}{84{\cdot}5}=0{\cdot}55^{\text{mm.}}$$

This is the size of the retinal image of an object 3 millimetres in height, placed at the *punctum remotum* of a myopic eye, which, without a spectacle glass, can see clearly only up to a distance of 98·25 millimetres from its cornea.

### *Negative Conjugate Focus—Convergent Incident Rays.*

The luminous rays which enter the eye are not always *divergent*, as we have supposed them to be in the preceding example.

They are sometimes *convergent;* as, for instance, when, under certain circumstances, they have passed through a convex lens.

In this case the rays do not come from a luminous point (L′) situated in front of the system, but *converge* toward a virtual point (−L′) situated behind the system, to the right in our example. The object is then on the same side as the image.

The object is *virtual*, and all the distances ($f'$, $g'$, and $l'$), which indicate its relations with the cardinal points, become negative.

We have only to introduce them with this sign (minus) in our formulæ, to be able to use them as well as in the case of a positive luminous point, *i.e.*, for divergent rays.

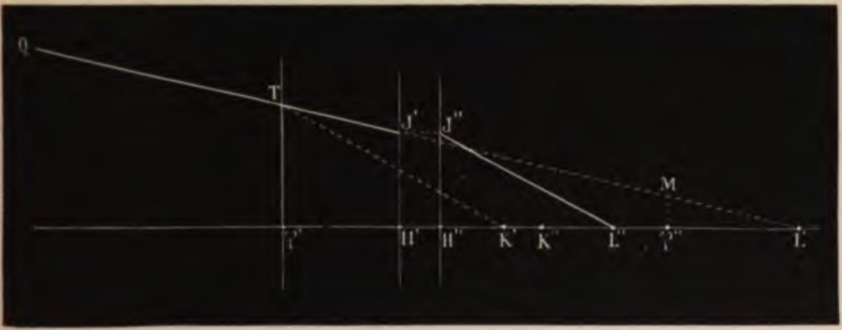

FIG. 46.

Figure 46 renders apparent the construction of the course of the rays which converge toward the point L′ on entering the system.

The plain line indicates the manner of finding the point L″, the image of L′; J′ J″ is parallel to the axis and J″ L″ is parallel to T K′.

The triangles L″ H″ J″ and K′ φ′ T are similar.

Hence we have—

$$\frac{f''}{\mathrm{H}''\,\mathrm{J}''}=\frac{\mathrm{K}'\,\phi'}{\phi'\,\mathrm{T}}$$

From the similar triangles L′ H′ J′ and L′ φ″ M we deduce—

$$\frac{f'}{\mathrm{H}''\,\mathrm{J}''}=\frac{f'-\mathrm{H}'\,\mathrm{H}''-\mathrm{F}''}{\phi''\,\mathrm{M}}$$

since

$$\mathrm{H}'\,\mathrm{J}'=\mathrm{H}''\,\mathrm{J}''.$$

Dividing the first formula by the second, we obtain—

$$\frac{f''}{f'} = \frac{\mathrm{K}'\,\phi' \times \phi''\,\mathrm{M}}{\phi'\,\mathrm{T}\,(f' - \mathrm{H}'\,\mathrm{H}'' - \mathrm{F}'')}.$$

The triangles L′ φ″ M and L′ φ′ T are also similar, and give us the equation—

$$\frac{\phi''\,\mathrm{M}}{f'' - \mathrm{H}'\,\mathrm{H}'' - \mathrm{F}''} = \frac{\phi'\,\mathrm{T}}{\mathrm{F}' + f''},$$

whence

$$\phi''\,\mathrm{M} = \frac{\phi'\,\mathrm{T}\,(f'' - \mathrm{H}'\,\mathrm{H}'' - \mathrm{F}'')}{\mathrm{F}' + f''}.$$

The substitution of this expression for φ″ M in the formula for $\frac{f''}{f'}$ and the replacement of K′ φ′ by F″ give—

$$\frac{f''}{f'} = \frac{\mathrm{F}''}{\mathrm{F}' + f''},$$

whence

$$\left.\begin{aligned} f'' &= \frac{f'\,\mathrm{F}''}{\mathrm{F}' + f'} \\ f' &= \frac{f''\,\mathrm{F}'}{\mathrm{F}'' - f''} \end{aligned}\right\} \quad \dots\dots\dots \quad \text{VIII. } a.$$

Substituting the values, beginning at the nodal points—

$$\left.\begin{aligned} g' &= \frac{g''\,\mathrm{G}'}{\mathrm{G}'' + g''} \\ g'' &= \frac{g'\,\mathrm{G}''}{\mathrm{G}' - g'} \end{aligned}\right\} \quad \dots\dots\dots \quad \text{VIII. } b.$$

Again, adding + F′ to the formula for $f'$—

$$\left.\begin{aligned} l'\,l'' &= \mathrm{F}'\,\mathrm{F}'' \\ l'' &= \frac{\mathrm{F}'\,\mathrm{F}''}{l'} \end{aligned}\right\} \quad \dots\dots\dots \quad \text{VIII. } c.$$

in which

$$l' = \phi'\,\mathrm{L}' = f' + \mathrm{F}'$$

$$l'' = \phi''\,\mathrm{L}'' = \mathrm{F}'' - f''.$$

For the *size* of the image, we have the constructions indicated by Figure 47.

Let A′ L′ be the virtual object, *i.e.*, the image which would be formed if the convergent rays reached their focus without encountering the system.

If the place, L″, where the image of L′ is formed, be known, it is necessary only to erect a vertical from this point and the point A″, where this vertical intersects the line K″ N, which is parallel with the ray of direction A′ K′, corresponds to the image of A′.

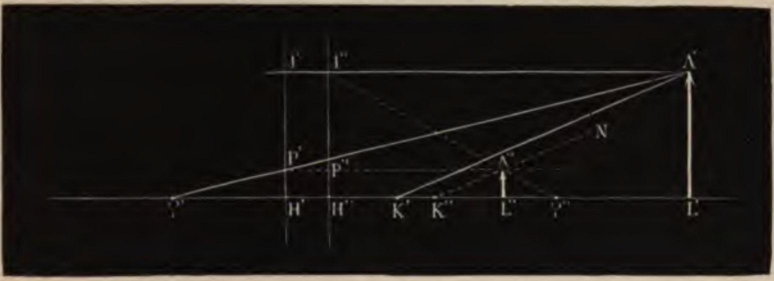

Fig. 47.

Hence L″ A″ is the image of L′ A′, and we have for its size the following equation, which results from the similarity of the triangles L″ A″ K″ and L′ A′ K′ :—

$$\left.\begin{array}{l} \dfrac{i}{o} = \dfrac{g''}{g'} \\[2ex] i = \dfrac{o\, g''}{g'} \end{array}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{IX. } a.$$

Again, we may use the ray which is directed from φ′ toward A′, and which meets the first principal plane at P′. It is parallel with the axis in the interior of the system, and meets the line K″ N, as well as the perpendicular erected upon the axis from L″, at A″.

The similar triangles H′ P′ φ′ and L′ A′ φ′ give—

$$\left.\begin{array}{l} \dfrac{i}{o} = \dfrac{F'}{l'} \\[2ex] i = \dfrac{o\, F'}{l'} \end{array}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{IX. } b.$$

Finally, the ray J″ A′, parallel to the axis, would, after having entered the system and passed through the second principal plane at J″, be directed toward the second focus φ″. It also meets the line K″ N at A″, and we have a third pair of similar triangles, L″ A″ φ″ and H″ J″ φ″, from which we derive—

$$\left.\begin{array}{l} \dfrac{i}{o} = \dfrac{l''}{F''} \\[2ex] i = \dfrac{o\, l''}{F''} \end{array}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{IX. } c.$$

### EXAMPLE.

Supposing the rays which enter the eye to be directed toward a point situated 100 millimetres behind the principal plane, then—

$$-f' = 100^{\text{mm.}}$$

$$f'' = \frac{100 \times 20{\cdot}7}{15{\cdot}5 + 100} = 17{\cdot}92^{\text{mm.}}$$

That is to say, the image is formed 17·92 millimetres behind the second principal point, or 1·75 + 0·365 + 17·92 = 20·035 millimetres behind the cornea.

Convergent rays are obtained by means of a convex lens. An eye which sees at a long distance with the aid of a convex lens, uniting rays hitherto parallel at a focus situated 10·75 millimetres behind the cornea, must have its retina situated 17·92 millimetres behind the second principal point, or, in other words, must be 20 millimetres in length (from the cornea to the retina). This, as we shall see farther on, is a hyperopic eye. Hence an eye that is hyperopic to this degree is 22·83 − 20·035 = 2·79 millimetres shorter than an emmetropic eye.

The same result is attained with formula VIII. *c*, which gives $l'' = \frac{15{\cdot}5 \times 20{\cdot}7}{115{\cdot}5} = 2{\cdot}79$ millimetres as the difference in length between the two eyes.

### THE REDUCED EYE.

We have already called attention to the fact that the formulæ given for a system composed of several refracting surfaces (p. 92) are the same as those for a single surface, except that in the first case there are two principal and two nodal points, while in the second case there is only a single principal and a single nodal point. In the compound system the incident rays are, so to say, displaced, relatively to the emergent rays parallel to the axis, by the interval between the two principal or between the two nodal points.

The values F′ and $f'$ and F″ and $f''$ are reckoned from two different points, H′ and H″, while in the simple system they are reckoned from one and the same point.

In the same way the values G′ and $g'$, G″ and $g''$ are reckoned from two nodal points, while, for a single surface, the lines, whose lengths they represent, meet in the single nodal point.

When in a compound system the two principal points, as well as the nodal points, are very near each other, and there is no necessity for being rigorously exact in the calculations, the two principal points may be supposed to be united in one, and the two nodal points may likewise be regarded as a single point. Thus the complex system becomes at once a single one, reduced to a single spherical surface, which separates the air from a more refractive medium.

The union of the principal points (H) corresponds to the summit of the surface, and the union of the two nodal points (K) to the centre of curvature of the surface.

These two points play the same *rôle*, in the simple system, as the four cardinal points in the compound system (see Figures 17, 18, 44, and 45).

According to the choice of the radius of curvature, $H\,K = r$, and of the index of refraction $n$, of the system, any desired absolute or relative values may be given to the focal distances $F'$ and $F''$.

We have only to bear in mind the relations which exist between these values for a surface separating the air (whose index of refraction is 1) from a medium having an index of refraction $n$:

$$F' = \frac{r}{n-1},$$

$$F'' = \frac{n\,r}{n-1},$$

and

$$\frac{F''}{F'} = n.$$

This reduction was made, for the eye, by Listing, upon the basis of his schematic eye.[1] The two foci retain their positions. He places the single principal point between the two principal points, and the single nodal point between the two nodal points, of the schematic eye, so that the ratio between the focal distances, or, what is the same thing, the ratio of the index of refraction of the first medium to that of the last medium, remains the same.

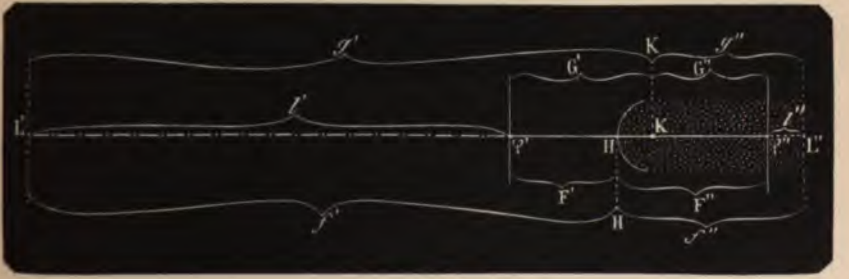

FIG. 48.

Thus the principal point of Listing's reduced eye is 2·3448 millimetres behind the anterior surface of the cornea, and the nodal point 0·4764 millimetres in front of the posterior surface of the crystalline.

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[1] Listing, *Dioptrik des Auges*, in Wagner's *Handwörterbuch der Physiologie*, iv., 473, and Helmholtz, *Handbuch der physiologischen Optik*, p. 60; Donders, *Anomalies of Acc. and Refr.*, p. 175

The radius of curvature of the refractive surface equals 5·1248 millimetres.

The first focal distance is $F' = 15·036$ millimetres; the second focal distance, $F'' = 20·113$ millimetres.

The index of refraction $n = \frac{20·113}{15·036} = \frac{103}{77} =$ the index of refraction of the vitreous body.

To still further simplify the calculations, Donders has suppressed the decimal fractions, and in this way has arrived at a reduced eye, which is represented in its actual size by Fig. 48.

The following are his optical constants, expressed numerically. He admits as:

radius of curvature of the refracting surface or cornea of the reduced eye $r = 5^{\text{mm.}} = \text{H K} = F' - F''$,

first focal distance, $F' = 15^{\text{mm.}} = \phi' \text{H} = \text{K}\, \phi'' = G''$,

second focal distance, $F'' = 20^{\text{mm.}} = \text{H}\, \phi'' = \text{K}\, \phi' = G'$,

hence the index of refraction $n = \frac{F''}{F'} = \frac{4}{3} = 1·3333$. This index of refraction corresponds to that of water.

Calculations with this eye, thus reduced, become so simple that they may be performed mentally. We have indeed already seen this on p. 19, where we chose, as examples, the data of Donders' diagrammatic eye.

This eye, reduced to the greatest simplicity, will assist us, in the course of this work, to render clear the problems of ocular refraction. We shall employ for this purpose the formulæ 8 *a*, 8 *b*, 9 *a*, 9 *b*, and 9 *c*. The values of these formulæ have, for the reduced eye, the following significations:

The second focal distance, $\text{H}\, \phi'' = F'' = 20$ millimetres, corresponds to the length of the *emmetropic reduced eye*, because $\phi''$ is the point of union of rays which were parallel before entering the system, and because emmetropia is characterised by the fact that the second principal focus of the system is situated on the retina.

The luminous rays given off from the first focus $\phi'$, are parallel in the interior of the eye.

$\text{K}\, \phi'' = G'' = 15$ millimetres, is the distance from the nodal point to the retina, in the emmetropic eye.

$f'$ represents the distance from an object to the cornea of the reduced eye, and will correspond to the distance of the *punctum remotum*, of the point to which the ametropic eye is adapted when in a state of repose.

$f''$ will be the distance from the image to the cornea. Hence it will give the length of the ametropic eye, whose *punctum remotum* is situated a distance of $f'$ from the cornea.

$g'$ will correspond to the distance from the *punctum remotum* to the nodal point;

$g''$ to the distance from the nodal point to the retina of the ametropic eye.

As long as $f'$ and $g'$ have positive values, *i.e.*, while the object is situated in front of the eye, and we are dealing with divergent rays, $f''$ and $g''$ will be greater than $F''$ and $G''$. This is the case in myopia.

If $f'$ and $g'$ have negative values, *i.e.*, if they be situated on the same side of the refractive system as the image, the rays with which we have to deal in this case, will be convergent; $f''$ and $g''$ become less than $F''$ and $G''$. This is the case in hyperopia.

In short, $l'$ corresponds to the distance from the luminous point to the first focus, $f' - F'$;

$l''$ to the distance from the image to the second focus, $f'' - F''$. Hence this $l''$ gives also the difference in length between the ametropic and the emmetropic eye.

If $f''$ be greater than $F''$, as in myopia, $l''$ is positive, and the eye is longer than in emmetropia. If $f''$ be shorter than $F''$, as in hyperopia, $l''$ becomes negative,—*i.e.*, the eye is shorter than in emmetropia.

Bearing in mind how the reduced eye originated, it will be easily understood that the refracting surface, which, for simplicity's sake, we call "the cornea of the reduced eye," is not identical with the cornea of the real eye. It corresponds to a plane situated about 2·5 millimetres behind the cornea.

For the size of images it will be seen, according to formula 9 *a*—

$$i = \frac{o\,g''}{g'},$$

that, for the same value of $o$ and $g'$, *i.e.*, for the same size of the object and the same distance from the latter to the eye, the size of the image is proportional to $g''$, *i.e.*, to the distance from the single nodal point of the reduced eye (from the second nodal point of the schematic eye) to the place where the image is formed.

In emmetropia $g'' = G'' = 15$ millimetres for the reduced eye, and 15·5025 millimetres in Helmholtz's schematic eye. Hence the retinal image is necessarily smaller in the first case than in the second. The proportion between them is as 15 is to $15{\cdot}5025 = 1 : 1{\cdot}0335$. It is by this number, therefore, that the size of the retinal image of the emmetropic reduced eye must be multiplied in order to obtain its actual size.

As to the value of $l''$, the difference in length between the emmetropic eye and an eye that is ametropic because of a difference in the length of its axis (*axial ametropia*), it results from the formula $l'' = \frac{F' F''}{l'}$.

The distance $l'$, from the object to the first focus, is the same for the reduced as for the schematic eye, but $F' \times F''$ is $15 \times 20 = 300$ for the reduced eye, and $15{\cdot}5025 \times 20{\cdot}719 = 321{\cdot}196$ for the schematic eye; hence $l''$ is 1·070 times smaller in the reduced than in the schematic eye.

It will be noticed that, in reality, the errors are not great, even for Donders' so considerably reduced eye: they do not, at least, exceed the limits of what is permissible in practice.

We feel justified, therefore, in using this eye for purposes of *demonstration.*

*Ametropia of curvature* is represented in the reduced eye by the change in length of the radius of its refracting surface.

In this case the length of the ametropic reduced eye is the same as that of the emmetropic eye ($f'' = F'' = 20$ millimetres), but the length of its radius of curvature ($r$) is no longer 5 millimetres; it will be greater for hyperopia and less for myopia. It may be found with the aid of the expression taken from the formula I I $c$—

$$r = \frac{f' f'' (n-1)}{f' n + f''},$$

in which

$$n = 1{\cdot}333.$$

*Example.*—What must be its radius of curvature, in order that an eye, retaining the length of the emmetropic eye, may present a *myopia* of $\frac{1}{0^{\mathrm{m}}{\cdot}250} = 4$ dioptries ?

We shall have—

$$r = \frac{250 \times 20 \times 0{\cdot}333}{250 \times 1{\cdot}333 + 20} = 4{\cdot}713^{\mathrm{mm.}}$$

The distance $g''$, from the nodal point to the retina of this myopic eye, will therefore be $g'' = f'' - r = 20 - 4{\cdot}713 = 15{\cdot}287$ millimetres.

The first principal focal distance of this eye will be, conformably with formula IV. $d$—

$$F' = \frac{r}{n-1} = 14{\cdot}15^{\mathrm{mm.}}$$

And, according to the formula IV. $b$., the second principal focal distance is—

$$F'' = \frac{n\,r}{n-1} = 18{\cdot}86^{\mathrm{mm.}}$$

Thus it is that we represent the state of *accommodation* of the emmetropic eye, which is, after all, a sort of *myopia of curvature.* The eye is, in fact, adapted for near vision by an increase in the curvature of one of its refractive surfaces, the length of the eye remaining the same.

By comparing the $g''$ of an axially myopic eye with the $g''$ of an eye myopic by increase of curvature, we obtain the difference in size between the retinal images received by these two eyes.

This difference is the same as that which exists between a myopic eye, adapted to near objects in consequence of an excessive length of its axis, and an emmetropic eye adapted to the same distance by an effort of its accommodation.

Let us now seek to determine the radius of curvature necessary to produce a curval hyperopia of 4 dioptries $= \frac{1}{0{\cdot}25 \text{ m.}}$

In this case $f'$ is negative, and the formula for the radius of curvature becomes—

$$r = \frac{f' f'' (n-1)}{f' n - f''}.$$

In our example—

$$r = \frac{250 \times 20 \times 0{\cdot}333}{250 \times 1{\cdot}333 - 20} = 5{\cdot}315 \text{ mm.};$$

hence

$$g'' = 20 - r = 14{\cdot}685 \text{ mm.}$$

$F'$ and $F''$ remain the same as in the preceding case.

Comparing here the $g''$ of the axially hyperopic eye with that of the curval hyperopia, the latter is found to be the greater. Hence the retinal image must be larger in the latter case than in the former.

An example of curvature-hyperopia is furnished by an eye which, hitherto emmetropic, is deprived of its crystalline. It finds itself, then, with regard to the size of retinal images, in better circumstances than another eye, hyperopic in the same degree, but whose hyperopia is due to a defect in the length of its axis.

## THE ARTIFICIAL EYE.

In the year 1875, I constructed an *artificial eye* whose proportions correspond exactly to the data of Donders' reduced eye.

The reader will now understand the signification and value of this little instrument. Being based upon Donders' reduced eye, which is itself only the classical simplification of the real eye, our artificial eye serves for the *experimental* solution of problems in physiological optics. The formulæ which are the basis of our knowledge of the refraction and accommodation of the eye, may, with its assistance, be verified; but it even answers, without formulæ or construction, the most important questions on this subject. It demonstrates how, by varying its length, an emmetropic eye may be made axially ametropic in any degree, and either myopic or hyperopic. It gives the size of the retinal image received by any such eye, with or without correction, and with any desired correction, *e.g.*, by spectacle glasses placed at any distance, optometers and other optical instruments, or, again, by means of the *accommodation.*

In fact, a convex glass, placed in immediate contact with the cornea of the artificial eye, has the same effect as an increase in the curvature of this refractive surface, equivalent to the accommodation of the normal eye.

*Curvature ametropia* is produced in the same way,—myopia by means of a convex, hypermetropia by means of a concave lens, applied to the refractive surface of the artificial eye.

But we may study, with the aid of this instrument, not only incident rays, but also light coming from the eye; hence it is of service also in *ophthalmoscopy* and for the solution of problems connected with it.

A little figure depicted on the retina forms the object of observation. We have only to light the fundus of the eye from behind in order to render this object visible through the pupil of the artificial eye. In this way, one can judge *directly*, and better than with the aid of the ophthalmoscopic mirror, how the *upright image* is produced, its size, the influence exercised upon it by different states of refraction, by correcting glasses, &c. &c.

In rendering the eye myopic, or by placing before it a strong convex lens, an *inverted image* is produced. This latter can be received upon a ground glass, moveable along the stem of the instrument. The distance at which this image is produced, its size for different forms and degrees of ametropia, and different convex lenses, are, thus, easy to study.

Should it be preferred to place one's self exactly in the conditions of clinical ophthalmoscopy, a small bit of opaque paper is applied at the back of the artificial eye, with the papilla, &c. painted upon it. It is lighted with the mirror of the ophthalmoscope and observed through the pupil of the artificial eye.

In this way the artificial eye has rendered us great service, both for our own instruction and for purposes of demonstration in our lectures.

We do not hesitate, therefore, to recommend it to those who desire to familiarise themselves perfectly with ocular refraction. Experimentation always speaks to us more clearly and vividly than formulæ or even drawings.

It is, however, essential that the instrument furnished by the optician should correspond exactly to our description. It is, above all, necessary that our artificial eye, filled with water, should have a focal length of just 20 millimetres,—*i.e.*, that the image of an object, situated at least 6 metres away, should be formed with perfect distinctness upon the posterior surface of the artificial retina, when this is 20 millimetres distant from the summit of the cornea. From this position it must be possible to shorten or lengthen the eye 3 millimetres, which will give an axial hypermetropia or myopia of 10 dioptries.

In the first case, the retinal image of the distant object is clearest when a convex glass of 10 dioptries is placed 10 millimetres in front of the cornea; in the second case (myopia), the image is most distinct when a concave glass of 10 dioptries is placed at the same distance.

Each artificial eye should be accompanied by a pamphlet containing a description, and several examples, of its workings.

Listing was the first, though not the only one, to propose a reduced eye.

In 1877, Stammeshaus, in a very valuable work, proposed to make the summit (H), of the refractive surface of the reduced eye, correspond to the *second* principal point, and its nodal point to the *second* nodal point of the schematic eye.

The radius of curvature, in the reduced eye suggested by him, equals 5·2152 millimetres, and the index of refraction is 1·3365 (as for the aqueous humor and the vitreous body). The first focal distance is 15·4983 millimetres, the second 20·7135 millimetres.

He thus obtains, for the size of retinal images, values which more nearly correspond to the actual values.

The distance $g''$, from the image to the second nodal point, being much less than that ($g'$) from the object to the first nodal point, differences in the former of these distances influence the size of the image more than differences in $g'$.

In 1879, von Hasner (*Das mittlere Auge*) proposed still another reduced eye. In order to assimilate more closely the values of the radius of curvature and the length of the real eye, he based his reduced eye upon the ratio $\frac{22{\cdot}5}{15}$, *i.e.*, $F'' = 22{\cdot}5$ millimetres, the length of his reduced eye in a state of emmetropia; $F' = 15$ millimetres, the index of refraction is $\frac{22{\cdot}5}{15} = \frac{3}{2} =$ the index of refraction for glass; and the radius of curvature equals $22{\cdot}5 - 15 = 7{\cdot}5$ millimetres. Hence $G'$ is, in von Hasner's reduced eye, as in that of Donders, equal to 15 millimetres.

## Combination of the Eye and a Lens.

It very often happens that the luminous rays do not enter the eye directly, but after having passed through a lens. The optical constants of the eye alone no longer suffice in this case for ascertaining the final direction of rays, or, in other words, for determining either the size of the image or its location. In this case we are dealing with a more complex system, made up of the lens and the eye. We must, therefore, determine the cardinal points and planes for this compound system, before we shall be able to enter upon a consideration of the images received by the retina under such circumstances.

In order to find the optical constants of the compound system, we shall proceed as we did when ascertaining those of the eye, which is itself composed of two dioptric systems. The optical constants will result from a combination of the constants of the two systems. These being once known, they will serve for the construction and calculation of the path of rays, exactly as in the eye (Figs. 44 and 45).

This case may present itself when it is a question of finding the absolute size of the retinal images formed by a magnifying glass. We shall then use the cardinal points of this glass, whose thickness is too great to be disregarded, and of the schematic eye. But it will oftenest be the influence exerted by spectacle glasses upon the size of retinal images that we shall wish to know, and not so much the *absolute* size of such images as the size of one combination relatively to that of another, or of those of two eyes to each other.

In this case the problem is much simpler. We may make use of the reduced eye with a single surface, and consider the lens as infinitely thin.

The formulæ for determining the optical constants of a composite system of this kind are always those (11, 12, 13, and 14) that we have already given.

It is only with a view to saving the reader the trouble of referring to them elsewhere that we repeat them, with an explanatory figure, here. He will, perhaps, often wish to use them simply to ascertain the influence of spectacle glasses upon the size of retinal images.[1]

The lens has, then, a single principal or nodal point at its centre ($a$).

Let F be its focal distance.

Let $e$ be the distance $a\ a$ between the principal point of the first system (the lens) and that of the second (the eye).

In the eye we call—

$a$, the principal point = the summit of the refractive surface;
K, the nodal point = the centre of curvature of the refractive surface;
$\phi'$, the first focus;
$\phi''$, the second focus;
$F' = a\,\phi'$, the first principal focal distance;
$F'' = a\,\phi''$, the second principal focal distance.

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[1] Compare Helmholtz, *Handbuch der physiologischen Optik*, p. 58; formulæ 11 $d$, 11 $e$, and 11 $f$.

In the *compound system* (the lens and eye together) we call—

$h'$, the first principal point;
$h''$, the second principal point;
$t'$, the first focus;
$t''$, the second focus;
$\mathfrak{F}' = h' t'$, the first principal focal distance $= k'' t'' = \gamma''$;
$\mathfrak{F}'' = h'' t''$, the second principal focal distance $= k'' t' = \gamma'$;
$k'$, the first nodal point;
$k''$, the second nodal point.

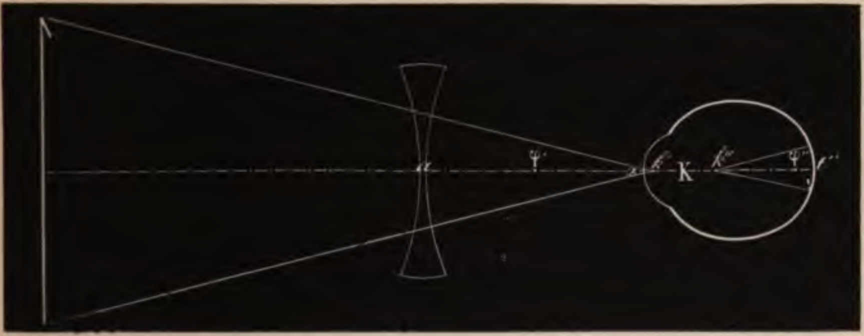

FIG. 49.

We know the optical constants of each of these two systems.

The following are the formulæ which serve to determine the optical constants of the compound system:

$$a h' = \frac{e \mathrm{F}}{e - \mathrm{F}' - \mathrm{F}}$$

$$a h'' = \frac{d \mathrm{F}''}{e - \mathrm{F}' - \mathrm{F}}$$

$$\mathfrak{F}' = \frac{\mathrm{F}' \mathrm{F}}{\mathrm{F}' + \mathrm{F} - e}$$

$$\mathfrak{F}'' = \frac{\mathrm{F}'' \mathrm{F}}{\mathrm{F}' + \mathrm{F} - e}.$$

The location of the points $k'$ and $k''$ results from the fact that $k'' t'' = \mathfrak{F}'$, $k' t' = \mathfrak{F}''$, and $k' k'' = h' h''$.

# CHAPTER II.

## THE REFRACTION OF THE EYE.

By "refraction of the eye" we understand—

1. The dioptric properties of the optical apparatus of the eye in a *state of repose.*
2. The relations of the retina to this dioptric system.

These two parts form the *static refraction* of the eye, or the *refraction* properly so called.

3. *The variations of which the dioptric apparatus is susceptible* in itself and in its relations to the retina.

These two last portions form the *dynamic* refraction (or *accommodation*) of the eye.

It is quite essential not to lose sight of the fact that the retina, destined simply to receive the images furnished by the dioptric apparatus, has no influence upon the formation of these images. The luminous rays are refracted by the dioptric apparatus: the images would be formed quite as well (indeed even better in certain cases), if the retina were not there. The dioptric apparatus and its action are absolutely independent of the retina; hence it will be well to commence by studying it separately.

After having become acquainted with it, we shall add the receptive and perceptive screen for the images—the retina—when we shall see in what manner the visual organ profits by the dioptric action of its refractive apparatus.

So long as the eye is at rest, and its dioptric apparatus, consequently, does not vary, we call the refraction of the eye "*static refraction.*"

But the dioptric apparatus of the eye is susceptible of changes which are produced under the influence of a muscular action (accommodation); this is the "dynamic refraction" of the eye. We shall study it also from the single point of view of the dioptric apparatus at first, and afterwards in its relations to the retina.

# I.—STATIC REFRACTION OF THE EYE.

## I.—DIOPTRIC APPARATUS OF THE EYE.

The CORNEA, the AQUEOUS HUMOR, the LENS, and the VITREOUS BODY form the dioptric apparatus of the eye.

This optical system consists, then, of the transparent media of the eye, and the refracting surfaces which separate them from the air and from each other.

At the beginning of this work we have explained what influence *refracting surfaces* of this kind exercise upon light which passes through them. We have seen that rays diverging from a luminous point are thus rendered less divergent, sometimes even convergent, so that they become reunited, and form behind their surface an *image* of the point. Since this happens with all points of which an object consists, such a dioptric system can produce an image of the whole object where the luminous rays meet.

We have, moreover, seen that the optical action of such an apparatus is stronger in proportion as its surfaces are more curved, and as its component substances are more refractive. The exponent of the degree, in which a transparent medium possesses refractive power, is called its *index of refraction.*

The reader may find the definition of this term, as well as the explanation of the refraction of light, in the first pages of this work, without being obliged to proceed farther in the mathematical part.

The curvatures, of the surfaces of the refracting media, have been determined by means of the reflected images which these surfaces produce. If two objects, having reflecting surfaces of different convexities, such as two convex lenses of different power, be taken, and the reflections of a window, for instance, be observed upon the two surfaces, it will at once be seen that the less convex surface produces the larger reflected image, while the smaller image corresponds to the lesser radius of curvature. There is, therefore, an intimate relation between the size of the reflected image and the curvature, or, to be exact, the *radius* of curvature, of a spherical surface.

It is an experiment of this nature which has been used to determine the curvature of the surfaces of the dioptric apparatus of the eye.

An object of known size has been placed at a given distance from the eye, and the images of reflection of this object, produced by the surfaces of the dioptric apparatus, have been measured.

From the relation existing between the size of the object and that of its reflected image, the length of the radius of curvature has been determined.

These measurements have been made with the aid of certain instruments, called, after Helmholtz, "ophthalmometers." The advantage of these instruments is, that they not only permit very precise measurements, but that the latter are not affected by the slight movements of the eye under examination.

I have given, in the chapter on *Ophthalmometry* (Graefe and Saemisch, *Handbuch der Augenheilkunde*, III., p. 204, and in de Wecker and Landolt, *Traité complet d'ophthalmologie*, I., p. 107), the description of Helmholtz's ophthalmometers,[1] of those of Coccius,[2] Mandelstamm and Schœler,[3] and of my own,[4] together with the methods of applying them, and the processes of measuring the various parts of the eye. MM. Javal and Schjötz[5] have since devised an instrument, based upon the same principle as that of Helmholtz, for determining the curvature of the cornea. Schœler[6] has recently indicated several new, simple, and ingenious methods, by which he has determined the properties of the dioptric apparatus of the eye.

It was Helmholtz who invented the first ophthalmometer. It was he also who made the first observations and calculations, and hence to him is due the credit of having laid the foundation of our knowledge of ocular refraction. His pupils and succeeding observers have merely extended his researches or increased the number of observations. Their labours are, however, none the less valuable, for such measurements are extremely delicate and difficult, and, on the other hand, the solidity of the superstructure can but be augmented by any increase in the number of accurate observations added to its foundation.

The reciprocal distances of the different surfaces of the eye, from the anterior surface of the cornea to that of the crystalline, and the thickness of the latter, have also been measured in the eye of the living subject by means of the ophthalmometer, as well as directly, in the eye after removal.

The indices of refraction of the different media of the eye have been determined by various methods. To obtain that of each of the liquid media, Helmholtz enclosed a certain quantity of the medium which he wished to examine between the concave surface of a plano-concave lens and a flat glass slide. Thus he obtained a plano-convex lens composed of the substance to be examined. The dioptric images, furnished by the lens thus formed, permitted the index of refraction, of the substance composing it, to be determined.

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[1] Helmholtz, "Ueber die Accommodation des Auges," *Arch. f. ophth.*, i. ; 2 p. t., 1854.

[2] Coccius, *Heilanstalt f. arme Augenkranke :* Leipzig, 1872.

[3] Mandelstamm u. Schœler, "Eine neue Methode zur Bestimmung der Optischen Constanten des Auges," *Arch. f. Ophth.*, xviii., 1, p. 155, 1872.

[4] Landolt, "Ophthalmometrie, in de Wecker et Landolt," *Traité complet d'ophthalmologie*, i., p. 770.

[5] Javal et Schjötz, *Ann. d'oc.*, July-August 1881.

[6] Schœler, "Bestimmung des physicalischen Baues des Auges," *Arch. f. ophth.*, xxx., 3, p. 301, 1884.

Other measurements, notably those of the indices of refraction of the various layers of the crystalline, have been made with an apparatus for which we are indebted to Professor Abbe (of Jena): this apparatus permits of making experiments with very small quantities of the substance in question.

Let us now review, with reference to their optical properties, the different parts of the dioptric apparatus of the eye.

*The Cornea.*—The cornea being always covered with a thin film of tears, this film should be regarded as the first refracting surface; but, according to Hirschberg,[1] the index of refraction of the tears differs only very slightly from that of the cornea and of the aqueous humor, and, moreover, this film is so thin that it may, without serious error, be disregarded or considered as one with the cornea.

Strictly speaking, the anterior and posterior surfaces, the substance and the thickness of the cornea, should be separately estimated; but in that portion which chiefly concerns us, viz., the centre, the posterior is nearly parallel with the anterior surface. Moreover, the index of refraction of its substance differs but very slightly from that of the aqueous humor. Hence the cornea may be regarded as forming, with the aqueous humor, one single medium, and its action as one with the action of its anterior surface.

The anterior surface of the cornea is the most important and active of all the surfaces of the dioptric system.

This is not on account of its having the greatest curvature. The curvatures of the posterior surface of the crystalline, and of those of nearly all its layers, are those of spheres of shorter radius; but the cornea separates two media, the air and the aqueous humor, whose indices of refraction differ from each other more than do those of any two contiguous media in the interior of the eye. For this reason the deviation, which luminous rays undergo in passing through the cornea, is greater than that to which they are subject even in passing through the crystalline. It is, moreover, of all the surfaces, the one most accessible for measurement, and the one which has been most thoroughly studied.

The form of the cornea resembles most nearly that of an ellipsoid of revolution, or an ellipsoid of triple axis. The antero-posterior axis is longest; the vertical and horizontal axes, or any other two axes which are perpendicular to each other, are shorter, and may differ quite considerably from each other. The summit of the ellipsoid is pretty near the centre of the cornea. (We shall speak of these differences at the end of this chapter.) Hence the curvature of the cornea is greatest at the centre, and diminishes gradually toward the periphery.

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[1] Hirschberg, *Centralblatt f. med. Wissensch.*, No. 13, 1874.

This form is more favourable to the formation of distinct retinal images than if the cornea were exactly spherical (*vide* Chapter I., pp. 12 and 13).

The radius of curvature of the central part of the cornea varies considerably in different persons, and often in the *two eyes* of the same individual.

We shall see, farther on, when speaking of astigmatism, that even in the same eye the curvature of the cornea is not always equal in all its meridians. In this case the cornea differs still more from an ellipsoid with short equal axes, and becomes more like an oblate ellipsoid, in which three axes, each differing from the other two, are to be taken into account.

But let us first consider the normal condition, in which we may regard the curvature as being the same for all meridians of the cornea.

Mauthner[1] published a case of zonular cataract with myopia of over 20 D and acuteness of vision of $\frac{5}{100}$, in which the cornea had a radius of curvature of only 6·33 millimetres in the axis of vision. He cites elsewhere another case[2] with 8·34 millimetres.

Helmholtz[3] has adopted 7·829 millimetres, in his schematic eye, as the average radius of curvature of the cornea.

Nagel[4] gives 7·6 to 7·7 millimetres as more nearly exact, being the result of measurements made by Donders and Mauthner.

Measurements of the curvature of the cornea have been made in considerable numbers. The most important and most numerous investigations are due to Helmholtz, Donders, Knapp, von Reuss, Mauthner, Woinow, Mandelstamm, Schœler, Reich, Weiss, Javal, and Schjötz.

According to Krause, the thickness of the cornea at its periphery is 1·12 millimetres. Hence the posterior surface is more curved than the anterior; the cornea forms a divergent menicus, but, for reasons which we have given above, its action may be disregarded.

The index of refraction of the *aqueous humor* and cornea is 1·3365.[5]

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[1] Mauthner, *Die opt. Fehler des Auges*, p. 617, 1876.

[2] Mauthner, *loc. cit.*, p. 599.

[3] Reich, *Arch. f. Ophth.*, xx., i., p. 107, 1874.

[4] Nagel, *Anomalien der Ref. u. Acc. in Handb. der ges. Augenhlk. in* Graefe u. Saemisch, vi., p. 282, 1880.

[5] Chossat, Brewster, Krause; comp Helmholtz, *Physiol. Optik*, p. 78.

The distance from the anterior surface of the cornea to the anterior surface of the crystalline is 3·6 millimetres. Deducting the thickness of the cornea (0·9 millimetre at its centre: Krause), we obtain 2·7 millimetres as the depth of what is called *the anterior chamber*.

*The Crystalline Lens.—The anterior surface* of the crystalline lens should, according to Krause, resemble a surface produced by an ellipse revolving about its shorter axis. But the central portion may still be regarded as spherical. Its curvature varies, the average radius of curvature being 10 millimetres (Helmholtz). Knapp has found a radius of 7·9 millimetres, and von Reuss one of 14·06 millimetres.

The *thickness* of the crystalline lens varies from 3·02 millimetres to 4·19 millimetres (von Reuss), the average being 3·6 millimetres (Helmholtz).

The posterior surface of the crystalline has a radius of curvature of 6·0 millimetres (Helmholtz). The greatest curvature found by Helmholtz showed a radius of 5·13 millimetres; the weakest is cited in a case of von Reuss', its radius being equal to 11·33 millimetres.[1]

The crystalline lens is not homogeneous, but is made up of a great number of superimposed layers, whose curvatures increase in passing from the surface toward the centre, so that the nucleus is nearly spherical (Fig. 50).

The index of refraction increases also from without inward, so that the crystalline lens may be regarded as being composed of so many divergent menisci, increasing in power proportionately to their nearness to the nucleus, which latter is a convex lens of a very short radius, and a very high index of refraction.

The effect of such a structure of the crystalline lens is to diminish spherical aberration, *i.e.*, to permit the formation of distinct images even by rays which enter the eye at a considerable angle with the optic axis.[2] A homogenous lens—a glass one, for instance—brings to a single focus only such rays as pass through it very near its axis. The lamellar structure of the crystalline lens peculiarly adapts it for indirect vision, in which retinal images are produced by rays which form considerable angles with the axis of the eye.[3]

The peculiar structure of the crystalline lens has still another effect, viz., to render the dioptric power of this lens greater than if it were homogeneous, indeed even greater than if the index of refraction for all its component parts were as high as that of its nucleus.

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[1] Nagel, *loc cit.*, p. 295.

[2] Helmholtz, *loc. cit.*, p. 72 of the original, p. 95; and Hermann, *Ueber schiefen Durchgang von Strahlenbündeln durch Linsen und über eine darauf bezügliche Eigenschaft der Kristallinse.* Zurich, 1874.

[3] Landolt u. Nuel, *Versuch einer Bestimmung des Knotenpunktes für excentrisch in das Auge fallende Lichtstrahlen* (*Arch. f. Ophth.*, xix., 3, 1873, and *Ann. d'oc.*, Jan. 1874.

It is easy to convince one's self of this. Decompose the crystalline lens into its nucleus and the two divergent menisci[1] which surround it, and whose index of refraction is lower than that of the nucleus (Fig. 51). These two menisci neutralise a part of the positive refractive power of the nucleus. This neutralisation is greater in proportion as the index of refraction of the cortical layers is higher, because the dispersive power of the menisci increases in like proportion.

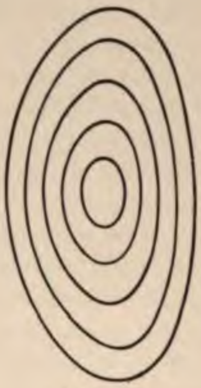

FIG. 50.

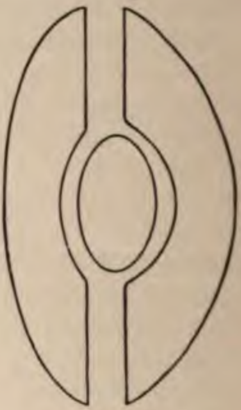

FIG. 51.

Hence the refractive power of the crystalline lens would be less if the index of refraction of the cortical laminæ were equal to that of the nucleus.[2]

The crystalline lens has, therefore, not one general *index of refraction*, but its index increases, as we have just seen, from the peripheral layers toward the centre.

It would lead to considerable complications, and would even be impossible to calculate the course of light successively through these different layers. The crystalline lens has, therefore, been considered as resolved into three parts, a peripheral, an intermediate, and a nuclear, and the index of refraction, for each, has been separately determined.

W. Krause has found, as the index of the external layer, 1·4053, for the intermediate 1·4294, and for the nucleus 1·4541. Woinow gives, for the external lamina, 1·3932; for the intermediate, 1·4199;

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[1] P. 113.

[2] Senff in Volkmann's Art. *Sehen* (*Handwoerterbuch der Physiologie*, vol. iii. 1, p. 290).

and for the central one 1·4315,[1] which he determined in a person sixteen years old.

We are indebted, for the first investigations of this matter, to Senff, Brewster and Helmholtz; and, for the latest ones, to Aubert and Matthiessen.[2]

By means of these data the focal distance of the crystalline lens may be calculated and, if these results be exact, this distance will be identical with the one found directly, experimentally, upon a crystalline lens removed from the eye.

By calculating the course of luminous rays through such a system, we may find what should be the index of refraction of a homogeneous lens, having the same curvature of its surfaces and the same thickness, in order that it may produce the same effect as the crystalline. Again, the focal distance of the crystalline lens, removed from the eye, may be directly determined, and from this, regarding the lens as homogeneous, may be deduced its index of refraction. It is in fact very advantageous, in point of the simplification of optical studies in ophthalmology, to regard the crystalline lens as homogeneous.

Its average index of refraction, thus determined, ought then, according to Helmholtz, to be 1·4371.[3] Aubert and Matthiessen find an index of refraction 1·4480, which is approximately the same as Listing's ($1·4545 = \frac{16}{11}$), which Helmholtz had adopted for his first schematic eye.[4]

In a state of rest the crystalline lens has, in the aqueous humor, a *mean focal distance* of 50·6168 millimetres.

It will be seen hereafter (in the chapter on accommodation) that the crystalline lens is susceptible of variation in form. The radii of curvature and the thickness do not always remain the same. The radius of curvature of its anterior surface changes from 10 to 6 millimetres. This surface advances 0·4 millimetre.

The radius of the posterior surface becomes 5·5 instead of 6 millimetres, but this surface does not change its position.

The thickness of the lens increases from 3·6 to 4 millimetres, and its focal distance, which in a state of rest is 50·6168 millimetres, becomes 39·074 millimetres in a condition of strong accommodation.

*The Vitreous Body.*—The vitreous body may, from an optical point of view, be regarded as homogeneous. Its index of refraction is

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[1] Stammeshaus, *loc. cit.*, p. 128.

[2] Aubert in von Graefe u. Saemisch, *Handbuch der ges. Augenhlkd.*, vol. ii., p. 410, and Matthiessen, *loc. cit.*, p. 181.

[3] Reich, *Arch. f. Ophth.*, xx., 1, p. 297, 1874.

[4] Helmholtz, *Physiol. optik*, p. 68. Compare also v. Zehender und Matthiessen: *Ueber die Brechungscoëfficienten cataractöser Linsensubstanz* (*Klin. Monatsbl.*, pp. 239 and 311, 1877).

1·3365 and is equal, consequently, to that of the cornea and of the aqueous humor. The vitreous body is situated posteriorly to the other media in the dioptric apparatus of the eye. It is in the vitreous body that luminous rays terminate their course, and this property specially distinguishes the dioptric system of the eye from ordinary systems, as lenses, microscopes, &c. In such instruments the light, after having passed through the dioptric system, finally re-enters the medium from which it started, *i.e.*, the air; while in the eye the light remains in another medium, of greater refractive power than the air. For this reason the eye is not directly assimilable to a convex lens, as it has been considered by those who have devised certain models of the eye, but resembles rather a convex surface separating the air from a more refractive medium, as, for instance, in the author's artificial eye (*vide* p. 98, reduced eye; and p. 103, artificial eye).

## ANGLE ALPHA AND ANGLE GAMMA.

Before terminating our study of the dioptric system of the eye, we have yet one question to answer, viz., Are the different surfaces, composing it, well centred? That is to say, regarding these surfaces as engendered by the rotation of ellipses or circles, do all their axes of rotation coincide so as to become merged in one line, or do they form angles with each other?

In the former case, all these surfaces, or at least the portion of their summits with which we are concerned, would be perpendicular to the common optic axis. In the latter, this would not be the case, at least not for all of them.

This investigation will make us acquainted with certain lines and angles, which have given rise to much discussion, controversy, doubt and confusion. It is well to know what to hold to, for the angles in question are of importance not only in physiological optics, but also in the study of the movements of the eyes. Hence we crave the indulgence of the reader if, in order to be explicit, we are obliged to enter into details which may, perhaps, appear somewhat minute and dry; but it is very important to give, once for all, a clear explanation of this matter.

Let A A′ (Fig. 52) be the *optic axis of the eye*, which may be regarded as passing through the centre of the cornea C and the posterior pole of the eye. It contains the cardinal points of the entire dioptric system (pp. 32 and 89), the principal points H′ and H″, the nodal points K′ and K″, the foci $\phi'$ and $\phi''$, and also the centre of rotation (M) of the eye. This last is situated 13·73 millimetres, as an average, back of the apex of the cornea (9·2 millimetres in front of the retina), hence 5·41 millimetres behind the second nodal point K″, and about 2 millimetres from the middle of the optic axis.[1]

The *line of vision*, O F, which connects the point of fixation with the *fovea centralis*, does not coincide with the optic axis. It has, in common with the latter, only the nodal points K′ and K″, where it crosses the optic axis. The visual line is, therefore, a secondary axis (p. 23).

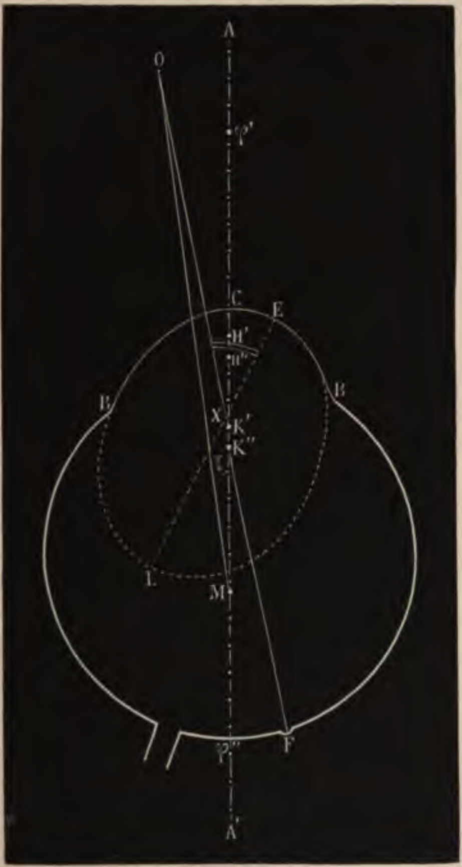

FIG. 52.

The figure is made very schematic, in order that the angles $\alpha$ and $\gamma$ may be rendered more apparent. A A′, optic axis; $\phi'$, anterior focus; $\phi''$, posterior focus; H′ H″, principal points; K′ K″, nodal points; M, centre of rotation; C, centre of the cornea; B B, base of the cornea; E L, major axis of the corneal ellipsoid; F, *fovea centralis;* O, point of fixation; K′ O, line of vision; M O, line of fixation; O X E, angle $\alpha$; O M A, angle $\gamma$.

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[1] Mauthner, *loc. cit.*, p. 646.

The line O M, which connects the object of fixation with the centre of rotation of the eye, is called the *line of fixation.* It is not identical with the line of vision, because of the distance which separates the centre of rotation from the nodal points.

The apex (E) of the corneal ellipsoid does not coincide with the centre (C) of the cornea. Neither does the major axis (E L) of the ellipse, therefore, coincide with the axis of the eye.

*The angle* (O X E, Fig. 52) *formed by the line of vision with the major axis of the corneal ellipse, is called the angle* $\alpha$.

Hence, the apex of this angle has nothing to do with the nodal points. It is only when the axis of the eye and the major axis of the corneal ellipse coincide, that its apex is at the first nodal point K′. This is quite possible, for the shape of the cornea varies considerably, and it may easily happen that the summit of the cornea and that of the ellipse coincide, or differ but very little.

The angle $\alpha$ is called positive (+) when the anterior portion of the corneal axis is situated to the *outer* (temporal) side of the line of vision, as in the figure, which represents a right eye seen from above. This is most frequently the case.

When the axis of the cornea and the line of vision coincide, there is no angle $\alpha$; in other words, it equals zero.

When the anterior portion of the corneal axis passes to the *inner* (nasal) side of the line of vision, the angle $\alpha$ takes the minus sign (−): it is then negative. The angle $\alpha$ may be as great as 12 degrees in the horizontal plane. The divergence of the two lines which form this angle is generally much less in the vertical plane.

*The angle formed by the line of fixation with the axis of the eye* is called the angle $\gamma$. This is the angle O M A in the figure.

Its apex is at the centre of rotation of the eye. It may be as great as 10 degrees, and may be positive, negative, or *nil*, according as the line of fixation is to the inner or outer side of the optic axis of the eye, or as the two coincide.

The angles $\alpha$ and $\gamma$ may differ considerably in the same eye. Oftentimes one of them is small, while the other is great (Dobrowolsky, Mauthner).[1]

The angle $\gamma$ is especially important to consider in determining the excursions of the eye, the angle of strabismus, &c.

If the optic axis and that of the corneal ellipse coincide or may be regarded as coincident, as is done in practice, the angles $\alpha$ and $\gamma$ increase and diminish together, and differ less from each other in

[1] Helmholtz, *loc. cit.*, p. 70 of the original; Donders, *loc. cit.*, pp. 184 and 248; Woinow, *Ophthalmometrie*, p. 48; Mauthner, *loc. cit.*, pp. 61 and 589; compare also Landolt, *Ophthalmométrie* in de Wecker et Landolt, *Traité complet, &c.*, vol. i., p. 748.

proportion as the object of fixation is farther removed from the eye. If we suppose the object O, in the figure, to be placed at an infinite distance from the eye, the two angles become equal.[1]

The *summit of the anterior surface of the crystalline* does not always coincide exactly with the corneal axis. Helmholtz[2] and Knapp[3] have found that the summit of the lens is sometimes situated as much as 2 degrees to the outer (temporal) side of the corneal axis.

The *centre of the pupil* is sometimes on the nasal (Helmholtz, Knapp), sometimes on the temporal side of the corneal axis (v. Reuss). The latter authority, moreover, has found the centre of the pupil always to the inner side of the *line of vision.*

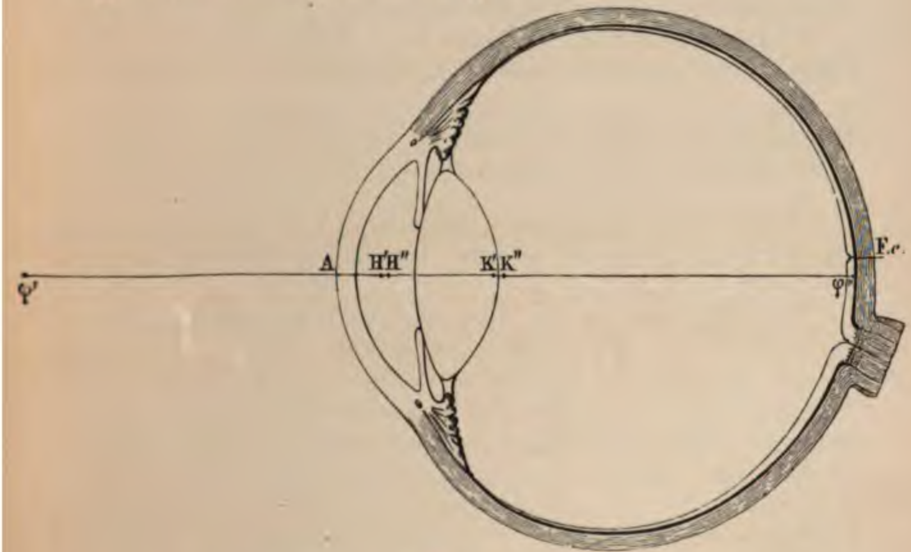

FIG. 53.

Schematic eye, three times natural size. $\phi'$, anterior or first principal focus; A, anterior surface of the cornea; H' and H", principal points; K' and K", nodal points; $\phi''$, posterior or second principal focus; F c, *fovea centralis;* $\phi'\ \phi''$, optic axis.

Recapitulation:

The average dimensions of the dioptric system of the eye, according to Helmholtz's most recent conclusions, are—

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[1] Besides the line of vision and that of fixation, a third line is known in physiological optics. It passes from the point of fixation through the centre of the pupil, and has been called "Visirlinie" by Helmholtz, *line of sight* by Knapp (*Arch. of Ophthal. and Otol.* iv., p. 373, 1874).

[2] Helmholtz, *Arch. f. Ophth.*, i., 2, p. 36, 1854; *Handbuch der physiologischen optik*, p. 86.

[3] Knapp, *Arch. f. Ophth.*, vi., 2, p. 1, 1860.

Index of refraction of the cornea, aqueous humor, and vitreous body, 1·3365.

Total index of refraction of the crystalline lens, 1·4371.

Radius of curvature of the cornea, 7·829 millimetres.

Radius of curvature of the anterior surface of the crystalline lens, 10·0 millimetres.

Radius of curvature of the posterior surface of the crystalline lens, 6·0 millimetres.

Distance from the apex of the cornea to the anterior surface of the crystalline lens, 3·6 millimetres.

Distance from the apex of the cornea to the posterior surface of the crystalline lens, 7·2 millimetres.

Thickness of the lens, 3·6 millimetres.

This is what is called the schematic eye.

We reproduce here (Fig. 53) the figure of the schematic eye, three times enlarged, which we have given heretofore.

## II.—THE DIOPTRIC SYSTEM OF THE EYE IN ITS RELATIONS TO THE RETINA—EMMETROPIA, AMETROPIA.

The preceding chapter has acquainted us with the construction of the dioptric apparatus of the eye. We shall now consider its action and purpose.

Bearing in mind the fact that the dioptric apparatus is composed essentially of a convex surface, which separates the air from a more refractive medium, and of a powerful convex lens, it is easy to understand that its action must be that of a collective system—such as a magnifying glass, for instance; that is to say, luminous rays may be rendered convergent by this system, and united in a real image. This is what actually takes place, at least for rays not too divergent, *i.e.*, rays emanating from objects not too close to the eye.

We have, in the first chapter, studied the course of luminous rays, through the dioptric apparatus of the eye, and have seen that such a system as the schematic eye unites parallel rays at a focus situated 23 millimetres (22·8236 millimetres exactly) behind the anterior surface of the cornea. This focus is called, as we know, the posterior principal, or second principal focus ($\phi''$, Fig. 53).

Parallel rays are such as come from a luminous point situated at infinity, or, in ophthalmological practice, at a distance of 5 metres or more. The angle of divergence, of rays coming from such a distance, may, for our purposes, be disregarded.

If the luminous point be situated on the axis of the eye, the rays given off from it will also be focused upon this axis behind the optical apparatus of the eye. If the luminous point be situated without the principal axis, its image will be formed upon a secondary axis, *i.e.*, on a straight line drawn from the luminous point through another point situated in the interior of the eye, and designated the *nodal point.*[1] In fact, on connecting, by straight lines, each point of which an object is composed with the corresponding point of the image, all these lines cross each other almost at the same point, which is therefore called the "nodal point."

This point, which corresponds to the union of the points K′ and K″ (Fig. 52), is situated about 7 millimetres back of the surface of the cornea; hence $23-7=16$ millimetres in front of the second focus. It is characterised by the fact that every luminous ray, which is directed toward it, passes through the dioptric system of the eye without undergoing deviation. A luminous point, situated *above* the principal axis, forms its image *below* this axis, in the interior of the eye; and, inversely, the image of a point *above* the principal axis will be formed *below* this axis, in the interior of the eye, inasmuch as the lines on which the images are formed intersect at the nodal point C (Fig. 12). If, instead of two points, we have an object, the latter may be considered as composed of an infinity of luminous points. All of them form their images between the two extreme points on the opposite side of the axis. This is why real images produced by the dioptric system of the eye, like those formed by every dioptric system, are *inverted* relatively to their objects.

## EMMETROPIA.

The purpose of the dioptric system of the eye is to form, upon the retina, images of external objects, and thus bring the individual into relation with the surrounding world. In order that this may be accomplished for distant objects, the retina must be located at the second principal focus of the dioptric system, or in the second focal plane, if we so term the plane passing, through this focus, perpendicularly to the axis.

This condition of the eye has been adopted as the standard condition of the eye, and is called *emmetropia* (Donders).

Hence we may say *emmetropia* is characterised by the fact that *the retina is situated at the focus of the dioptric system of the eye;* or, *the emmetropic eye is one which focuses parallel rays upon its retina;*

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[1] Those who have read the first chapter know that there are in reality *two* nodal points, the first of which is 6·9 millimetres, and the second 7·3 millimetres, behind the anterior surface of the cornea; and that rays directed toward the first one continue their course in a direction parallel to that which they first took, and as if they emanated from the second nodal point. But in practice we may suppose the two nodal points to be merged in a single one.

or, again, *one which, in a state of repose, is adapted to infinite distance.* (An eye is said to be *adapted* to a distance when it receives upon its retina distinct images of objects placed at that distance).

It has been shown (p. 11) that the same effect, that is produced by the complicated dioptric system of the eye, may be produced by means of a single convex surface separating the air from a more refractive medium. Thus is obtained the reduced eye of which the author's artificial eye is a faithful reproduction. Its refractive surface has a radius of curvature of 5 millimetres. It is filled with aqueous or vitreous humor, or with water, all of these substances having the same index of refraction. Under these conditions its principal focal distance H $\phi''$ (Fig. 54) equals 20 millimetres.

Hence emmetropia is represented in the eye upon which, because of its simplicity, we base the following discussions, when its retina is 20 millimetres behind its refractive surface, at the focus $\phi''$. This is the condition represented by Figure 54.

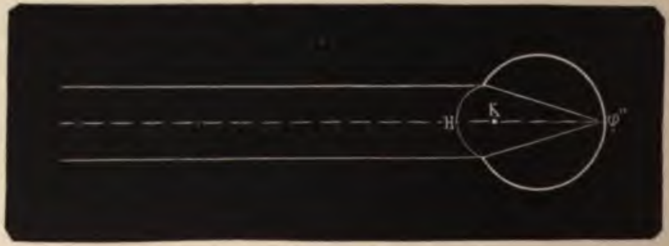

FIG. 54.

We have said, when speaking of the form of the crystalline lens, that the latter becomes more convex upon quitting, under the influence of muscular action, a state of rest. In this case the refractive power of the eye is necessarily increased. In a state of rest it has its least refractive power, and is adapted to the most distant point that it can see.

The name *punctum remotum* is given to the point to which the eye is adapted when at rest. Hence we may also say—

*The emmetropic eye is one whose* punctum remotum *is situated at infinity.*

The schematic, emmetropic eye ought, evidently, to measure 22·8 millimetres from the summit of the cornea to the *fovea centralis* of the retina. This is the distance from the second focus to the first refractive surface. Adding to this the combined thickness of the choroid and sclerotic, we obtain $22·8+1·3=24·1$ millimetres as the total length of emmetropic, schematic or average eye, from the summit of the cornea to the posterior surface of the sclerotic. As a matter of fact, the dimensions of emmetropic eyes differ but slightly from these, for the reason that the dioptric system is fairly constant.

But it results from our definition that emmetropia does not consist in a given power of the dioptric apparatus, nor in an invariable length of the eye, but in a *relation* between the refractive power and the length of the eye. An emmetropic eye might have a much more powerful dioptric system than that of the schematic eye; instead of a focal distance of 23 millimetres it might have, for example, one of only 20 millimetres, provided it were proportionately shorter, so that, having a focal distance of only 20 millimetres, its focus fell upon the retina. Conversely, we may conceive of a longer emmetropic eye, whose dioptric system is correspondingly weaker. In short, an eye is always emmetropic if its principal focus fall upon its retina.

A well-known law of optics teaches that when rays emanating from a point A are focused, by a dioptric system, at a point B, rays which are given off from B will, inversely, be focused at A; in other words, the image may be substituted for the object, and the object for the image, indifferently.

This may easily be verified. Take a convex lens of 20 dioptries, for instance. Place on one side of it a small, lighted candle at a distance of 8 centimetres, and on the other a paper screen. Move the latter until the image of the candle flame appears distinct upon it, which will occur, in our example, when the screen is about 13 centimetres from the lens. Substitute, now, the candle for the screen (*i.e.*, the object for the image), and *vice versâ*, and it will at once be seen that the image is now produced at exactly the place where the candle formerly stood.

Applying this law to the emmetropic eye, it is plain that, if a luminous point be located upon the retina, rays emanating from it will be parallel after leaving the eye, since, in order to form a distinct image on the retina, rays must be parallel before entering the eye, *i.e.*, must come from an object situated at infinity.

As the luminous point is brought nearer the dioptric apparatus, its image recedes. Indeed, rays emanating from the luminous point L′ (Fig. 55) situated at a finite distance from the eye, are no longer parallel, but divergent in inverse ratio to the distance of the object from the eye. The more divergent they are before reaching the dioptric system, the farther away from it will they be focused. The point of reunion L″ recedes farther and farther behind the principal focus the nearer the object approaches. This is easy to verify with the lens, the light, and the screen above mentioned. The nearer we bring the flame, the more we have to remove the screen from the lens, in order to obtain a distinct image. Inasmuch as the retina cannot follow this movement of the image, but remains in its position, a luminous point, brought nearer the emmetropic eye, no longer forms a well-defined image upon its retina, but a circle of diffusion. One of its diameters, A B, is shown in the figure.

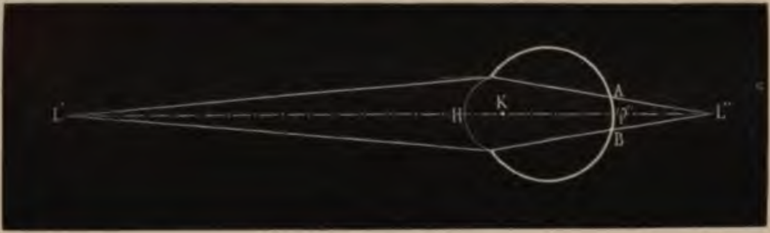

Fig. 55.

The image of diffusion of a luminous point formed by spherical surfaces is a circle, since that which takes place in the plane of the paper takes place similarly in the other planes of the eye. Hence the figure must be imagined as having been revolved about the line L′ L″ as an axis, in order that one may obtain a correct idea of the luminous cones corresponding to L′ and L″, and of the circle of diffusion.

That which occurs in the case of a single point takes place as well in the case of every point situated at a *like* distance from the eye. As we have already stated, an object may be regarded as made up of an infinity of luminous points. If it be situated at a finite distance from an emmetropic eye, each of these points forms upon the retina a circle of diffusion; consequently the entire object forms a diffused, and not a distinct, retinal image.

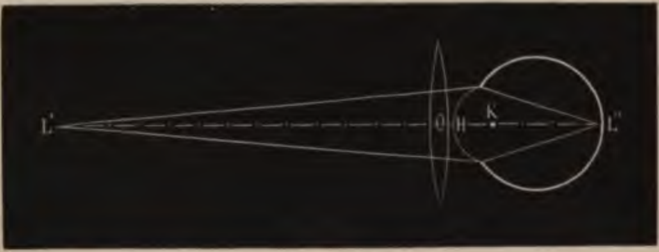

Fig. 56.

In order that an emmetropic eye may see clearly at the short distance of L′, the rays emanating from this point must be rendered parallel, since only parallel rays are focused by an emmetropic eye in a state of rest, upon its retina. This can be effected by means of a *convex lens.* However, the lens must be of such a power, and so placed, that its focus shall coincide with the point L′. It will then give two rays, emanating from L′, a parallel direction, as in the case of all rays emanating from its focus (compare p. 123).

Let us suppose L′ to be ½ metre in front of the emmetropic eye, and that we place a convex glass very near the eye. This glass should have a focal distance of ½ metre, if it is to render parallel the rays which come from L′, or, what amounts to the same, if it is to adapt the emmetropic eye to the distance of L′.

We may consider the matter in still another way, and say that the dioptric apparatus of the emmetropic eye is too weak, when at rest, to permit of distinct vision at so short a distance, and that we increase its power by adding to it a lens having a refractive power of $\frac{1}{1^{m}/_{2}} = 2$ dioptries (see paragraph on the dioptry, in Chapter I.).

## AMETROPIA.

Every eye which is not emmetropic is called *ametropic.* In *ametropia, the retina is either* BEHIND or IN FRONT OF *the principal focus of the eye;* parallel rays are focused either in front of or behind the retina.

Hence the *punctum remotum of the ametropic eye is not situated at infinity.* The ametropic eye does not see distant objects distinctly.

Conversely, *rays coming from the retina of an ametropic eye are not parallel.* Let us take the first case in point.

## MYOPIA.

*Definition.—Myopia is that state of refraction of the eye, in which the retina is situated* BEHIND *the principal focus of the eye.*

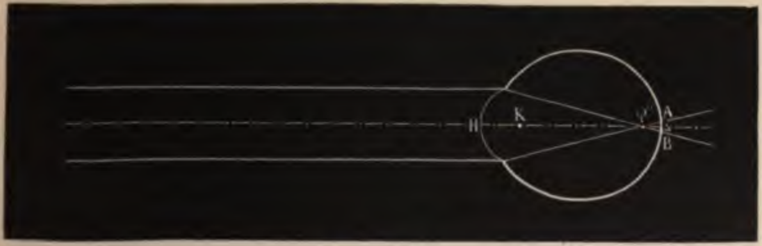

FIG. 57.

Parallel rays, which have already been united at the focus φ″ (Fig. 57), continue their course, now a divergent one, and, when they reach the retina, form, upon it, a circle of diffusion A B, instead of a well-defined image.

In order that this image may become distinct, the luminous point must be brought nearer the eye. Its image then recedes from the principal focus, as was shown above, and finally reaches the retina.

Thus, upon bringing the luminous point from infinity to the point R (Fig. 58), its image moves from $\phi''$ to N, and is then just on the retina. Since it forms, here, a distinct image, the point is now seen distinctly.

An eye whose retina is beyond the focus of its dioptric system is, therefore, adapted to a luminous point situated at a *finite distance in front of it.*

This condition of the eye is called *myopia.* This is an old name, dating from a pre-ophthalmological epoch. It bears no direct relation to the refractive condition which it is made to designate. Its etymology is μυεῖν (to half-close, to wink), and ὤψ (eye). Donders[1] proposed the name *brachymetropia*, because these eyes are adapted to a short distance; but this new name has not succeeded in replacing the old one, which had been too long in use to be easily set aside. It would, however, be more like the common expressions of similar signification, "*short-sighted,*" *vue basse, kurzsichtig*, and would indicate, in a measure, the nature of the anomaly which it designates.

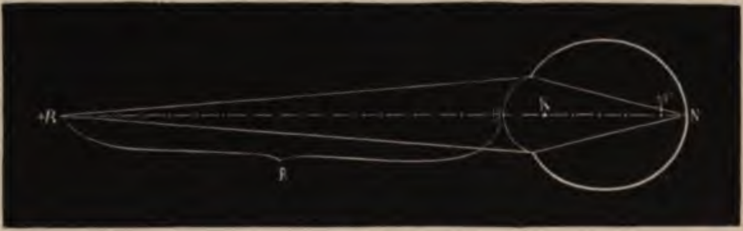

FIG. 58.

*Different forms of Myopia.*—It will be noticed that the definition of myopia does not say anything either of the absolute refractive power of the eye or of its length. It states only that the retina is situated beyond the principal focus, or that the focus falls in front of the retina, in consequence of which the eye is adapted to a finite distance.

Hence we may imagine a myopic eye having exactly the same refractive power as the schematic, emmetropic or normal eye, but which is, as it is represented in the preceding figures, longer. The myopia of such an eye is therefore due to an excess in the length of its axis, and may be designated by the name of *axial myopia*, $M^a$.

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[1] Donders, *Anomalies, &c.*, p. 82.

On the other hand, the same optical effect may be produced without change in the length of the normal eye, by increasing the refractive power of the dioptric apparatus, either by an increase in the convexity of the cornea or the surfaces of the crystalline lens, or by advancing the lens, or by an increase in its index of refraction, or in that of the aqueous humor, or, finally, by a diminution in that of the vitreous body.

In all these cases parallel rays are focused nearer the dioptric apparatus, *i.e.*, in front of the retina.

Cases of this kind have been observed, especially of myopia due to excessive curvature of the refractive surfaces. Hence we may speak of a *myopia of curvature*, $M^c$.

At other times we notice an increase in the refractive power of the eye, in consequence of a change in the index of refraction of the crystalline lens (increase), or in that of the vitreous body (diminution). Myopia, consequent upon such a change, may be called *myopia due to alteration in index of refraction*, $M^i$.

Axial myopia is by far the most frequent form. We shall find abundant proof of this in the clinical chapter. Cases of myopia of curvature, or by alteration of the index of refraction, are rather exceptional and, although mixed forms may be met with, it is safer to regard the dioptric apparatus as constant, and the axis of the eye as variable. Hence a myopic eye may be regarded as having the same dioptric power with, but as being longer than, an emmetropic eye, and this assumption will generally be correct.

*Degree of Myopia.*—A myopic eye is evidently more myopic in proportion as it differs from emmetropia; in other words, as its retina is situated farther behind the focus of its dioptric system, or, again, in proportion as the point to which it is adapted, its *punctum remotum*, is nearer.

It is, indeed, *the distance of this "farthest point"*[1] *from the eye* which affords the simplest and most rational measure of the *degree of ametropia.* Myopia increasing as this distance diminishes, *i.e.*, as the farthest point approaches, the degree of myopia is not directly, but *inversely* proportional to the distance of the *punctum remotum* from the eye. If the *punctum remotum* is situated 25 centimetres, or ¼ of a metre away, the myopia equals $\frac{1}{25^{\text{cm.}}}$ or $\frac{1}{1^{\text{m.}}/_4} = 4$ dioptries; if it is 50 centimetres, or half a metre away, the myopia is $\frac{1}{\cdot 50^{\text{cm.}}} = \frac{1^{\text{m.}}}{1^{\text{m.}}/_2} = 2$ dioptries; if the point be 1 metre distant, the myopia will be $\frac{1}{1^{\text{m.}}} =$ 1 dioptry.

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[1] The farthest point of distinct vision.

Mathematically speaking, it is not sufficient to say "the distance of the *punctum remotum* from the eye," since the eye is not a point. It must be specified from what point of the eye we commence to compute the distance of the *punctum remotum*, the inverse of which gives us the measure of the ametropia. It being a question of mathematics, we choose one of the cardinal points of the eye, and not an anatomical point—as, for example, the anterior surface of the cornea, whose position does not enter into any of the calculations concerning the dioptric apparatus (see page 89).

Donders reckons this distance from the anterior nodal point K′. He takes, therefore, to measure the degree of ametropia, not $f'$, but the value which we have called $g'$ in Chapter I. This facilitates calculation of the size of images.

We prefer to reckon from the *first principal point* H′. Hence we use the value $f'$ to express the degree of ametropia. Our reason for so doing is that the position of the principal points in the eye is more constant than that of the nodal points. The latter varies under the influence of the accommodation. Moreover, the first principal point is nearer to an accessible part of the eye, only 1·75 millimetres separating it from the anterior surface of the cornea, while the first nodal point is 7·32 millimetres from the cornea. Hence, if in practice we calculate the distance of the *punctum remotum* from the cornea, we do not thereby commit a serious error.

Moreover, in aphakia, where there is but one principal point, *i.e.*, the summit of the cornea, the theoretical and the practical method coincide, whereas the nodal point, that is to say, the centre of curvature of the cornea, is 7·8 millimetres farther back, and not accessible to direct measurement. Donders has, indeed, abandoned, in aphakia, the nodal point as the starting-point in measuring the degree of ametropia, and prefers in this case the principal point.[1] In the reduced, as also in our artificial eye, we therefore commence at the *refractive surface* to compute the distance of the *punctum remotum*.

Let us designate, once for all, the distance from the *punctum remotum* to the eye by R (*R* H, Fig. 58). This is the length which in preceding discussions we have called $f'$. The degree of myopia will be $\frac{1}{f'}$ or $\frac{1}{\text{R}}$.

We see, then, that the expression of the degree of myopia is identical with that of the refractive power of a lens. Instead, therefore, of speaking of a myopia of $\frac{1}{25^{\text{cm.}}}$, we measure the distance R with the metre, and call the myopia one of $\frac{1}{1^{\text{m.}}/_4} = 4$ dioptries. For $\text{M} = \frac{1}{0{\cdot}50}$ we write $\text{M} = 2$ dioptries, and for $\text{M} = \frac{1}{1}$, we say a myopia of 1 dioptry.

The *punctum remotum* of a myope being 2 metres distant, his myopia will equal $\frac{1}{2^{\text{m.}}}$, or 0·5 D.

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[1] Donders, *Anomalies, &c.*, p. 311.

For emmetropia, in which the far point is at infinity, R is infinitely great, and we obtain, according to the same principle, the expression $E = \frac{1}{R} = \frac{1}{\infty} = 0$.

The analogy, between a lens and the degree of myopia, becomes still more evident when we compare the adaptation of the myope with the emmetrope's adaptation to a short distance. Let us take the case of a myopic person whose *punctum remotum* is 25 centimetres distant ($M = \frac{1}{1^{m}/_{4}} = 4$ D). In order to see at the same distance, an emmetrope needs a convex glass having a focal distance of 25 centimetres, or $\frac{1}{4}$ of a metre, and placed close to the eye.

Hence myopia has the same effect upon *the adaptation of the eye* as a convex glass of a focal distance equal to the distance of the *punctum remotum*, this glass and the eye being considered as forming one whole.

Since the myopic eye is comparable to an emmetropic eye provided with a convex glass, it is but logical to express the degree of myopia by the strength of the convex glass which renders an emmetropic eye myopic. We say, therefore, that an eye has a myopia of 5 D, when it is adapted to the focal distance of 5 D $= \frac{1^{m.}}{5}$ or $\frac{100^{cm.}}{5} =$ 20 centimetres. The *punctum remotum* of a myope of 16 D, is $\frac{1^{m.}}{16}$ or $\frac{1000^{mm.}}{16} =$ 62 millimetres in front of the cornea.

The analogy between a myopic eye and an emmetropic eye furnished with a convex lens placed almost in contact with the cornea, is perfect only in so far as the adaptation of the eye is concerned. A reservation must be made with regard to the size of retinal images.

Thus an emmetropic eye, to the cornea of which is applied a convex lens of 4 D, is adapted to 25 centimetres by the increase in the refractive power of its dioptric system. It has not changed in length, and, nevertheless, it focuses divergent rays upon its retina. Hence it is comparable to an eye myopic from increase in refractive power, for instance, from excessive curvature, but not to an eye myopic from prolongation of its axis.

The latter is adapted to the same distance as an eye myopic from excessive curvature, or an emmetropic eye provided with a convex glass. The retinal images are well defined in each of the three cases; but they are not of the same size. We shall see this in the chapter which treats of the influence of correcting glasses upon the size of retinal images.

To those who do not care for new calculations, the author's artificial eye will render these differences at once apparent. Place, for instance, an object 15·3 millimetres in diameter, 115 millimetres from the cornea of this eye. In a condition of emmetropia, it is necessary, in order to obtain a distinct image, to place a convex lens, having a focal distance of 115 millimetres, in contact with its refractive surfaces. This lens produces the same effect as if the emmetropic eye had become myopic from excessive curvature, or as if it made an effort of accommodation. The retinal image will have a diameter of 2 millimetres.

In order to cause the eye to see at the same distance, without changing its refractive power, we are obliged to increase its length 3 millimetres. It has now an *axial myopia* of $\frac{1}{0^{m.}115}$. The retinal image is still distinct, but is 2·3 millimetres in diameter.

The farther we remove the convex lens from the emmetropic eye, the greater becomes the size of the retinal image. In order to obtain, for the emmetropic eye, an image of the same size as that of an axially myopic eye, the convex glass must be placed 15 millimetres in front of its refractive surface, at the point $\phi'$ (see p. 89).

In order to cause a myopic eye to see clearly at a distance, parallel rays must evidently be rendered as *divergent* as if they came from the *punctum remotum* of that eye.

This is possible with the aid of a concave lens. Such a lens renders parallel rays as divergent as if they emanated from its focus. Let us place before the myopic eye a concave lens whose focus ($F$) coincides with the far point ($+R$) of the eye. Parallel rays, after having passed through the lens, will diverge as if they came from $R$. The myopic eye will focus them upon its retina, and will, consequently, see as well at a distance as the emmetropic eye.

This lens may therefore be said to have *corrected* the ametropia of this eye.

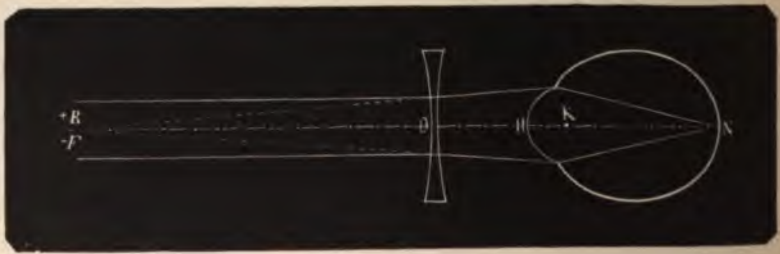

FIG. 59.

What should be the focal distance of this *correcting lens?* This depends evidently upon the distance, from the eye, at which we place it. The focal distance must be less in proportion as the lens is placed farther from the eye, *i.e.*, nearer to its *punctum remotum*, since the focus of the lens must coincide with this point.

Let the distance ($R$ H) from the *punctum remotum* to the eye equal 50 centimetres. If we place the correcting glass at a distance of O H = 30 centimetres from the eye, it will be at $R\,\mathrm{H} - \mathrm{O\,H} = \mathrm{O}\,R$, $50 - 30 = 20$ centimetres from the *punctum remotum*. Hence its focal distance ought to be 20 centimetres, or $\frac{1\text{ m.}}{5}$. Its refractive power is, therefore, 5 D.

If it be placed 10 centimetres from the eye (O H = 10 centimetres), it will need to have a focal distance of $50-10=40$ centimetres, and a refractive power of $\frac{100}{40}=2{\cdot}5$ D.

Placed 5 centimetres from the eye, it is $50-5=45$ centimetres from the far point. To this distance corresponds a refractive power of $\frac{100}{45}=2{\cdot}2$ D.

Finally, if we bring the lens close to the eye, its focal distance (F) must be just equal to that of the far point ($R$), *i.e.*, 50 centimetres, or $\frac{1\text{ m.}}{2}$, and its refractive power will be 2 D in our example. Thus, when the concave lens is sufficiently near the eye, so that the two may be regarded as united, its focal distance must be equal to the distance of the *punctum remotum*, and its refractive power is equal to the positive focal distance of the convex lens which would render an emmetropic eye myopic in the same degree.

From this it results that the correcting glass, *i.e.*, the one which adapts the myopic eye for distant vision, may serve to determine the degree of the myopia; with the condition, however, that the glass be placed quite close to the eye. An eye which sees at a distance only with the aid of the concave lens number 12 D, has a myopia of 12 D, and its *punctum remotum* is situated $\frac{1}{12}$ of a metre, or $\frac{1000\text{ mm.}}{12}=$ 83 millimetres in front of it.

We repeat that, in order that this conclusion be admissible, the correcting glass must be near enough to the eye so that we may regard them as united. Now, in practice, we generally place the correcting glasses about 15 millimetres in front of the eye.[1] Hence the focal distance of the correcting concave glass will always be 15 millimetres less than the distance of the *punctum remotum* from the eye. The focal distance being the inverse of the refractive power, the concave lens will, therefore, always be stronger than the myopia which it corrects.

Let the distance of the *punctum remotum* from the myopic eye be 125 millimetres ($M=\frac{1000}{125}=8$ D). The correcting glass, placed 15 millimetres in front of the eye, will need to have a focal distance of $125-15=110$ millimetres, which corresponds to 9 D. The concave number 9 therefore corrects, under these conditions, a myopia of 8 D. A difference of 1 D, or 15 millimetres, in 125 millimetres of focal distance, is certainly not to be disregarded.

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[1] We admit 15 millimetres in front of the first principal point H′, which is equivalent to 13 millimetres in front of the cornea.

This difference, between the degree of the correcting glass and that of the myopia, becomes greater in proportion as the *punctum remotum* is nearer the eye—that is to say, as the degree of myopia increases. Thus, the *punctum remotum* of an eye whose myopia amounts to 20 D, is 50 millimetres in front of it. The correcting glass, placed 15 millimetres from the eye, must therefore have a focal distance of $50-15=35$ millimetres, which gives $\frac{1000}{35}=28{\cdot}5$ D.

On the contrary, the weaker the myopia, the less the difference between its degree and the strength of the correcting glass becomes, because the distance of the *punctum remotum* from the eye increases. The 15 millimetres become less and less noticeable in proportion to this distance. A myopia of 5·5 D ($R=182-15=167$ millimetres) would demand a glass having a focal distance of 167 millimetres, *i.e.*, one of nearly 6 D. In this case, therefore, the difference is one of only half a dioptry. From this degree on, the difference will decrease still more rapidly. At 2 D it amounts to only 0·06 D, and may be entirely disregarded.

In order to avoid these errors when treating of the higher degrees of myopia, one should always make it apparent whether the degree spoken of is the real degree of the myopia, or that of the lens which corrects it.

The glass placed 15 millimetres in front of the eye is usually designated the correcting glass. Hence, in order to determine the real degree of myopia, 15 millimetres should be added to the focal distance of the glass which serves to correct it.

*Example.*—A concave lens of 18 D, placed at a distance of 15 millimetres, corrects the myopia of an eye. This glass has a focal distance of $\frac{1000}{18}=55$ millimetres; hence the *punctum remotum* of the eye is situated $55+15=70$ millimetres from it. This gives a myopia of $\frac{1000}{70}=14{\cdot}28$ D.

Taking the distance $H' R=f'$ as the expression of the degree of myopia, the correcting glass ought to be at the first principal point H′—that is to say, in the interior of the eye, 1·75 millimetres behind the anterior surface of the cornea. In this case, its focal distance would be exactly equal to $f'$. In taking $f'$ as the focal distance of a glass *very near* the eye, the error thereby committed is generally so slight that it may be disregarded.

## HYPERMETROPIA OR HYPEROPIA.

When the retina is situated *in front of the focus* of the dioptric system of the eye, parallel rays, before being focused at $\phi''$, form upon the retina a circle of diffusion of which A B (Fig. 60) is one diameter.

Where is the *punctum remotum* to which such an eye is adapted? That is to say, where must the luminous point be placed, the conjugate focus of which shall be upon the retina, which is itself situated in front of the principal focus of the eye?

For the myopic eye, we had to bring the point from infinity to a finite distance from the eye. As the object was brought nearer, its image receded from the principal focus, and finally fell upon the retina, which, in the myopic eye, is situated behind the focus. In other words, it was necessary to give the luminous rays a *divergent* direction.

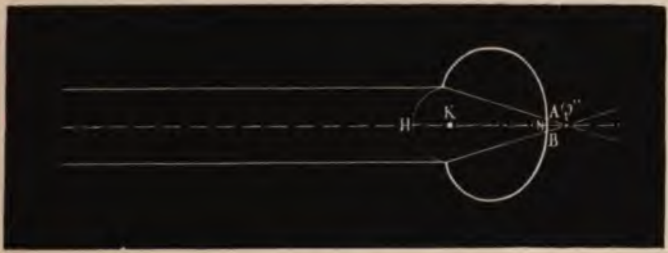

FIG. 60.

In hyperopia, the case is inverted. The eye is *too short*, since the retina is situated in front of the focus, nearer the dioptric system. Hence it would be necessary, in order that the image might advance from the focus toward the dioptric system, to carry the object *beyond infinity*. This is not possible in reality, but the same result may be obtained by rendering the incident luminous rays *convergent* (Fig. 61). Indeed, rays coming from a *finite* distance, are *divergent* (Fig. 58), those coming from *infinity*, are *parallel* (Fig. 54), hence those coming from *beyond infinity* must be *convergent* (Fig. 61).

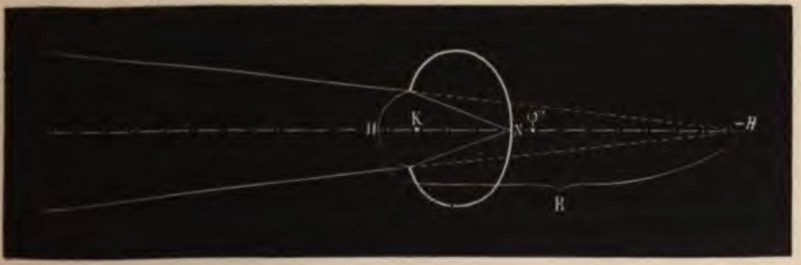

FIG. 61.

Inasmuch as no objects are beyond infinity, convergent rays do not exist in nature. Hence an eye, whose retina is situated in front of the principal focus, does not see distinctly at any distance. It is adapted to a point which does not exist in reality. The *punctum remotum* of the hyperopic eye is, then, the point toward which the luminous rays ought to converge, in order to be focused upon the retina. Its *punctum remotum* ($R$) is situated *behind* it. It is virtual (*negative*), because it results only from the prolongation of the convergent luminous rays which the eye needs in order to see distinctly.

Thus Figure 61 represents again the same eye as Fig. 60. It is shorter, by N $\phi''$, than the emmetropic eye. In order to be focused at N, upon the retina, the rays must already converge before they reach the eye. In our example, they are directed towards the point $R$, where they would meet if the eye did not intercept them. Its optical apparatus, by rendering them still more convergent, focuses them at N.

It is the point $-R$ which represents the *punctum remotum* for this sort of eyes. It is therefore situated on the same side of the optical system as its image. This is the reason why the distance R, between the *punctum remotum* and the eye, bears the negative sign ($-R$).

An eye so constructed being, so to say, adapted to a point situated beyond infinity, its state of refraction is called *Hypermetropia* or *Hyperopia.*

The simplest means of procuring convergent rays for the hyperopic eye, is the convex lens. We know, in fact, that, by rendering them convergent, a convex lens brings parallel rays together at its focus.

*Definition.—Hyperopia is characterised by the fact, that the retina is situated between the dioptric system and the principal focus of the eye;* or, *that eye is hyperopic, which, in a state of rest, requires convergent rays, in order to be able to focus them upon its retina;* or *whose* punctum remotum *is situated beyond infinity,* i.e., *behind the eye, at the point where the convergent rays would meet, if they were not intercepted by the eye;* or, *a hyperopic eye is one which, in order to see distinctly at a distance, requires a convex glass.*

*Different forms of Hyperopia.*—Here, again, the definition does not inform us as to the cause of the retina being situated in front of the focal plane of the eye. In fact, hyperopia may be due to lack of length in an eye, whose dioptric system is as powerful as that of the average emmetropic eye. This is called *axial hyperopia* ($H^a$).

The same anomaly may also be due to a diminution of the refractive power of the dioptric system, the length of the eye being normal. This lessening of refractive power may arise from a *lack of convexity* of the refractive surfaces (*curvature hyperopia,* $H^c$), or from a decrease in the index of refraction of the aqueous humor and crystalline, or, finally, from an increase of that of the vitreous body, *hyperopia from change in the index of refraction* ($H^i$).

All these forms of hyperopia have been observed; but, like myopia, hyperopia is generally due to an anomaly in the length of the eye, the dioptric system being more constant, and scarcely differing from that of the emmetropic eye. Hence we regard *axial hyperopia* as the rule, and the other forms as exceptions.

*Degree of Hyperopia.*—The hyperopia is evidently stronger in proportion as the condition of the eye differs more from emmetropia, *i.e.*, as the retina is further from the focus, or nearer to the dioptric apparatus. The more the retina advances, the more the luminous rays must converge, in order to be united upon it, and the more they do converge, the nearer does the *punctum remotum* $R$ approach to the eye.

The distance of the *punctum remotum* here, again, serves to determine the degree of the ametropia. The degree of hyperopia, as that of myopia, is in inverse ratio to the distance of the *punctum remotum*.

If we take $R$H (Fig. 62) as this distance, and let R[1] to be its expression in terms of the metre, the degree of hyperopia will be $\frac{1}{\mathrm{R}}$, or $r$ dioptries.

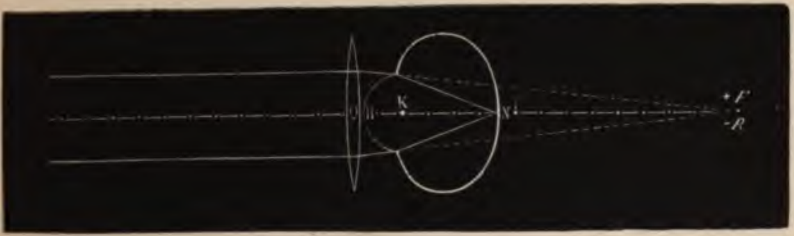

FIG. 62.

How is the distance between the hyperopic eye and its farthest point to be ascertained? In myopia we had only to bring a small object toward the eye until it was seen distinctly. It was then at the *punctum remotum*, and the distance which separated it from the eye was directly measurable.

But the *punctum remotum* of hyperopia cannot be so directly determined, since we cannot carry an object beyond infinity, and, in every other condition, the rays which emanate from an object are divergent, or, at most, parallel.[2]

We are therefore obliged to have recourse to a convex lens, which will furnish us convergent rays. Such a lens, whose focus coincides with the *punctum remotum* of the hyperopic eye, causes parallel rays to converge toward its *punctum remotum*. Hence it gives the convergence which is necessary, in order that the hyperopic eye may focus them upon its retina.

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[1] R corresponds again to $f'$ of our formulæ in Chapter I., only it is negative, being on the same side of the dioptric system as $f''$.

[2] It will be noticed that the designation of the degree of the ametropia, whether myopia or hyperopia, being based solely upon the distance of the *punctum remotum*, is absolutely independent of the *cause* of the ametropia.

If this lens is placed in close contact with the eye, its focal distance is necessarily the same as the distance which separates the *punctum remotum* from the eye.

Let $R$ (Fig. 62), again represent the *punctum remotum* of the hyperopic eye. The lens O causes parallel rays to converge towards this point $R$, if the latter corresponds with the focus $F$ of the lens. If the lens is close enough to the eye, so that one may regard its optical centre O and the point H of the eye as coincident, the focal distance O $F$, or F, of the former, is equal to the distance H $R$ = R, which we seek.

The degree of hyperopia ($\frac{1}{R}$) is then equal to the power of this lens, which may also be expressed by the reciprocal of its focal distance F.

If R equals 2 metres, the degree of the hyperopia is $\frac{1}{2} = 0{\cdot}5$ D, and will demand, for its correction, a convex lens of 0·5 D, placed close to the eye. If R equal 16·6 centimetres or $\frac{1^{m}}{6}$, the hyperopia is $\frac{6}{1^{m}}$ = 6 D, the refractive power which the correcting glass must have.

In fact, one may consider the hyperopic eye as being corrected by this lens, since it is thus adapted to the parallel rays which emanate from infinity. It sees distinctly at a distance just as the emmetropic eye does.

By adding a convex lens to a dioptric system, we increase its refractive power. Hence we might consider the hyperopic eye as being too weak to focus parallel rays upon its retina, and hyperopia as a want of refraction, which is corrected by a convex glass.

The degree of hyperopia thus expresses how many units of refraction (dioptries) the hyperopic eye lacks of being emmetropic. It must therefore have the negative sign (−). Indeed, in order to supply this refractive deficiency, a *positive* glass of the same power is necessary. The negative dioptries of the hyperopia are thus neutralised by the positive dioptries of the correcting glass, and the static refraction of the corrected eye may be expressed by zero, as in emmetropia; in which, R being equal to infinity (∞), the refraction is $\frac{1}{\infty} = 0$. If we say of an eye that it has a hyperopia of 6 dioptries, that might signify that it is 6 dioptries weaker than the emmetropic eye, and that, consequently, + 6 D are needed to render it emmetropic.

Although this way of regarding hyperopia is, strictly speaking, correct only in the case of *curvature hyperopia*, it has, however, the advantage of simplifying most of the practical considerations of it. It may be used without error in the determination of the numbers of correcting glasses for any distance. But it must always be borne in mind that, in reality, hyperopia is more frequently a defect in length than a defect in refractive power.

It is not possible, in practice, to place the correcting lens so near to the eye that its focal distance may be exactly equal to the distance between the *punctum remotum* and the eye. That, however, does not prevent its being used with safety in determining the exact degree of the hyperopia. It is only necessary to make allowance for the distance between the glass and the eye. In fact, at whatever point we place a convex lens, we can always find one whose focus coincides with the *punctum remotum* of the eye. It will cause parallel rays to converge toward this point, and will correct the hyperopia by enabling the eye to focus parallel rays upon its retina, and see at a distance.

Suppose an eye (Fig. 63) whose *punctum remotum* is $\mathrm{H}R = 111$ millimetres behind it. It has a hyperopia of $\frac{1000}{111} = 9$ D. If we place the correcting glass O in immediate contact with the eye, as in figure 62, it must also have a focal distance of 111 millimetres, hence a refractive power of 9 D. But if we place it where spectacle glasses usually are placed, *i.e.*, 15 millimetres in front of the eye, it is $\mathrm{O\,H} =$ 15 millimetres farther from the *punctum remotum*, and must have a focal distance of $111 + 15 = 126$ millimetres ($R\mathrm{H} + \mathrm{H\,O} = R\mathrm{O}$ or $F\mathrm{O}$), hence a refractive power of 8 D.

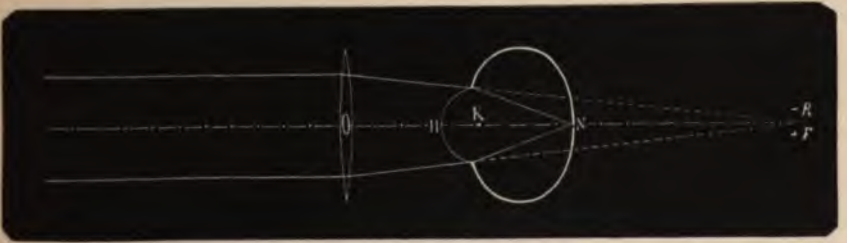

FIG. 63.

If we place the glass still farther from the eye—55 millimetres, for instance—its focal distance must be $111 + 55 = 166$ millimetres, hence its refractive power 6 D. If it be at 111 millimetres from the eye, its focal distance must become $111 + 111 = 222$ millimetres, and its refractive power 4·5 D.

There would thus really be an infinite number of correcting glasses for the same degree of hyperopia (just as in myopia), but the name *correcting glass* is reserved to the lens which, when placed where spectacle glasses usually are, *i.e.*, 15 millimetres from the eye,[1] corrects the ametropia.

In the lower degrees of hyperopia this difference may be disregarded, on account of the great distance between the eye and its *punctum remotum*. If we have to deal with a hyperopia of 1 D, *i.e.*, an eye whose *punctum remotum* is 1000 millimetres behind it, the spectacle glass, placed 15 millimetres in front of it, ought, strictly speaking, to have a focal distance of 1015 millimetres = 0·985 D. It would then be weaker by 0·015 D, than the actual degree of hyperopia. But, in practice, an approximation of a quarter or half dioptry is sufficiently exact. We may therefore, without hesitation, speak of a hyperopia of 1 D, or 2 D, if we can correct it with the spectacle glass of corresponding number.

However, from the moment that the *punctum remotum* is as near as 166 millimetres to the eye, *i.e.*, in cases of hyperopia of 6 D or more, the 15 millimetres between the correcting glass and the eye can no longer be disregarded, even in practice. This hyperopia of 6 D would be corrected by a glass having a focal distance of 166 + 15 = 181 millimetres, hence 5·5 D of refractive power.

For a hyperopia of 20 D (R = 50 millimetres), the difference becomes very great, since, at 15 millimetres from the eye, a glass having a focal distance of 65 millimetres is necessary. Such a glass has a refractive power of 15·3 D, hence it is 20 − 15·3 = 4·7 D weaker than the hyperopia.

It will be noticed that the further the correcting glass is removed from the hyperopic eye, the weaker it must be. The reason for this is easily seen: since its focus must always coincide with the *punctum remotum*, which is at a fixed distance behind the hyperopic eye, the focal distance of the correcting glass must be greater in proportion as it is carried away from the *punctum remotum*, and, consequently, away from the eye. The power of the glass, being the increase of its focal distance, diminishes as the latter increases.

The *punctum remotum* and its retinal image are evidently conjugate foci. The distance from the *punctum remotum* to the first principal point, and that from the retina to the second principal point, are, therefore, conjugate focal distances. Consequently, being acquainted with the dioptric system of the eye, one can easily calculate the corresponding length for any degree of axial myopia or hyperopia, by means of the formula 8 *a*, p. 20, or V *a*, p. 91:

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[1] This means 15 millimetres from the principal point, 13 millimetres from the cornea. Here again, in fact, just as in myopia, the lens which expresses the degree of the ametropia ought to be at the principal point (H′) of the eye, because we reckon the distance of the *punctum remotum* from this point. This can only be realised in aphakia, or with the artificial eye, in which cases the two principal points become a single one, which is situated at the summit of the refractive surface, to which the correcting lens may be applied. In any other case, the distance between the first principal point and the lens must be taken into consideration and *deducted* from the focal distance of the latter.

$$f'' = \frac{f' F''}{f' - F'}$$

in which $f''$ equals the distance from the second principal point to the retina. To this we have only to add 2·11 millimetres in order to obtain the total length of the eye. $f'$ is the distance from the *punctum remotum* to the first principal point, or the distance which expresses the degree of ametropia. It will be negative for hyperopia, so that, in this case, the formula becomes—

$$f'' = \frac{f' F''}{f' + F'}$$

$F''$ is the second principal focal distance of the eye = 20·7136 millimetres.

$F'$ is the first principal focal distance of the eye = 15·4983 millimetres for the schematic eye.

If it is a question only of knowing the difference in length between an ametropic and the emmetropic eye, their dioptric systems being equal, we use the formula 8 *a*, p. 20, or V *e*, p. 92—

$$l'' = \frac{F' F''}{l'}$$

in which $l'' = f'' - F''$, that is to say, the difference in length between the ametropic and emmetropic eye;

$$F' F'' = 15{\cdot}4983 \times 20{\cdot}7137 = 321{\cdot}026 \text{ millimetres,}$$

and

$$l' = f' - F'$$

(or $f' + F'$ when $f'$ is negative, as in hyperopia).

$l'$ is the difference between the distance from the *punctum remotum* to the first principal point, and that from the anterior focus to the first principal point of the eye. In other words, $l'$ is the distance from the *punctum remotum* to the anterior focus of the eye. The latter is situated 15·4983 − 1·7532 = 13·7451 millimetres in front of the cornea. This is nearly where spectacle glasses, the glasses used for the correction of ametropia, are placed. Their focal distance is, therefore, always shorter, by $F'$, than $f'$ in myopia, and longer, by $F'$, than $f'$ in hyperopia. Hence $l'$ in the formula is, in this case, equal to the focal distance of the correcting glass placed at the anterior focus $\phi'$.

From this it results that, in order to ascertain the difference in length between an axially ametropic eye and the schematic emmetropic eye, one has only to divide the constant quantity 321 by the focal distance of the correcting glass expressed in millimetres.

This difference, added to the length of the normal eye (E = 22·824), will give the length of the myopic eye; subtracted from the length of the

the normal eye, it will give the length of the hyperopic eye, for any position of the *punctum remotum*, that is to say, for any degree of axial ametropia.

By performing these calculations, the following table is obtained :[1]—

## TABLE FOR AXIAL AMETROPIA.

| MYOPIA. | | | | | HYPEROPIA. | | |
|---|---|---|---|---|---|---|---|
| I. Degree of the ametropia, in dioptries. $\frac{1\text{ m.}}{R}$ or $\frac{1\text{ m.}}{f'}$ | II. Distance between the *punctum remotum* and the first principal point $f'$ = focal distance corresponding to the dioptries. R or $f'$ | III. Dioptries of the concave correcting lens, placed at $\phi'$, its focal distance being $l' = f' - F'$. $-\frac{1\text{ m.}}{l'}$ | IV. Excess in length of the myopic eye— $+l'' = \frac{F'.F''}{l'} = \frac{321}{l'}$ $+l''$ | V. Total length of the myopic eye; that of the emmetropic eye being— E = 22·824 millimetres. $E + l''$ | VI. Dioptries of the convex correcting lens, placed at $\phi'$, its focal distance being— $l' = f' + F'$. $+\frac{1\text{ m.}}{l'}$ | VII. Defect in length of the hyperopic eye— $-l'' = \frac{F'.F''}{l'} = \frac{321}{l'}$ $-l''$ | VIII. Total length of the hyperopic eye; that of the emmetropic eye being— E = 22·824 millimetres. $E - l''$ |
| 0 | ∞ | 0 | 0 | 22·824 | 0 | 0 | 22·824 |
| 0·5 | =2000 | − 0·503 | + 0·16 | 22·98 | ÷ 0·46 | − 0·16 | 22·67 |
| 1 | 1000 | 1·015 | 0·32 | 23·14 | 0·98 | 0·31 | 22·51 |
| 1·5 | 666·6 | 1·53 | 0·49 | 23·31 | 1·46 | 0·47 | 22·35 |
| 2 | 500·0 | 2·06 | 0·66 | 23·48 | 1·93 | 0·62 | 22·20 |
| 2·5 | 400·0 | 2·60 | 0·83 | 23·65 | 2·40 | 0·77 | 22·05 |
| 3 | 333·3 | 3 14 | 1·01 | 23·83 | 2·86 | 0·92 | 21·90 |
| 3·5 | 285·7 | 3·70 | 1·19 | 24·01 | 3·32 | 1·06 | 21·76 |
| 4 | 250·0 | 4·26 | 1·37 | 24·19 | 3·76 | 1·21 | 21·61 |
| 4·5 | 222·2 | 4·83 | 1·55 | 24·37 | 4·20 | 1·35 | 21·47 |
| 5 | 200·0 | 5·42 | 1·74 | 24·56 | 4·64 | 1·50 | 21·32 |
| 5·5 | 181·8 | 6·01 | 1·93 | 24·75 | 5·06 | 1·62 | 21·20 |
| 6 | 166·6 | 6·64 | 2·13 | 24·95 | 5·49 | 1·76 | 21·06 |
| 6·5 | 153·8 | 7·23 | 2·32 | 25·14 | 5·90 | 1·90 | 20·92 |
| 7 | 142·8 | 7·85 | 2·52 | 25·34 | 6·31 | 2·03 | 20·80 |
| 7·5 | 133·3 | 8·48 | 2·73 | 25·55 | 6·72 | 2·16 | 20·66 |
| 8 | 125·0 | 9·13 | 2·93 | 25·75 | 7·11 | 2·28 | 20·54 |
| 8·5 | 117·6 | 9·79 | 3·14 | 25·96 | 7·51 | 2·41 | 20·41 |
| 9 | 111·1 | 10·46 | 3·35 | 26·17 | 7·89 | 2·53 | 20·29 |
| 9·5 | 105·2 | 11·14 | 3·58 | 26·40 | 8·28 | 2·66 | 20·16 |
| 10 | 100·0 | 11·83 | 3·80 | 26·62 | 8·6 | 2·78 | 20·04 |
| 10·5 | 95·2 | 12·54 | 4·03 | 26·85 | 9·03 | 2·90 | 19·92 |
| 11 | 90·9 | 13·2 | 4·26 | 27·08 | 9·4 | 3·02 | 19·80 |
| 12 | 83·3 | 14·7 | 4·73 | 27·55 | 10·1 | 3·25 | 19·57 |
| 13 | 76·9 | 16·2 | 5·23 | 28·05 | 10·8 | 3·47 | 19·35 |
| 14 | 71·4 | 17·8 | 5·74 | 28·56 | 11·5 | 3·39 | 19·13 |
| 15 | 66·6 | 19·5 | 6·28 | 29·10 | 12·18 | 3·91 | 18·91 |
| 16 | 62·5 | 21·27 | 6 83 | 29·65 | 12·8 | 4·11 | 18·71 |
| 17 | 58·8 | 23·09 | 7·41 | 30·23 | 13·4 | 4·32 | 18·50 |
| 18 | 55·5 | 25·0 | 8·03 | 30·85 | 14·08 | 4·52 | 18·30 |
| 19 | 52·6 | 26·9 | 8·65 | 31·47 | 14·6 | 4·71 | 18·11 |
| 20 | 50·0 | 28·98 | 9·31 | 32·13 | 15·2 | 4·90 | 17·92 |

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[1] Nagel in Græfe und Sæmisch, Handbuch VI., 10, p. 271.

We have found, between the product of the focal distances, as we have calculated them, and the corresponding value given by Nagel, a difference too slight to influence the number which expresses the lengthening or shortening of the axis. Nevertheless, our values in columns V. and VIII. differ a little from those given by Nagel (22·834), because we took as the length of the emmetropic eye the number (22·824 millimetres) which we obtained by our calculations in Chapter I.

If we make use of the reduced eye (p. 98), we shall have to employ the same formulæ. However, $f'$ and $f''$, as well as $F'$ and $F''$, touch each other at the refractive surface, since the principal points $H'$ and $H''$ coincide with the latter.

$F''$ corresponds to the length of the emmetropic reduced eye, and equals 20 millimetres; $f''$ to that of the ametropic reduced eye.

$F'$ is the anterior focal distance of the reduced eye, and equals 15 millimetres; $f'$ the distance from the *punctum remotum* to the refractive surface of the ametropic reduced eye.

If we again designate by $l'$ the focal distance of the correcting glass, placed at $\phi'$, the value $f'$ is $f' = l' + F'$ in myopia, $l' - F$ in hyperopia, *i.e.*, 15 millimetres longer than the focal distance of the correcting glass, in myopia, and 15 millimetres shorter than it, in hyperopia.

The product $F' \times F''$ becomes $15 \times 20 = 300$ millimetres, and the difference in length ($l''$) between the ametropic and the emmetropic eye is obtained by dividing 300 by the focal distance of the correcting glass, expressed in millimetres: $l'' = \frac{300}{l'}$ (see p. 20 *et seq.*).

The calculation becomes, in this way, so simple that it may be readily performed mentally, without the results differing greatly from that of the schematic eye. Thus, a myopic eye corrected by the concave 5 D, of which the focal distance ($l'$) equals 200 millimetres, is longer, by $l'' = \frac{300}{200} = 1·5$ millimetres, or, to be more exact, by $\frac{321}{200} = 1·51$ millimetres, than the emmetropic eye.

For a correcting glass of 1 dioptry ($l' = 1000$ millimetres), we obtain a difference in the length of the eye, amounting to:

$$l'' = \frac{321}{1000} = 0·321^{\text{mm.}}$$

Conversely, we may ask, what change in the refractive condition is produced by a variation of 1 millimetre in the length of the eye?

We say, 0·321 millimetres of difference in length correspond to 1 dioptry; hence 1 millimetre corresponds to $\frac{1}{0·321} = 3·1$ dioptries. This means that each millimetre added to the length of an emmetropic eye produces an increase of myopia which is corrected by 3·1 dioptries, the glass being placed at $\phi'$. Each millimetre of shortening of the emmetropic eye requires a convex glass 3·1 dioptries stronger, supposing it to be placed at the anterior focus $\phi'$ of the eye, which has thus become hypermetropic.

When it is a question of *ametropia of curvature*, under which head belongs the eye in a state of accommodation, also the problem changes.

The length of an eye, ametropic by increase in curvature of its refracting surfaces, is the same as that of the emmetropic eye: $f'' = F'' = 20$ millimetres in the case of the reduced eye. It is the radius of curvature ($\rho$) of its refractive surfaces which varies so as to produce the ametropia, *i.e.*, the difference of $f'$.

In this case, we make use of the formula for this radius of curvature, which has been given on page 16; $\rho = \frac{f' f'' (n-1)}{f' n + f''}$ for curvature myopia;

$$\rho = \frac{f' f'' (n-1)}{f' n - f''} \text{ for curvature hypermetropia.}$$

The variations in the radius of curvature, necessary to produce different degrees of curvature ametropia, are contained in the following table[1]:—

## TABLE FOR CURVATURE-AMETROPIA.

| DEGREE IN DIOPTRIES. $\frac{1^{\text{m.}}}{\text{R}}$ or $\frac{1^{\text{m.}}}{f'}$ | MYOPIA. Radius of curvature of the reduced eye, $\rho$ in millimetres. | HYPEROPIA. Radius of curvature of the reduced eye, $\rho$ in millimetres. |
|---|---|---|
| 0 | 5 | 5 |
| 1 | 4·926 | 5·076 |
| 2 | 4·854 | 5·154 |
| 3 | 4·784 | 5·235 |
| 4 | 4·719 | 5·319 |
| 5 | 4·651 | 5·405 |
| 6 | 4·587 | 5·494 |
| 7 | 4·520 | 5·586 |
| 8 | 4·464 | 5·681 |
| 9 | 4·405 | 5·780 |
| 10 | 4·348 | 5·882 |
| 11 | 4·291 | 5·988 |
| 12 | 4·237 | 6·097 |
| 13 | 4·182 | 6·211 |
| 14 | 4·132 | 6·329 |
| 15 | 4·081 | 6·451 |
| 16 | 4 032 | 6·578 |
| 17 | 3·983 | 6·711 |
| 18 | 3·937 | 6·849 |
| 19 | 3·891 | 6·993 |
| 20 | 3·846 | 7·142 |

To ascertain the influence produced upon the refraction of the eye by variations in curvature of the different refractive surfaces in particular, or of the indices of refraction of the refractive media of the eye, the formulæ for the schematic eye may be used, giving to the signs on page 79 their respective values.

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[1] Nagel, *loc. cit.*, p. 270.

## II.—DYNAMIC REFRACTION OF THE EYE ACCOMMODATION.

Heretofore we have considered the eye as being adapted to one single distance, that of its *punctum remotum*. When the luminous point was at any other distance, it formed upon the retina a diffuse image. Thus, on bringing the object between the eye and its *punctum remotum*, each of its component points forms upon the retina a circle of diffusion, and from the sum of these circles a diffuse image is produced.

Daily observation, however, teaches us that the same eye can see distinctly at very different distances. Only we do not see, simultaneously and distinctly, both a distant and a near object; we see either the one or the other.

If, for instance, we approach a window, before which hangs a muslin curtain, we can sometimes see the threads of the curtain, and sometimes objects outside of the window, but never both at one time; or again, holding a finger between the eye and a book, we sometimes see the finger distinctly, and at other times the text of the book; but both are never, simultaneously, distinctly visible.

From this it results that distinct images of objects situated at very different distences from the eye must be formed upon the sensitive layer of the retina. But, at the same time, the condition of the eye must have changed, since it cannot see near objects, when it is adapted to those at a great distance, nor distant objects, when it is adapted to near ones.

Now we know that, being given any dioptric system, and a screen upon which this system may produce images, as the dioptric apparatus of the eye upon the retina, the image is distinct for but one single distance of the object.

If the object be brought nearer to the screen, its image recedes from it, and the screen receives only a diffuse image. In order to obtain a well defined image, the screen must likewise be withdrawn from the dioptric system, or the power of the latter must be sufficiently increased so that it may focus the rays, emanating from the object, upon the screen, in spite of the approach of the object. If, then, the same eye sees an object at 100 metres as distinctly as it does another object which is only 20 centimetres distant, a change must be possible, either in the position of its retina, or in the refractive power of its dioptric system.

The former hypothesis is not admissible. In order to change the position of its retina, the eye would have to be able to vary its length, and vary it considerably, too, in a fraction of a second of time. We have seen, indeed, that, in order to change its adaptation from infinity to 75 millimetres, the eye would have to become 5 millimetres longer. This number represents the difference in length between an emmetropic eye and one which has become myopic by elongation, its *punctum remotum* being at a distance of 75 millimetres (13·3 D).

The eye possesses no apparatus capable of producing such a change in its length. The dioptric system of the eye, therefore, must vary when the object of fixation changes its distance. Its refractive power must *increase* when the object approaches and *diminish* when the latter recedes. This is, in fact, what takes place.

The faculty of the eye, to give its dioptric apparatus the strength necessary to see, clearly, objects situated at different distances, is called *accommodation*. This accommodation is produced by a *change in the convexity of the crystalline lens.*

The following experiments show how accommodation is produced:

It is well known that convex mirrors produce erect and diminished images of objects situated in front of them. Concave mirrors produce inverted images of such objects. These images are smaller in proportion as the curvature of the mirror (whether convex or concave) is stronger, *i.e.*, as its radius is shorter.

One may easily convince one's self of this with the aid of biconvex lenses of different curvatures. The convex surface, turned toward the observer, forms a convex mirror; and the air, in contact with the opposite surface, forms a concave mirror. Turning one's back to a window, one immediately sees the two images of the latter, which are produced by the two surfaces of the lens. One image is due to the convex surface, and is situated behind it; the other, due to the concave surface, is inverted, smaller, and nearer. These images are

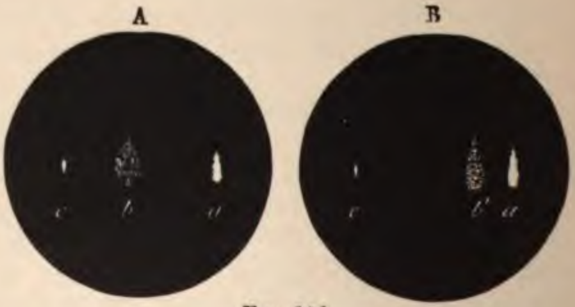

FIG. 64.[1]

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[1] Figures A and B, 64, are after those in Donders' work (*Anomalies, &c.*, p. 13), but are rectified, the images *a* and *b* being upright, while *c* is inverted.

smaller in proportion as the lenses, by which they are produced, are stronger, *i.e.*, as the surfaces of these lenses are more strongly curved.

If an eye be given a definite direction, and a bright light be placed beside it, so that a line drawn from the light to the eye shall form an angle of about 30° with the visual line, and the eye be looked at from the other side, at an angle, too, of about 30°, three small images of the light are seen on the pupil, as is represented in Fig. 64. These are the reflected images furnished by the cornea and the anterior and posterior surfaces of the crystalline lens, or, to be quite exact, by its anterior surface and the anterior surface of the vitreous body, which is in contact with the posterior surface of the crystalline lens.

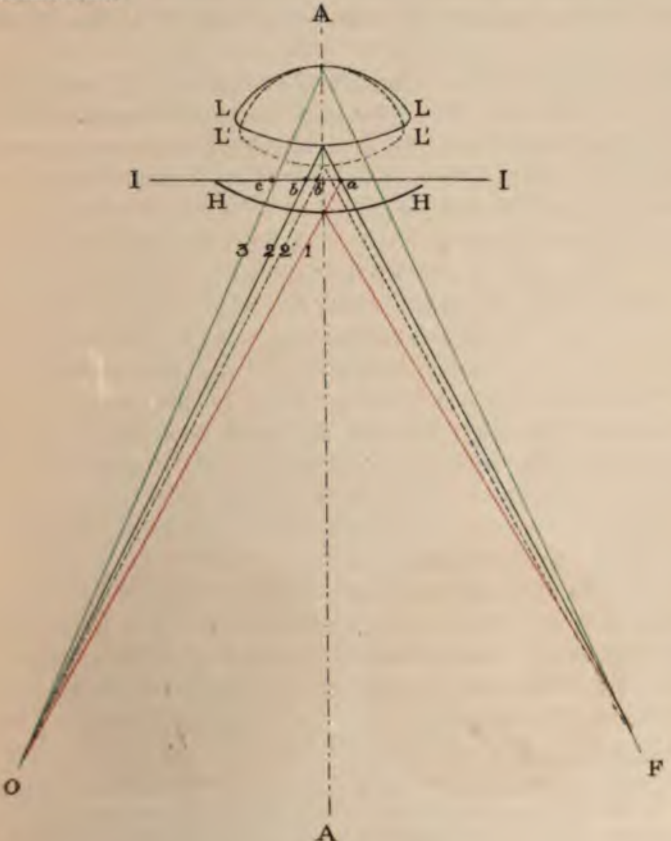

FIG. 65.

The image *a*, situated to the side nearest the flame, is the one furnished by the cornea. It is the brightest one, because the difference between the indices of refraction of the air and of the cornea, or of the aqueous humor, is greater than between those of the aqueous humor and the crystalline lens, or between those of the crystalline lens and the vitreous body. It is the medium one in point of size, because the curvature of the cornea is greater than that of the anterior, and less than that of the posterior surface of the lens. The influence which the refraction, by the dioptric media, exerts upon the two other images is not sufficiently great to change entirely the relations, as to size, of the reflected images.

Fig. 65 shows how these reflected images are formed.[1] A A is the axis of the cornea, which may here be supposed to be coincident with the visual line, *i.e.*, with the direction which has been given to the eye. H H represents the cornea, L L and L′ L′ the crystalline lens.

Let us consider, for the moment, the full line (L L) only, and disregard the dotted one. The light is at F and the observer at O.

Let I I represent the plane upon which the observer sees the reflected images furnished by the different surfaces of the eye.

The rays given off from the flame (F) are reflected by the apex of the cornea in the direction of the red line 1. Hence the observer sees at *a* the image of the flame reflected by the cornea. The anterior surface of the crystalline (L L) reflects the rays in the direction of the full line 2. The corresponding image appears at *b*. The posterior surface of the crystalline reflects rays along the green line, in the direction of the line 3, and its image is seen at *c*.[2]

The image *b*, situated between the other two, is due to the anterior surface of the crystalline. It is the largest, because the reflecting surface which produces it is least convex when the eye is at rest. It is, moreover, somewhat magnified by the meniscus formed by the cornea and the aqueous humor.

Finally, the image *c*, from the posterior surface of the crystalline lens, nearest to the observer, is the smallest and the least luminous, because it is due to the surface of strongest curvature, and because the light, before reaching this surface and before leaving the eye, has passed through rather thick layers of refractive media. This last image is, moreover, inverted, because it is furnished by a concave surface. For the same reason, its movements are in directions opposite to those in which the flame moves.

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[1] Donders' *Anomalies, &c.*, p. 14.

[2] It is self-evident that only the cornea reflects the rays directly, as is indicated by the figure. Those which enter the eye, to be reflected by the anterior surface of the crystalline lens, are deviated, when they enter and when they leave the cornea, by the aqueous humor; those of them which pass through the crystalline lens, and are then reflected by its posterior surface, undergo an additional refraction by this lens, through which they pass twice. This refraction may be disregarded in the schematic representation, but not when, to obtain the position of the surfaces of the crystalline, one desires to determine the place where the reflection is actually produced, nor when, from the sizes of the reflected images, one wishes to deduce the curvature of these surfaces.

Fig. 64 A, and the full lines in Fig. 65, show the position and formation of these images of reflection, while the eye fixes the most distant object that it can see distinctly, or, in other words, while it is adapted to its *punctum remotum, i.e.*, is at rest.

If we bring the object of fixation in the direction A A nearer and nearer to the eye, without the latter ceasing to distinguish it clearly, the following phenomena are observed in the reflected images (Fig. 64, B):

The image (*a*) from the cornea does not vary.

The image (*b*), from the anterior surface of the crystalline lens, approaches the one from the cornea and becomes smaller. This proves that the anterior surface of the crystalline has approached the cornea. A glance at Fig. 65 suffices to show that, if the reflected image of the flame appears at *b*′, instead of at *b*, *i.e.*, if it follows the dotted line 2′, in coming toward the observer's eye, the surface, by which it is reflected, must have advanced. The surface represented by the dotted line L′ L′ has in this case replaced the full one L L. The fact, that the image is smaller in the second case than in the first, shows that the corresponding surface has become more convex.

The third image (*c*) does not seem to change its position; but it becomes a trifle smaller. Hence the curvature of the posterior surface of the crystalline lens has increased somewhat, but the surface itself has not changed place.

The thickness of the crystalline lens must evidently increase, by an amount equal to the advancement of its anterior surface, when, under the influence of the adaptation to near objects, the convexity of its surfaces increases.

All these phenomena are more easily observed when, instead of a single flame whose contours are not well defined, we take, as the object, two luminous squares. They are cut in a screen covered with ground glass, and are strongly illuminated from behind by means of gas or electric light. The reflexes from the dioptric surfaces of the eye, when it is adapted to distant and to near objects, are then represented by A and B, Fig. 66, in which the letters have the same signification as before.

Each surface reflects two images corresponding to the two objects. The distance between the two images of each pair is proportional to the size of the image. This size may be readily measured by means of the ophthalmometer.

When an eye passes from the fixation of the most distant to that of the nearest object which it can see distinctly, it has evidently undergone all the change in optical adaptation, *i.e.*, accommodation, of which it is susceptible. We may say, then, as a recapitulation of the discussion which immediately precedes:

*The accommodation of the eye is produced by a change in the form of the crystalline lens, in virtue of which the anterior surface of this lens advances and becomes more convex, while the convexity of its posterior surface increases but little, and this surface itself does not change position.*

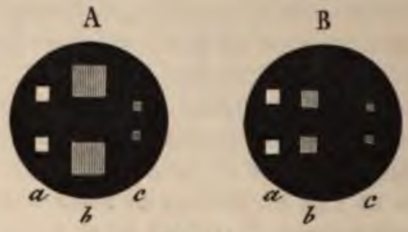

FIG. 66.

**A, state of repose. B, state of accommodation. *a*, corneal reflexes, invariable; *b*, reflexes from the anterior surface of the crystalline, smaller, and consequently nearer each other during accommodation (B), and nearer to those from the cornea; *c*, reflexes from the posterior surface of the crystalline lens, least luminous of all, maintaining their position, and becoming but little smaller during accommodation.**

No other modification has been observed either in the dioptric apparatus or in the length of the eye. Moreover, Knapp[1] has proved, by direct measurements and by calculation, that the change of curvature, which the crystalline lens undergoes, suffices to explain all the accommodation of which an eye is susceptible.

Let us recall the numerical data of the changes produced in the optic apparatus of the eye during accommodation.

In an eye adapted to infinity, the radius of curvature of the cornea is, approximately, 8 millimetres long, that of the anterior surface of the crystalline lens 10 millimetres, and that of its posterior surface 6 millimetres. The distance from the anterior surface of the cornea to the anterior surface of the crystalline lens is 3·6 millimetres, to the posterior surface 7·2 millimetres.

When the eye is accommodated to 135 millimetres (5″), these values become the following:

The radius of curvature of the anterior surface of the crystalline lens becomes equal to 6 millimetres, and that of its posterior surface to 5·5 millimetres. Its anterior surface comes 0·4 of a millimetre nearer the cornea, while its posterior surface does not change position; the thickness of the crystalline lens increases by 0·4 of a millimetre.

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[1] Knapp, *Arch. f. ophth.*, vi., 2, pp. 1-52, 1860.

| | At rest. | During accommodation. |
|---|---|---|
| Radius of curvature of anterior surface of crystalline lens, . . . . | 10·0 $^{\text{mm.}}$ | 6·0 $^{\text{mm.}}$ |
| Radius of curvature of posterior surface of crystalline lens, . . . | 6·0 $^{\text{mm.}}$ | 5·5 $^{\text{mm.}}$ |
| Distance from anterior surface of cornea to anterior surface of lens, . . | 3·6 $^{\text{mm.}}$ | 3·2 $^{\text{mm.}}$ |
| Distance from anterior surface of cornea to posterior surface of lens, . . | 7·2 $^{\text{mm.}}$ | 7·2 $^{\text{mm.}}$ |
| Thickness of the crystalline lens, . | 3·6 $^{\text{mm.}}$ | 4·0 $^{\text{mm.}}$ |

*Historical.*—The history of accommodation is very interesting. It would, of itself, constitute a course of applied optics. Every change, in the form of the eye, in its refracting surfaces or in its dioptric power, which might possibly produce the change in refraction which constitutes accommodation, has been brought forward as explaining this phenomenon. The practical nature of this treatise will not unfortunately permit us to make the excursion into the past, which would be necessary to a review of these various hypotheses. Those of our readers who are interested in it will find the full history of the theory of accommodation well treated in Helmholtz's classical work.[1]

We shall here limit ourselves to indicating the ground gone over from the time the proper explanation of accommodation was first known, up to the period when the theory was virtually proved.

The first to ascribe this variation, in the refraction of the eye, to a change in the form of the crystalline lens, seems to have been the Jesuit, Scheiner[2] (1619). After him came Descartes (1637),[3] Pemberton[4] (1719), Camper[5] (1746), and Hunter[6] (1794).

Th. Young[7] (1801) not only advanced, as probable, the change of form in the crystalline lens, but he also brought direct proofs to support this opinion. His experiments and theory, however, passed unnoticed.

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[1] Helmholtz, *Handbuch der physiologischen Optik*, p. 118.

[2] Christophorus Scheiner, *Occulus, hoc est: fundamentum opticum*, &c.: Oenipont (Innspruck), 1619.

[3] Cartesius, *Dioptrice:* Lugduni Batavorum (Leyden), 1637.

[4] Pemberton, *De facultate oculi*, &c.: Lugduni Batavorum, 1719.

[5] Camper, *Dissertatio physiolog. de quibusdam oculi partibus:* Lugd. Bat., 1746.

[6] Hunter, *Philos. Transactions*, 1794, p. 21.

[7] Th. Young, *Philos. Transactions*, 1801, I., p. 53, and *Philos. Transactions*, 1805, vol. xcii., p. 23. *Miscellaneous Works of the late Th. Young*, edited by G. Peacock, vol. i., p. 12: London, 1855.

Purkinje,[1] Graefe,[2] Th. Smith,[3] Hueck,[4] Stellwag von Carion,[5] and Forbes[6] pronounced in favour of the same theory without being acquainted with Young's works, or bringing irrefutable proofs of it.

The reflexes, which were afterwards to serve to elucidate this important question, were discovered in 1823 by Purkinje, and Sanson[7] employed them for the diagnosis of cataract (1837).

But it was only in 1849 that Max Langenbeck[8] had the happy idea of using these reflex images to make sure as to what changes of form are produced in the refractive surfaces of the eye during the accommodation. Although observing with the unaided eye and from a very unfavourable position, he ascertained, nevertheless, the variations which these catoptric images undergo, and from them he drew the just conclusion that the anterior surface of the crystalline lens increases in curvature during accommodation for near vision.

Langenbeck's work, though not generally known to the physicians and physiologists of that time, was, however, noticed by Donders[9] and Cramer.[10] The latter undertook to repeat Langenbeck's observations, and found at what angle the eye should be observed in order that the reflected images may be seen under the most favourable conditions. He invented an instrument (the phakoidoscope), which, by magnifying, enables one not only to observe clearly, but also to measure, these reflected images. Hence Cramer's remarkable work leaves no doubt as to the cause of accommodation.

At the same time, and independently of Langenbeck and Cramer, Helmholtz[11] found the same fact, and added that the posterior surface of the crystalline is also modified, though but slightly. This had escaped the notice of the other observers.

Helmholtz finally indicated his ophthalmometer, by means of which observations of the form, and of variations in form, of the dioptric surfaces of the eye have been rendered so precise.

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[1] Purkinje, *Beobachtungen und Versuche zur Physiologie der Sinne :* Berlin, 1825.

[2] Graefe, *Reit's Archiv f. Physiologie*, ix., 231, 1804.

[3] Th. Smith, *Philosophical Magazine*, 1838, v., 3, No. 13.

[4] Hueck, *Bewegung der Kristallinse :* Leipzig, 1841.

[5] Stellwag von Carion, *Zeitschrift der k. k. Gesellschaft der Aerzte in Wien*, 1850. 3 and 4.

[6] Forbes, *Comptes rendus*, xx., 61, 1845.

[7] Sanson, *Leçons sur les maladies des yeux*, publ. by Bardinot and Pigne, Paris, 1837.

[8] Max Langenbeck, *Klinische Beitræge aus dem Gebiete der Chirurgie und Ophthalmologie :* Gœttingen, 1849.

[9] Donders, *Nederl. Lancet*, Second Series, D. V., pp. 135 and 147.

[10] Cramer, *Het Accomodatievermogen physiologisch toegelicht :* Haarlem, 1853.

[11] Helmholtz, *Monatsberichte der Berliner Academie*, February 1853, p. 137. Compare also Donders, *Anomalies of Accommodation and Refraction*, p. 10, and Mauthner, *Die optischen Fehler des Auges*, p. 28.

We may say that the discovery of the part played by the crystalline lens, in the act of accommodation, is one of the finest achievements of modern physiology, and the theory of its working is certainly one of the most firmly established; for not only have *savans* furnished lucid and mathematical proofs of its correctness, but all other theories, which have been advanced as explaining accommodation, have been easily and entirely overthrown. Thus, the doctrine of accommodation forms the perfect complement of the doctrine of the refraction of the eye in a state of repose.

The fact that the eye is accommodated, for near vision, by an increase in the curvature of its crystalline lens, is, then, incontestably proved. We are now interested to know how this change of form is brought about. In order to solve this problem, we must first bear in mind the topographical anatomy of the crystalline lens, and then observe attentively all the changes that can be shown to take place in these parts, when the form of the crystalline lens is modified during accommodation.

## ANATOMY OF THE CILIARY REGION OF THE EYE.

The portion of the ocular globe which is concerned in the act of accommodation is the region comprised between the base of the cornea and the *ora serrata*. This is where the character of the sclerotic tissue changes, becomes transparent—in short, becomes cornea, from the venous space known as Schlemm's canal (*c S*, Fig. 67).

Posteriorly, the choroid is distinguished from its anterior zone by the *ora serrata* (*O s*), which is situated about 6 millimetres from the circumference of the cornea. This posterior portion of the region is called the *ciliary circle*.

Anteriorly, this zone acquires a more considerable thickness, is continuous with the iris, and enters into close relation with the sclerotic. It may be regarded as composed, at this point, of two concentric rings; one internal, plaited, black one, corresponding by its inner face to the crystalline lens, and described by the name of the *ciliary crown;* the other external, concentric with the first, and known as the *ciliary muscle*. The ciliary crown and ciliary muscle, together, constitute the *ciliary body*, the anterior portion of the choroid zone.

At the border of the anterior chamber, *i.e.*, in the angle included between the base of the iris and that of the cornea, Descemet's membrane is found to be converted into a plexus of fine fibrils, which are in direct and intimate relation with those of the sclerotic, the ciliary muscle, ciliary body, and of the iris.[1]

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[1] Waldeyer, in Graefe and Saemisch, *Handbuch*, I., p. 226.

The lacunæ of this tissue, which anastomose with each other and communicate directly with the anterior chamber, form *Fontana's space* (*e F*, Fig. 67, and *e*, Fig. 68).

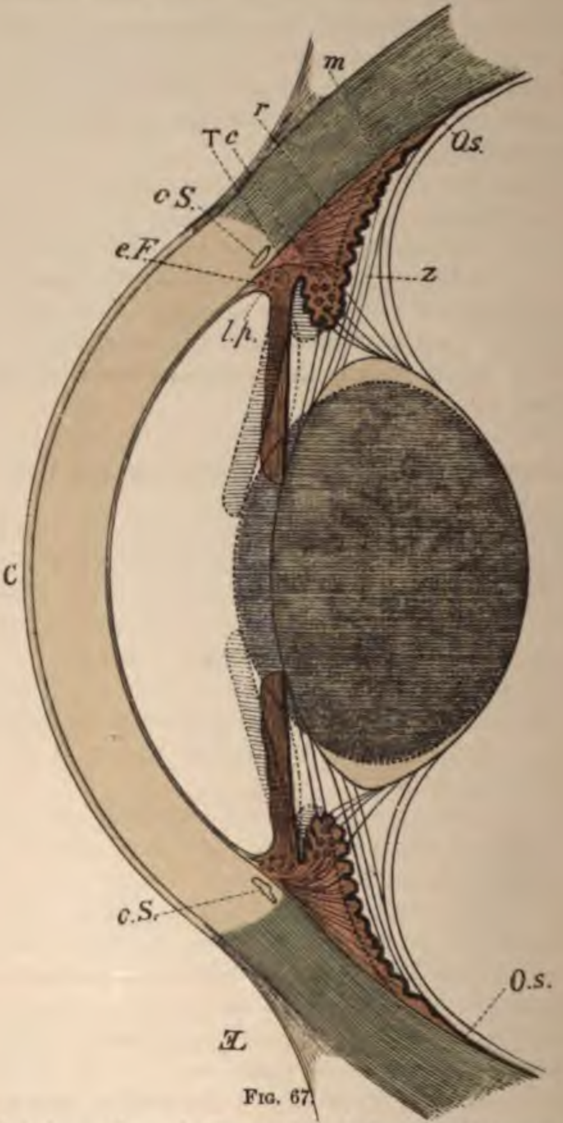

FIG. 67.

Anterior portion and ciliary region of the eye. *C*, cornea; *c S*, Schlemm's canal; *O s*, *ora serrata*; *l p*, pectinated ligament; *e F*, Fontana's space; *T*, tendinous ring; *m*, meridional fibres, *r*, radiating fibres, *c*, circular fibres of the ciliary muscle; *Z*, zone of Zinn.

The full lines indicate the crystalline lens, iris, and ciliary body in a state of rest; the dotted lines show the same in a state of accommodation.

Some of the above-mentioned fibrils are reflected upon the anterior face of the iris, and constitute the pectinated ligament (*l p*). The others penetrate the ciliary muscle, or lose themselves in the walls of the vascular plexus known as *Schlemm's canal.*[1]

At the junction of the posterior surface of the cornea with the sclerotic, between the fibrous border of Schlemm's canal, the insertion of the iris and the anterior portion of the ciliary body, is found a *tendinous ring*, described by Gerlach[2] (T, Fig. 67). The connective and elastic tissue, of all the parts here met with, contribute to its formation. This firm ring constitutes the tendinous origin of the ciliary muscle.

The ciliary muscle forms another ring, external to the ciliary crown. It has the shape of a triangular bundle, upon meridional section (Figs. 67 and 68). Its shortest side corresponds, in front, to Schlemm's canal, and forms, with its outer, sclerotic side, a right angle, in the normal eye. Its inner face corresponds to the ciliary crown, its apex is directed backward.

The fibres of the ciliary muscle may be divided into three parts:—
1. The most powerful layer (*m*, Fig. 67, and *f*, Fig. 68) is nearest the sclerotic. It is composed of a thick layer of fibres having a meridional direction, and forms lamellæ parallel with the sclerotic. These muscular fibres extend backward into the choroid. Here they become dissociated, change direction, and terminate principally in the conjunctival lamellæ of the *lamina fusca*. This portion of the choroid therefore represents the tendon of the meridional fibres of the ciliary muscle.

2. The second part (*r*, Fig. 67, and *g*, Fig. 68) of this muscle contains fibres which are less intimately connected with each other. Their direction deviates more and more from parallelism with the surface of the eyeball from those which are nearest the meridional to those which approach the shortest side of the muscular triangle, and radiate toward the centre of the globe.

Hence the fibres of this portion of the muscle terminate near the posterior surface of the ciliary body.

3. The third part of the ciliary muscle is represented by the annular muscle of Müller (*c*, Fig. 67, and *h*, Fig. 68). It is composed of circular fibres, which form a ring parallel with the base of the cornea, and of which a section is seen in our figures. They correspond to the shortest side of the triangle formed by the ciliary muscle, and give, to the sclerotico-ciliary angle of the muscle, a shape proportionately less acute as they are more numerous.

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[1] See, on this topic, Kölliker, *Elements of Human Histology*.

[2] Gerlach, *Beitraege zur normalen Anatomie des menschlichen Auges:* Leipzig, 1880.

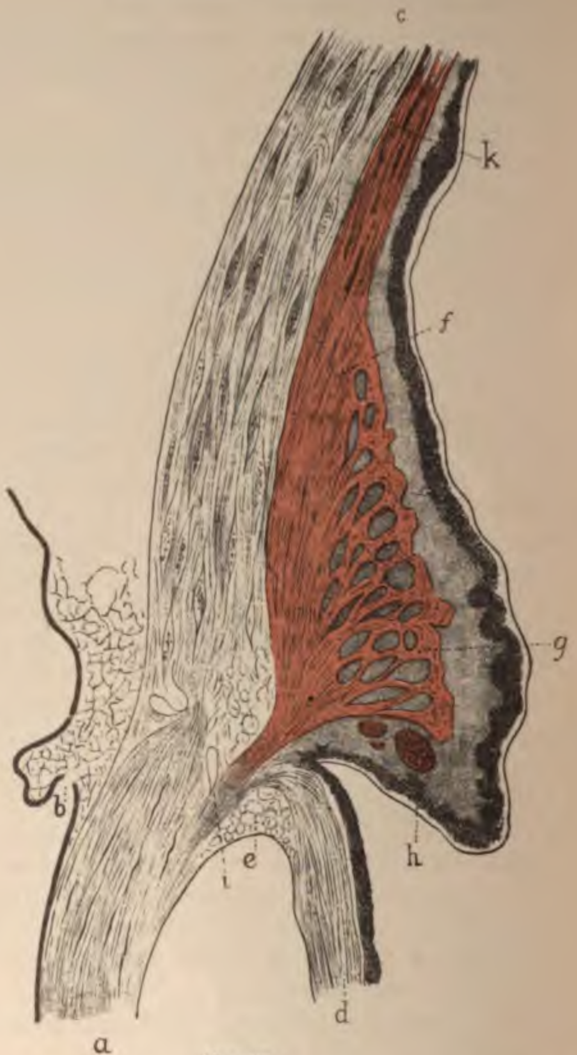

FIG. 68.

Ciliary muscle, after Iwanoff (Graefe and Saemisch, *l. c.*, I., p. 271). *a*, cornea ; *b*, corneal limb ; *c*, sclerotic ; *d*, iris ; *e*, Fontana's space ; *f*, meridional portion of the ciliary muscle ; *g*, radiating portion of the ciliary muscle ; *h*, annular portion, or circular muscle of Müller ; *i*, anterior tendon of the ciliary muscle ; *k*, posterior tendon of the meridional portion of the ciliary muscle.

This part of the ciliary muscle is the most variable. Iwanoff,[1] from whose writings we take, in part, this description, has ascertained that in certain myopic eyes the circular fibres may be entirely lacking

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[1] Iwanoff in Graefe and Saemisch, *Handbuch, &c.*, I., p. 276 (Fig. 6).

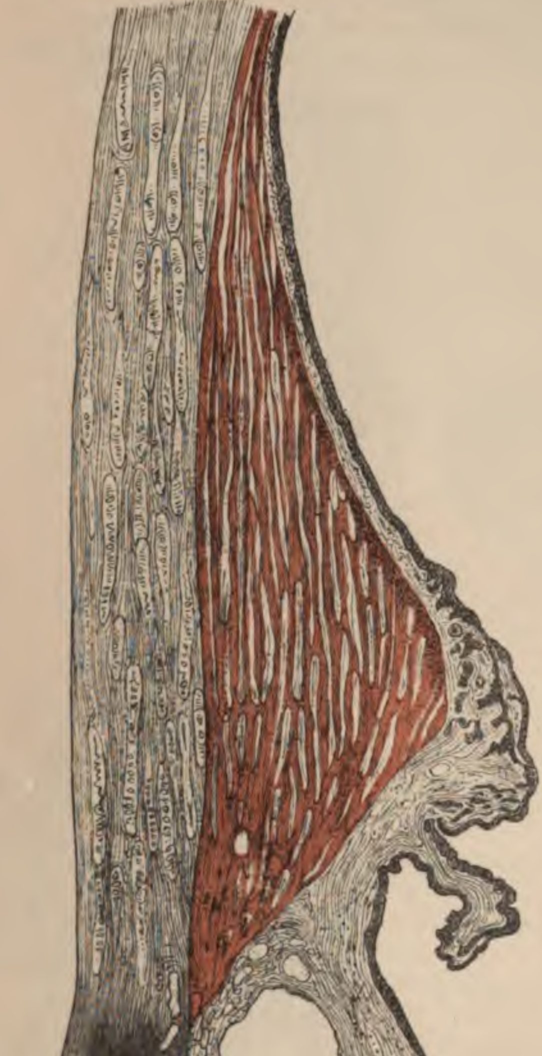

FIG. 69.

Ciliary muscle of a strongly myopic eye (after Iwanoff).

(Fig. 69). In this case the sclerotico-ciliary angle becomes acute and all the more so when, in the most pronounced cases, the radiating

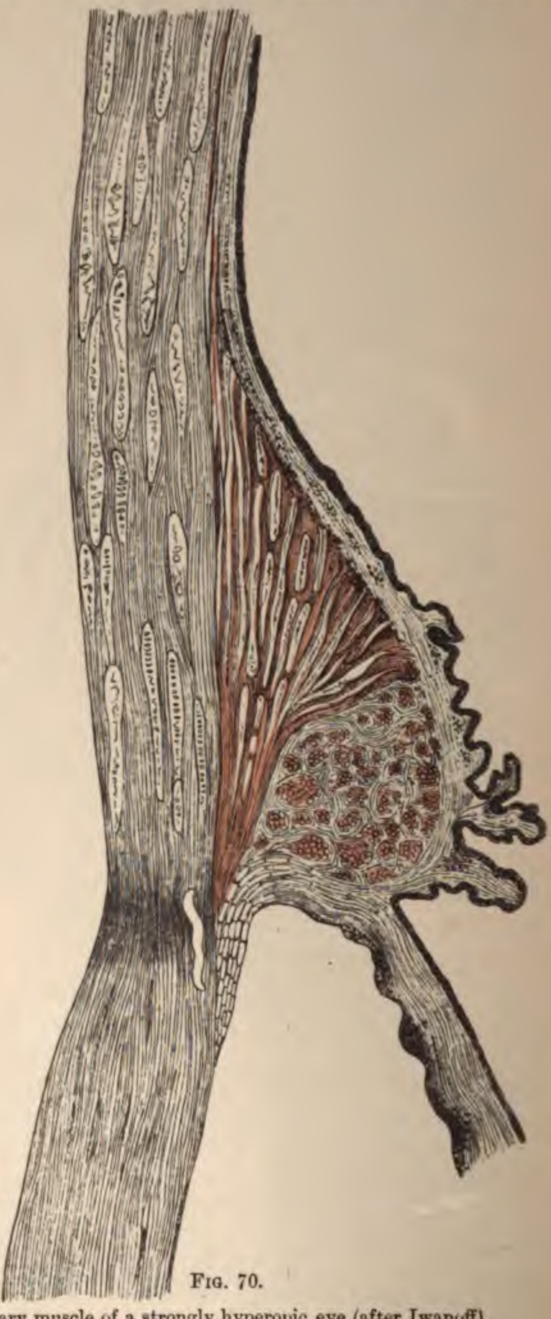

Fig. 70.

Ciliary muscle of a strongly hyperopic eye (after Iwanoff).

fibres have likewise diminished in number. This peculiarity of the muscle would, according to that author, be characteristic of axial myopia.

The other extreme is represented by those eyes in which the circular fibres are so highly developed that they form a third of the ciliary muscle. In this case the sclerotico-ciliary angle becomes obtuse [1] (Fig. 70). According to Iwanoff, it is especially in hyperopic eyes that this form of the ciliary muscle is met with.[2]

The entire *ciliary muscle* is, as we have said, included in the *ciliary body*. The inner portion of the latter is formed by the ciliary processes. These processes, about 78 in number, take the shape of a crown of radiating folds directed forward toward the axis of the eye (P *c*, Fig. 71).

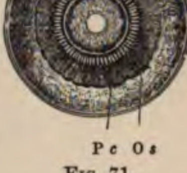

FIG. 71.

Equatorial section of the human eyeball; anterior portion, seen from behind. — P *c*, ciliary processes, forming the ciliary crown; O *s*, *ora serrata* (after Merkel, Graefe and Saemisch. *Handbuch*, VIII., p. 211).

The *crystalline lens* is suspended in the ring formed by the ciliary body, and is kept in place by the zone of Zinn.

The *zone of Zinn* is not a membrane, as is sometimes said, but an agglomeration of fibres having the nature of connective tissue. They take their origin, partially, at the *ora serrata*, from the ciliary portion of the retina, more especially from its internal limiting membrane (Merkel and Gerlach).

Most of the fibres of the zone of Zinn, however, arise from the intervals between the ciliary processes, and a few of them from the ciliary processes themselves (Fig. 67). Those fibres, of the zone of Zinn, which are situated farthest forward, direct themselves at once upon the anterior surface of the crystalline lens. The shorter ones all have a meridional direction, and attach themselves to both surfaces of the crystalline lens, where they become part of the capsule (Fig. 72).

But, with the exception of those situated farthest forward, which pass directly to the anterior face, and the most posterior ones, which go directly to the posterior surface of the lens, these fibres intercross in such a way that those coming from behind are attached to the anterior, while those arising in front are attached to the posterior surface of the crystalline.

Hence no circular canal exists about the equator of the crystalline lens, as would be the case if the zone of Zinn were represented by two membranes corresponding to the two surfaces of the lens. There are only lacunæ left open between the fibres, which intercross at a greater or less distance from the lens. We may therefore, at most, speak of Petit's space, as we do of Fontana's space at the base of the anterior chamber (Fig. 67), but not of Petit's canal.

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[1] Iwanoff, *loc. cit.*, p. 278, Fig. 7.

[2] We must guard against generalising, and assuming that, what Iwanoff has found to be true in cases of extreme ametropia, is indiscriminately true of all ametropic eyes. Our eminent friend, Dr. Loring, has made some very appropriate observations on this subject (Internat. Med. Congress, Philadelphia, 1876; and, *Is the Human Eye changing its Form, &c.?* New York, 1878), and we agree with him that it is advisable to suspend judgment concerning the anatomy of ametropic eyes, until more extended investigation of the matter has been made.

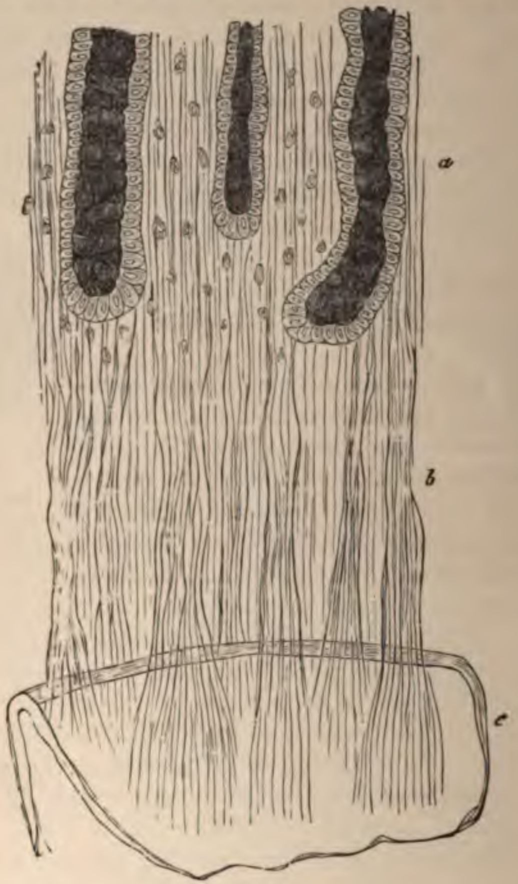

FIG. 72.

Insertion of the zone of Zinn upon the crystalline lens, seen from in front. The pigment of the detached ciliary processes has remained adherent to the non-plicated portion (*a*) of the zone of Zinn. The fibres of the plicated portion (*b*) are directed toward the equator of the lens, where they find insertion, as a bundle, into the anterior and posterior capsule (*c*). (Arnold, in Graefe and Saemisch, *Handbuch*, I., p. 307.)

The fibres of the zone of Zinn are inserted upon the anterior face of the crystalline, about 3 millimetres from the axis, and form a denticulate circle. This denticulation is due to the fact that the insertion is made by bundles of fibres, which have very minute interstices between them (Fig. 72). In this way is explained the communication which has been proved by Schwalbe and others to exist between the anterior chamber and Petit's space. There are, here and there, between the zonular fibres, lacunæ, which, in certain places, communicate with each other.

The insertion of the zone of Zinn into the posterior capsule seems to be situated somewhat nearer the periphery of the lens.[1] The union of this zone with the capsule of the lens is a genuine fusion, the tissue of the zonular fibres having the same histo-chemical properties as that of homogeneous membranes.

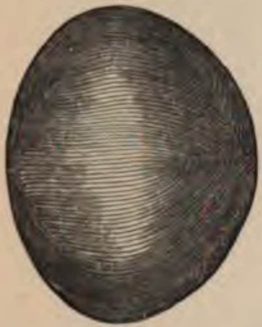

FIG. 73.

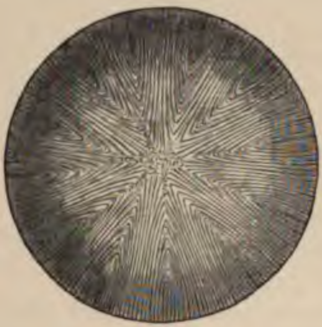

FIG. 74.

We do not need to review the well-known structure of the crystalline lens, composed of flat fibres, joined together by the denticulation of their edges and passing, strongly curved at the equator, from one surface to the other. Fig. 73, taken from Arnold's classical work, represents the crystalline lens of a new-born child. It is a side view, the anterior surface being turned toward the left, the posterior surface to the right. Fig. 74, from the same work, shows the posterior surface of the crystalline lens of an adult.[2]

The crystalline lens, in its normal condition, is compressible, elastic, and capable of bulging, or of becoming more or less flattened. In the living eye, the crystalline lens has its least convex form when in a state of repose, because it is then subject to the traction of the zone of Zinn, which flattens it. When it is removed from the eye, or simply released from the influence of the zone of Zinn, as occurs when the latter is ruptured, or the lens luxated, it yields to its elasticity, and assumes its most convex form.

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[1] Gerlach, *loc. cit.*, pp. 55-68.

[2] Arnold, in Graefe and Saemisch, *Handbuch*, I., p, 299, Fig. 9.

## MECHANISM OF ACCOMMODATION.

After having found the principle of accommodation, Helmholtz conceived a most ingenious theory of its mechanism, which he made known in 1867 in his classical work on physiological optics. It is based essentially upon the elasticity of the crystalline lens. The eminent physiologist admitted that the same thing that takes place pathologically, when the lens is released from the action of the zone of Zinn, occurs also in the normal condition, in consequence of the relaxation of its suspensory and tensor ligament. This relaxation of the zone of Zinn would, according to Helmholtz, be produced by the ciliary muscle, whose firm origin is at the base of the cornea (Gerlach's tendinous ligament). It draws the ciliary body toward this point. Hence the origin of the zone of Zinn is brought forward, and the traction exercised by it is diminished. The crystalline lens, abandoned to its elasticity, becomes more strongly curved, in proportion as the contraction of the muscle is stronger, the zone of Zinn more advanced and relaxed.[1]

The correctness of this theory has since been confirmed by the experiments of Hensen and Voelckers, and the observations of Coccius and Hjort.

Hensen and Voelckers[2] have operated on dogs, cats, monkeys, and upon a freshly enucleated human eye. In the living animal the eyeball was laid bare on the outer side, in order to be able to reach the ophthalmic ganglion. The ciliary nerves arising in this ganglion were irritated by an electric current.

The following are the results of such irritation, as observed by these two investigators:

The pupil contracts forcibly; the peripheral border of the iris is retracted, while its pupillary border becomes bulged.

The ciliary muscle contracts and draws the choroid toward the cornea, which forms, therefore, its firm origin. This important observation was made, in part, by means of an opening cut in the junction of the cornea and the sclerotic, opposite the ciliary muscle. Thus the contraction of that muscle and the advancement of the choroid were viewed directly.

Again, pins stuck perpendicularly into different parts of the choroid indicated, by the swaying of their free ends, that by the contraction of the ciliary muscle, this membrane is drawn toward the anterior portion of the globe.

[1] Helmholtz, *Handbuch der physiologischen Optik*, p. 110.

[2] Hensen and Voelckers, *Experimentaluntersuchung über den Mechanismus der Accommodation*, Kiel, 1868, and *Archiv f. ophth.*, xix., 1, 1873.

When only one ciliary nerve is irritated, the contraction of the iris and ciliary muscle, and the advancement of the choroid, take place only in one isolated portion.

The changes of form in the crystalline lens, produced by the irritation of the ciliary nerves, were the same as those described by Langenbeck, Cramer, and Helmholtz as the cause of accommodation. Hensen and Voelckers simply add, to the variations of images reflected from the surfaces of the crystalline lens, other conclusive proofs in support of an increase of curvature in that organ.

They introduced, through the sclerotic and cornea, needles, one of whose ends, in each case, was in contact with either the anterior or posterior surface of the lens.

Upon electrizing the ciliary muscle, the free extremity of the anterior needle was carried backward, that of the posterior needle forward. This proves conclusively that the curvature, of both surfaces of the lens, increases during the act of accommodation. The increase of curvature was greater for the anterior than for the posterior surface. The latter was, however, marked enough to be observed directly through the opening made in the sclerotic.

The equator of the lens not being accessible to direct examination, needles were again employed. These, being implanted in it, indicated that, by a contraction of the ciliary muscle, it is caused to advance.

The retina accompanies the choroid in its movements, as one may convince one's self by observation of its vessels. These movements of the retina, under the influence of the accommodation, seem to be proved also by the accommodation-phosphene—a phenomenon observed by Purkinje and Czermak. When, in a dark room, we suddenly relax our accommodation, we notice the appearance of a luminous circle at the periphery of our field of vision. This sensation of light within the eye is to be ascribed to an irritation of the peripheral portions of the retina. The experiments of Hansen and Voelckers seem adequate to explain its production. At the moment when the traction of the ciliary muscle ceases (relaxation of the accommodation), the zone of Zinn quickly renews its tension, while the crystalline lens yields more slowly to the traction thus brought to bear upon it. In this way, the border of the retina, being closely connected with the choroid zone, is stretched and irritated by the sudden relaxation of the accommodation, until the lens has resumed its flattened form.

The movement of the choroid, which has been pointed out by these two investigators, is of the highest practical importance. It explains the injurious influence which the accommodation must exert in affections of the deeper membranes of the eye. We see it especially in the production and progress of myopia, which is so often due to choroiditis.[1] Again, these experiments explain the favourable, calming influence which mydriatics, exert in such affections, by paralysing the ciliary muscle and suppressing the mobility of the affected membranes.

The zone of Zinn was examined in a similar way, *i.e.*, by means of a glass thread introduced through the sclerotic and cornea, and applied to these fibres. The free extremity of the thread moved backward each time the ciliary nerves were irritated. This proves that the zone is drawn forward by the contraction of the ciliary muscle.

With regard to the *ciliary processes*, they were not clearly visible through the sclerotic opening, and the conclusion was rather indirectly reached by the experimenters, that these processes *advance* during accommodation.

This deficiency has, however, been largely supplied by Coccius and Hjort. These *savans* performed their experiments in a way entirely different from that in which those of Hensen and Voelckers were made.

Coccius observed eyes upon which large peripheric iridectomies had been performed. Through these openings the ciliary processes, zone of Zinn and the equator of the crystalline lens became accessible to direct observation. In order to increase the exactness of his observation, Coccius used, at times, a microscope furnished with an ophthalmoscope-mirror, and, at other times, a binocular ophthalmoscope, with oblique illumination.

The light and instrument being well directed, he had the person under examination fix, first, a distant object: then he caused an effort of accommodation by bringing the object nearer to the eye.

Our schematic figure (75) represents the phenomena which Coccius was able to observe when the eye passed from the state of rest (R) to that of accommodation (A). We have submitted it to the criticism of that author, who has kindly furnished the amplest details, and informed us, by letter, that the facts published by him several years ago have been confirmed in every respect by observations which he has since made. The following are their results:

During accommodation the ciliary processes (*p p*, Fig. 75) advance, and increase in size. The advancement is due to the traction which the contraction of the ciliary muscle exerts upon the ciliary body; the enlargement probably to a compression, of the veins, which accompanies it.

The zone of Zinn advances; the space included between the equator of the lens and the periphery of the zone is broadened (C J and C′ J′).

The equatorial edge of the crystalline lens (C C and C′ C′) becomes slightly broader, which fact has been noticed also by O. Becker.

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[1] Landolt, *On Myopia*, Royal London Ophth. Hosp. Reports, Dec. 1879.

The zonule is seen to have the form of a series of radiating bandelets, alternately light (*a*) and dark (*b*). The former correspond to the fibres of the zonule, which are situated most anteriorly, and hence exposed to the light in oblique illumination; the dark ones represent the re-entering folds of the zonule, which are not illuminated.

The folds of the zonule are not in direct relation with the ciliary processes. The latter are entirely free in the aqueous humor of the posterior chamber; they are, moreover, less numerous than the folds of the zonule.

When the eye makes an effect of accommodation, the darker striæ become broader, the brighter ones narrower.

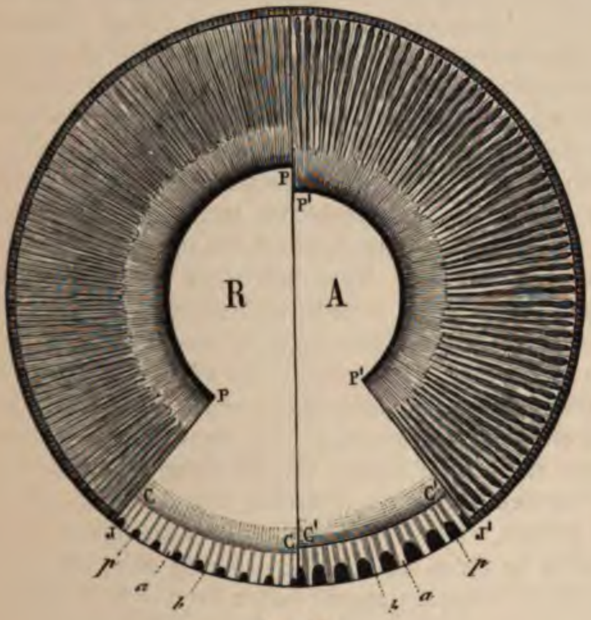

FIG. 75.

Schematic representation of the changes observed by Coccius in the eye during accommodation. Left half (R), state of repose. Right half (A), state of accommodation. P P and P′ P′, pupillary border; P J and P′ J′, edge of the iridectomy; C C and C′ C′, equator of the crystalline; *p*, ciliary processes; *a*, advanced portion of the zone of Zinn (lighted up); *b*, re-entering fold of the zone of Zinn (in the shadow).

The change in form, which the equator of the crystalline undergoes, manifests itself by the broadening of the dark circle which characterises its border (C′ C′), and which is due to total reflection of the light. The equator of the crystalline always remains at a certain distance from the ciliary processes. Hence there can be no question of compression exerted by the ciliary processes upon the edges of the crystalline.

Professor Hjort of Christiania[1] has had an even more favourable opportunity of observing the mechanism of accommodation in the living subject. A man, thirty years old, had his iris entirely removed by an explosion of dynamite. It must have escaped through a small wound in the cornea. The eye recovered, after a short time, so well, that the acuteness of vision became normal, and the range of accommodation equal to 5·8 D—that is to say, little below that of the other eye, which amounted to 7 D.

The whole of the ciliary region was accessible to observation. This was made partly with the naked eye, partly with the aid of Bücke's lens, and frequently repeated. Even during voluntary accommodation, the above-described phenomena were easy of verification; but they became particularly clear under the influence of Calabar extract.

Hjort's observations correspond very well with those of Coccius. The edge of the crystalline lens becomes more obtuse, and approaches the centre; the ciliary processes project, swell, &c. There is, however, one little point on which the two observers differ. While, according to Coccius, the space, between the equator of the lens and the ciliary processes, increases, Hjort has never observed any change in its diameter. This quite accords with Helmholtz's theory, whereas Coccius' observation does not seem to agree entirely with it,—at least if we are to regard an increase of the ciliary space as implying a stronger tension of the suspensory ligament of the lens. The stronger curvature of the latter ought, on the contrary, to be caused by a relaxation of the zone of Zinn. But the disagreement of the accounts given by the two authors is, probably, only apparent. If the ciliary processes project, while the crystalline lens diminishes in a direction perpendicular to them, an observer, looking obliquely into the eye, may find the ciliary space appear larger, even if, in reality, it remains unaltered. In other words, it may be that the space, between the lens and the ciliary processes, appears, in perspective, more shortened during rest than during accommodation.

There is still another change produced in the eye, as an effect of accommodation, which is easier to perceive than the phenomena we have just been describing. *The pupil contracts* during accommodation (P′ P′, Fig. 75), and dilates when the latter is relaxed (P P). If we have not previously spoken of this phenomenon, which was long ago noticed, it is because, contrary to the supposition which formerly prevailed, the iris plays only a very unimportant *rôle* in accommodation.

The proof of this is, that the faculty of adapting the eye for near vision is entirely lost when the crystalline is removed, or withdrawn from the influence of the ciliary muscle, even though the iris remains entirely intact; while, on the other hand, accommodation may exist, even in the case of aniridy, when the iris is entirely absent (von Graefe, Hjort, and others).

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[1] Hjort, *Klin. Monatsbl. f. Augenheilk.*, p. 205, 1876.

It might be supposed that the variations in the diameter of the pupil, though not taking part directly in the accommodation, properly so-called, possessed, nevertheless, some utility in modifying the luminous intensity of the retinal image, proportionately to the distance of the object of fixation. But Olber's calculations[1] prove that, when the object is brought toward the eye from a distance of 756 to one of 108 millimetres, pupillary contraction diminishes the luminous intensity in the ratio of 37·3 to 17·5, whereas this intensity would increase only from 1·012 to 1·073 if the pupil remained immobile.

Donders[2] has, moreover, observed that the contraction of the pupil is not absolutely synchronous with accommodation, but takes place somewhat later.

It has been proved, experimentally, that a little more time is required for the production of accommodation than for its relaxation. Thus Vierordt[3] finds that the change in adaptation from 18 metres to 0·1 of a metre requires 1·18 second of time, while the corresponding relaxation is brought about in 0·87 of a second. Aeby[4] estimated two seconds as the time he needed to change his accommodation from adaptation for a distance of 430 to that for 115 millimetres, and only 1·2 second as necessary for the production of the inverse change.

All experiments and observations confirm Helmholtz's theory as to the mechanism of accommodation, of which theory we repeat the statement, as follows:—

*The contraction of the ciliary muscle causes the zone of Zinn to advance, and thus diminishes the traction exerted by the latter upon the crystalline. The crystalline, abandoned to itself, assumes the form which the elasticity of its fibres naturally give it, and becomes more convex, especially its anterior surface.*

*When the innervation ceases, the ciliary muscle is relaxed; the ciliary processes become tense, and stretch the zone of Zinn, which, in turn, flattens the crystalline by exerting upon it a traction in the direction of its equator.*

The *nerves of accommodation, i.e.*, of the ciliary muscle, belong to the third pair (*motor oculi communis*), which contains, as well, the fibres distributed to the sphincter of the pupil. Their origin, according to Hensen and Voelckers,[1] seems to be in the floor of the fourth ventricle.

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[1] Olbers, *De mutationibus oculi internis. Diss. inauguralis:* Göttingen, 1780.

[2] Donders, *Pupilbeweging bij Accommodatie. Arch. v. Genees- en Natuurk.*, ii.

[3] Vierordt, *Arch. f. physiol. Heilk.*, N. F. i., p. 17, 1857.

[4] Aeby, *Zeitschrift f. rat. Medicin.*, N. F. xi., p. 300, 1861.

Irritation of the *anterior* portion of the latter produces *accommodation;* that of a portion situated *farther back* causes *contraction of the pupil*, and, when the part, where the fourth ventricle passes into the *aquæductus Sylvii*, is irritated, contraction of *rectus internus* is produced.

The fibres which control *dilatation* of the pupil arise from the cervical portion of the *great sympathetic.* According to Hensen and Voelckers, not all of these fibres pass through the ophthalmic ganglion.

## MEASURE OF THE ACCOMMODATION—THE RANGE OR AMPLITUDE OF ACCOMMODATION.

Accommodation is, as we have seen, due to a muscular effort, in consequence of which the refractive power of the eye is increased.

In a state of *rest*, the dioptric system of the eye presents its *minimum of refractive power;* the eye is adapted to the most distant point that it can see distinctly, *i.e.*, to its *punctum remotum.*

During a condition of *greatest possible accommodation*, on the contrary, the dioptric system has acquired the *maximum of refractive power* of which it is susceptible, and the eye is adapted to the nearest point which it can see distinctly, *i.e.*, to its *punctum proximum.*

The *total accommodative power* which an eye can bring into play—the *range of accommodation*—is, therefore, represented by the difference between its refraction when at rest, and the refraction when under the influence of the maximum effort of accommodation.

We have seen, above, that the refraction of the eye is inversely proportional to the distance of the object which it sees distinctly. The refraction must be stronger in order to focus, upon the retina, rays coming from the object, in proportion as the object is nearer.

If we let R represent the distance of the *punctum remotum* from the eye, the refraction, at rest, is—

$$r = \frac{1}{R}.$$

This is the *static refraction* of the eye, which we shall hereafter designate by $r$. It is, as we know, the minimum of refraction of the eye.

---

[1] Hensen and Voelckers, *Arch. f. Ophth.*, xxiv., 1.

If P be the distance between the *punctum proximum* and the eye, the refraction, in a condition of *maximum accommodation*, is—

$$p = \frac{1}{\mathrm{P}}.$$

This represents the maximum of refractive power possessed by an eye.

The difference between the two, the *range of accommodation*, is therefore—

$$\frac{1}{\mathrm{P}} - \frac{1}{\mathrm{R}}$$

or, if P and R be measured with the metre,

$$p - r \text{ dioptries.}$$

The accommodation has the same effect as a convex lens added to the eye. Let us suppose an eye devoid of the power of accommodation: it is adapted to its *punctum remotum* *R* (Fig. 76), and will not be able to see anything nearer than that—an object at *P* for instance. The rays emanating from this point are too divergent to be focused upon the retina of such an eye. To render vision, at the distance of *P*, possible, the divergence of rays coming from *P* must be diminished until they have such a direction as will make them appear to come from the point *R*, to which the eye is adapted.

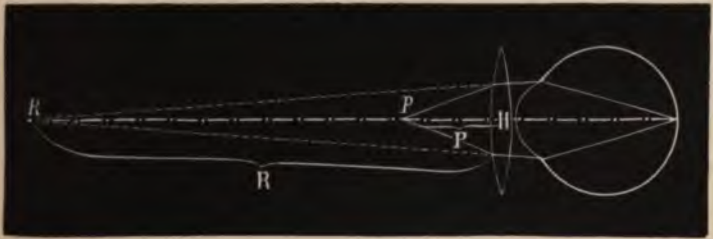

FIG. 76.

Or, what amounts to the same, the refractive power of the eye must be increased sufficiently to cause rays, which diverge from a point nearer than its *punctum remotum*, to reunite upon its retina. This effect is obtained by adding to the eye a convex lens, which diminishes the divergence of the rays given off from *P*, just enough so that they seem to come from *R*, or, in other words, a lens which supplies the deficit in the refractive power of the eye.

If *P* is the *punctum proximum* of the eye, the lens which makes vision at the distance H *P* possible has, therefore, the same effect as the accommodation, and nothing is more natural than to express the amplitude of accommodation by the lens which is its equivalent.

The analogy between the accommodation and a convex lens is all the more complete as the increase in curvature of the crystalline, during accommodation, may be likened to a convex meniscus added to the crystalline at rest, and applied to its anterior surface (Fig. 67).

It is true that, in order to be exactly comparable to the amplitude of accommodation, the lens ought to be placed *within* the eye. However, no great error is committed by applying it directly to the cornea, so that it may be regarded as forming one whole with the dioptric system of the eye. In reckoning its focal distance, then, we take as starting-point the cornea, or, strictly speaking, the first principal point H′ of the eye, as we did in the case of refraction of the eye at rest (p. 128).

Being comparable to a lens, the amplitude of accommodation may therefore be expressed as a lens, either by the inverse of its focal distance, or in units of refraction, *i.e.*, in *dioptries.*

Let us represent by A the focal distance of the lens whose effect is equal to the amplitude of accommodation, and we shall have, for the latter, the following formula:—

$$\frac{1}{A} = \frac{1}{P} - \frac{1}{R}$$ [1]

This is the form given by Donders to the expression of the range of accommodation.[2]

Hence the three terms of this equation represent lenses.

$\frac{1}{R}$ is the lens which corresponds to the state of rest of the eye when it is adapted to its *punctum remotum*, $\frac{1}{P}$ is the lens which corresponds to the eye when it is fixing its *punctum proximum*, and $\frac{1}{A}$ the lens which must be added to the former to obtain the power of the latter —the lens which changes adaptation for the *punctum remotum* into adaptation for the *punctum proximum*, the total change of refraction of which the eye is susceptible—*the range or amplitude of accommodation.*

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[1] Those of our readers who have read the first chapter will see at once that this formula is identical with the one for the conjugate foci of lenses (15 *b*, p. 52, and 15 *c*, p. 58). The *punctum remotum* and *punctum proximum* are, in fact, conjugate foci. In the example selected (myopia), the lens $\frac{1}{A}$ does not cause rays coming from the *punctum proximum* P to converge toward the *punctum remotum* R; but it gives to rays emanating from P the direction which they would have if they came from R, which, in principle, amounts to the same. Only, in this case, both conjugate foci are situated on the same side of the lens, and for this reason the third term of the equation takes the *minus* sign. In hyperopia, on the contrary, we shall find P and R oftenest on opposite sides of the system, and all three terms of the equation positive.

[2] Donders, *Anomalies, &c.*, p. 28.

A, P, and R are linear values which can be expressed in either millimetres, centimetres, or metres. If we express them in *metres,* we obtain the value of the lenses expressed in dioptries (p. 71). In this case we have for the *range of accommodation* the formula—

$$a = p - r$$[1]

in which $a$ equals the number of dioptries represented by the range of accommodation; $p$ equals the number of dioptries represented by the eye when adapted to its *punctum proximum,*—that is to say, the maximum of refraction of which an eye is susceptible; and $r$ equals the number of dioptries represented by the eye when it fixes its *punctum remotum,*—that is to say, when it is at rest, and has its minimum of refracting power.

Let us take an example:—

An eye, when at rest, sees distinctly at a distance of 50 centimetres, —in other words, its *punctum remotum* (*R*, Fig. 76) is 50 centimetres distant, or the distance R equals 50 centimetres $= \frac{1^{\text{m.}}}{2}$ By bringing all its accommodative power into action, it distinguishes, clearly, small objects just at a distance of 9 centimetres, but not nearer. This is the location of its *punctum proximum,* P. Hence $\text{P} = 9^{\text{cm.}} = \frac{1^{\text{m.}}}{11}$

The range of accommodation, therefore, in this case is equivalent to a lens $\frac{1}{\text{A}} = \frac{1}{9} - \frac{1}{50} = \frac{1}{10{\cdot}97}$. These 10·97 centimetres represent the focal distance (A) of the lens which expresses the range of accommodation.

Expressing our distances in terms of the metre, we obtain for 9 centimetres $\frac{1^{\text{m.}}}{11}$, hence $p = 11{\cdot}1$ D; and for 50 centimetres $\frac{1^{\text{m.}}}{2}$, hence $r = 2$ D, and $a = 11{\cdot}1 - 2 = 9{\cdot}1$ dioptries.

An eye which, when at rest, is adapted to a distance of $\frac{1^{\text{m.}}}{2}$, has a myopia of 2 D. If, with its maximum of accommodation, it is adapted to 9 centimetres, it represents 11·1 dioptries of positive refraction. The difference in refractive power, between the two, is the amplitude of accommodation.

*Range of accommodation in myopia.*—Suppose that a myope of three dioptries sees distinctly as near as 111 millimetres, *i.e.,* $\frac{1^{\text{m.}}}{9}$; what is his range of accommodation?

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[1] Landolt, *The Introduction of the Metric System in Ophthalmology,* London, 1876; and in De Wecker et Landolt, *Traité complet,* vol. i., p. 662.

In a state of rest, his refraction is represented by $r = 3$ D; in a state of maximum accommodation, by $p = 9$ D.

Whence we have:

$$a = 9 - 3 = 6 \text{ D.}$$

Inversely, what degree of myopia has an eye which, with a range of accommodation of 4 dioptries, sees objects distinctly as near as 5 centimetres? 5 centimetres correspond to 20 D. Hence we may put the question as follows:—What must be the refraction of an eye at rest, if, upon adding 4 D to it, by the maximum effort of accommodation, it has become equal to 20 D? Evidently its refraction was four dioptries less: $r = 20 - 4 = 16$ D. That is to say, the eye was myopic by 16 D.

We have solved the problem by means of our formula $r = p - a$. We can also do it with that of Donders: $\frac{1}{A} = \frac{1}{P} - \frac{1}{R}$. The focal distance (A) of the lens, which corresponds to an amplitude of accommodation of 4 dioptries, is 25 centimetres.

Hence we write—

$$\frac{1}{R} = \frac{1}{5^{\text{cm.}}} - \frac{1}{25^{\text{cm.}}} = \frac{1}{6 \cdot 25^{\text{cm.}}}$$

Performing the calculation: $\frac{1^{\text{m.}}}{6 \cdot 25^{\text{cm.}}}$, we obtain, in fact, 16 dioptries.

The distance A, in the formula for accommodation, has no direct meaning. It only represents the converse of the refractive power which is equivalent to the accommodation; but it is not to be confounded with the distance between the *punctum remotum* and the *punctum proximum*. This latter distance ($R\,P$ in Fig. 76), which has been called by Donders the *region of accommodation*, represents the entire extent controlled by the eye, within which the eye can distinguish objects clearly, from a state of rest up to that of maximum accommodation. But this distance does not give a proper idea of the accumulative power of the eye. Thus, an emmetrope who sees distinctly objects between infinity and 25 centimetres, has an enormous region of accommodation, since his *punctum remotum* ($R$) is at infinity. His range of accommodation, on the other hand, is only $a = p - r$, in which $p = 4\,d$ and $r - 0$; hence $a = 4$ dioptries. A myope of 6 D, whose *punctum remotum* is 16 centimetres distant, and who, by bringing his entire power of accommodation into play, sees objects as near as 5 centimetres, controls only $16 - 5 = 11$ centimetres of region of accommodation; but his range of accommodation is $a = 20 - 6 = 14$ D, hence much greater than that of our emmetrope. More refractive power is necessary to change the adaptation from 16 to 5 centimetres than to change it from infinity to 25 centimetres.

*Range of accommodation in Emmetropia.*—When the eye is *emmetropic*, its *punctum remotum* is at infinity ($R = \infty$), and the lens by which the refraction of the eye, at rest, may be expressed, becomes $r$ or $\frac{1}{R} = \frac{1}{\infty} = 0$.

Hence we have, for the *range of accommodation in emmetropia:*

$$\frac{1}{A} = \frac{1}{P} - \frac{1}{\infty}$$

$$\frac{1}{A} = \frac{1}{P} - 0$$

$$\frac{1}{A} = \frac{1}{P}$$

or

$$a = p.$$

That is to say, the range of accommodation is, in this case, equal to the refraction presented by the eye, when it is adapted to its *punctum proximum.*

We may express the same thing in still another way.

The *punctum remotum* of emmetropia being at infinity, the lens which supplies the place of the accommodation in adapting the eye to its *punctum proximum,* ought to give rays coming from the *punctum proximum* a parallel direction, as if they came from infinity. This is possible only when the focus of the lens and the *punctum proximum* coincide. Hence the focal distance (A), of the lens which represents the range of accommodation, and the distance (P), of the *punctum proximum,* are, in this case, equal : $A = P$. Consequently, the refractive power of the lens ($p$), which represents the eye when adapted to its *punctum proximum,* is equal to the range of accommodation : $a = p$.

Let us take an example :—

An emmetrope who sees at a distance of 125 millimetres, or $\frac{1^{m.}}{8}$, has a range of accommodation of $\frac{1^{m.}}{8} = \frac{1000^{mm.}}{125}$ or 8 dioptries:

$$a = p = 8 \text{ D};$$

and an emmetrope who has an amplitude of accommodation of 5 dioptries sees at the focal distance of $5 \text{ D} = \frac{1^{m.}}{5}$ or 20 centimetres.

*Range of accommodation in Hyperopia.*—Let us now take the case of a hyperope, and determine his *range of accommodation.* Suppose that his *punctum proximum,* like that of the emmetrope in the preceding case, be 125 millimetres or $\frac{1^{m.}}{8}$ distant.

His range of accommodation must necessarily be greater than that of the emmetrope. In order merely to render himself emmetropic, the hyperope has been forced to exert an effort of accommodation to fill the place of such a convex lens as would have corrected his ametropia. After this he must still add to his dioptric system the same lens as the emmetrope needs to adapt his eye to its *punctum proximum.*

The first of these lenses is equal to $\frac{1}{R}$ or $r$, R being the distance of the *punctum remotum* from the hyperopic eye; and the second lens is $p$ or $\frac{1}{P}$. We have, then, for the *amplitude of accommodation of the hyperope,* the formula—

$$\frac{1}{A}=\frac{1}{P}+\frac{1}{R}$$

or

$$a=p+r.$$

We may consider the matter in still another way.

In hyperopia the *punctum remotum* is behind the eye. Hence R is negative. The expression $\frac{1}{R}$ or $r$ is, therefore, also negative. This is why we have said that hyperopia is comparable with a concave (negative) lens added to the eye. The formula for the amplitude of accommodation becomes, then:

$$\frac{1}{A}=\frac{1}{P}-\left(-\frac{1}{R}\right)=\frac{1}{P}+\frac{1}{R}$$

or

$$a=p-(-r)=p+r.$$

*Examples.*—The *punctum proximum* of a hyperope of 3 dioptries is 125 millimetres distant. What is his amplitude of accommodation? When adapted to the point *P*, this eye represents a refractive power of $\frac{1}{125^{\text{mm.}}}$ or $\frac{1}{1^{\text{m.}}/8}=8$ dioptries. But, before being able to see at this short distance, the hyperope has been obliged to correct his ametropia, by making an effort of accommodation corresponding to the degree of such ametropia. Hence the total of his range of accommodation is $3+8=11$ D.

Or, again:

The *punctum remotum* of the hypermetrope of 3 D is $\frac{1^{\text{m.}}}{3}$ or 333 millimetres behind the eye, and the formula for accommodation gives—

$$\frac{1}{A}=\frac{1}{125}+\frac{1}{333}=\frac{1}{9{\cdot}08}=11 \text{ D}$$

$$\frac{1}{A}=\frac{1}{1^{\text{m.}}/_8}+\frac{1}{1^{\text{m.}}/_3}=\frac{1}{1^{\text{m.}}/_{11}}=11 \text{ D}$$

or

$$a=8+3=11 \text{ D}.$$

Let us take another case, and ask: Where is the *punctum proximum* of a hyperope of 3 D, who commands 8 D of range of accommodation? We shall reason as follows:—

If he have a hyperopia of three dioptries, he must first expend 3 dioptries of his accommodative power before his eye will focus parallel rays upon its retina. There remain, therefore, 5 dioptries with which to change his adaptation to that for a finite distance. This distance will be equal to the focal distance of 5 dioptries, *i.e.*, $\frac{100^{\text{cm.}}}{5} = 20$ centimetres, or with our formula—

$$a = p + r.$$

We deduce from this—

$$p = a - r = 8 - 3 = 5 \text{ D},$$

or, again, with Donders' formula:

$$\frac{1}{A} = \frac{1}{P} + \frac{1}{R}$$

$$\frac{1}{P} = \frac{1}{A} - \frac{1}{R} = \frac{1}{12{\cdot}5^{\text{cm.}}} - \frac{1}{33{\cdot}3^{\text{cm}}} = \frac{1}{20^{\text{cm.}}}$$

In hyperopia it may also happen that, although possessing a certain amount of accommodative power, the eye does not succeed in seeing at a finite distance. This is the case when the range of accommodation is less than the degree of the hyperopia. Here the accommodative power succeeds only in diminishing the degree of the hyperopia—in increasing the *negative* distance of the point to which the eye is adapted and in bringing it *toward* infinity, but not in causing it to pass to the *front* of the eye, at a *finite* distance.

Thus, an eye which is hyperopic by 5 D, having a range of accommodation of 3 D, will succeed in correcting a considerable portion of its hyperopia. But its refraction will always be 2 D weaker than emmetropia; it will always be hyperopic, negative, *i.e.*, adapted to rays converging toward a point behind the eye. Its power of refraction, in a state of rest, is expressed by $r = -5$ D; its range of accommodation by $a = 3$ D, and its refraction, during adaptation to its *punctum proximum*, by—

$$p = a + (-r) = a - r = 3 - 5 = -2 \text{ D}.$$

The *punctum proximum* will therefore be at $-\frac{1^{\text{m.}}}{2}$, *i.e.*, 50 centimetres *behind* the eye. The same result is obtained by Donders formula:

$$\frac{1}{P} = \frac{1}{A} + \left(-\frac{1}{R}\right) = \frac{1}{A} - \frac{1}{R} = \frac{1}{33{\cdot}3} - \frac{1}{20} = -\frac{1}{50};$$

hence

$$P = -50 \text{ centimetres.}$$

## THE INFLUENCE OF AGE UPON REFRACTION AND ACCOMMODATION.

A. *Static Refraction.*—The refractive power of the eye *at rest*—the position of the *punctum remotum*, in other words—does not change until toward the age of fifty-five years. An emmetrope always sees at infinity; a myope, whose *punctum remotum* is 25 centimetres or $\frac{1^{\text{m.}}}{4}$ in front of the eye, always needs a concave glass of 4 D, in order to see at a distance, and the *punctum remotum* of a hyperope of 2 D is invariably $\frac{1^{\text{m.}}}{2}$, or 50 centimetres behind his eye.

It is only from this age on that we find a change in the optical condition of the eye, of such a kind that its *refractive power diminishes.* The (positive) *punctum remotum* recedes from the eye, $+\text{R}$ becomes greater and $+\frac{1}{\text{R}} = r$ becomes less; the negative *punctum remotum* of the hyperope comes nearer to the eye, $-\text{R}$ diminishes, $-\frac{1}{\text{R}} = -r$, *i.e.*, the degree of the hyperopia, increases.

The emmetrope commences to get hyperopic (acquired hyperopia), the myope notices a decrease, in his myopia, proportionate with the recession of his *punctum remotum*, and the hyperope is conscious of an annoying increase in his hyperopia.

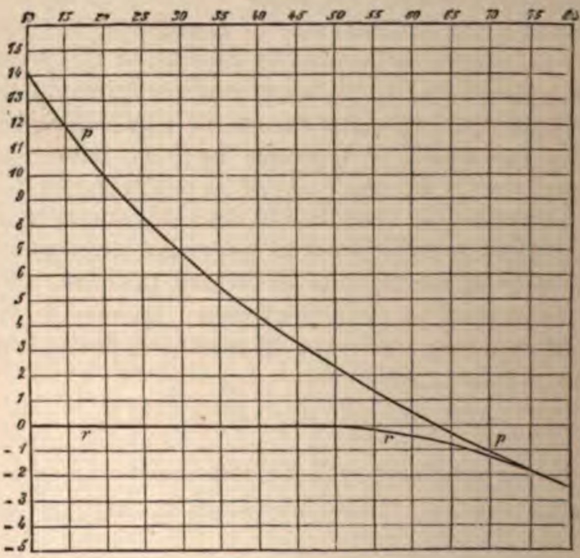

FIG. 77.

This diminution of the refractive power of the eye is, indeed, independent of the nature of its refraction. It affects the emmetrope in the same degree as the myope or hyperope. It depends solely upon the age, and follows a well-defined law which Donders has pointed out.[1] This law is explained by the curve $r\,r$ in the diagram (Fig. 77).

The figures placed horizontally at the top of the diagram, corresponding to the ordinates, indicate the *age;* the figures at the left, opposite the abscisses, stand for numbers of *dioptries.* The curve $r\,r$ corresponds to the refraction of the eye when at rest. As has been said above, $r = \frac{1}{\mathrm{R}}$, which designates the static refraction of the eye, is equal to zero, in emmetropia, because, in this case, $\mathrm{R} = \infty$. It is for emmetropia that this scheme was originally drawn.

Hence the line $r$ commences at the zero-point of the division. It is straight and coincident with the zero-line until just before reaching the ordinate 55, where it commences to descend and enter the negative portion of the diagram. This indicates that, between fifty and fifty-five years of age, the refraction begins to diminish, and becomes negative for the emmetrope, *i.e.*, he becomes hyperopic. His *punctum remotum* changes its position and passes from infinity to the rear of the eye; R becomes negative, and, consequently, $\frac{1}{\mathrm{R}} = r$ also.

At sixty years of age, the refraction has diminished one-third of a dioptry. The *punctum remotum* is about 3 metres behind the eye. At the age of sixty-five, the static refraction is $\frac{3}{4}$ths of a dioptry, at sixty-eight one whole dioptry, and at eighty years of age, 2·25 below what it was before the age of fifty-three years. At eighty years of age a person who was emmetropic up to the age of fifty-three, is hyperopic by 2·25 dioptries, and his *punctum remotum* is $\frac{1^{\text{m.}}}{2{\cdot}25} = 444$ millimetres behind the eye.

In the case of *myope*, the curve $r\,r$ is exactly the same as for the emmetrope. The diminution of static refraction is the same; only, the position of the *punctum remotum*, in the two cases, is not identical.

Let us suppose a person to have a myopia of 3 dioptries. We shall only have to carry the curve $r\,r$ into the positive portion of the diagram (since myopia represents an excess of refractive power), and apply it to the abscissa 3. We shall then notice that, up to the age of fifty-three, the myopia remains equal to 3 dioptries; that thereafter it diminishes in like proportion as the acquired hyperopia of the emmetrope increased; and that, at the age of eighty, the curve $r\,r$ is one-fourth of a division below the line $+ 1$, *i.e.*, the myopia is of 0·75 dioptry. It has diminished by $3 - 0{\cdot}75 = 2{\cdot}25$ dioptries, like the refraction of the emmetrope.

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[1] Donders, *loc. cit.*, p. 204.

Hence a myope of 2·25 dioptries would be emmetropic at this age, and a person who is myopic by less than 2·25 dioptries will become at first emmetropic and then hyperopic. Upon applying the curve *r r* to the abscissa 1, for instance, as for a myopia of one dioptry, it will cross the zero-line and descend into the negative portion of the scheme, between sixty-five and seventy years.

In the case of *hyperopia* the entire curve *r r* will be below the zero, in the *negative* part of the diagram, because hyperopia may be regarded, as was shown above, as deficiency of refractive power. The decrease in refraction, which is due to advancing age, is added to the original hyperopia, and increases it.

Let us take the case of a hyperope of 2 D. His refraction does not change before the age of fifty-three years; but, from this time on, he will notice that his hyperopia increases comformably to the curve *r r* applied to abscissa number $-$ 2. At the age of eighty, when the curve is 2¼ divisions below the horizontal, the hyperopia will amount to $2 + 2{\cdot}25 = 4{\cdot}25$ dioptries.

*The diminution in the refractive power of the eye, which corresponds to the increase of age, is attributable to a change in the constitution of the crystalline.*

We have shown (p. 113) that the different layers, of which this lens is composed, do not all have the same index of refraction, but that the refractive power of the crystalline substance increases from the peripheral layers towards the centre, attaining its maximum in the nucleus.

We also know, from investigations made by Seuff, Listing, and Helmholtz, that this constitution of the crystalline has the effect of giving it a greater refractive power than if it were homogeneous, or even wholly composed of a substance having the density of the nucleus.

The index of refraction, of the peripheral layers, increases with age. One proof of this, among others, is the very pronounced reflex from the crystalline, in aged persons, by oblique illumination. This reflex is sometimes intense enough to simulate a cataract; examination with transmitted light, however, shows the lens to be perfectly transparent. The reflex is due solely to an increase in the index of refraction of the cortical laminæ of the lens. In consequence of this, the difference in refraction between the aqueous humor and the crystalline lens is greater, and hence the reflection from the anterior surface of the latter increases. The index of refraction of the peripheric layers of the crystalline increasing, without that of the centre undergoing any appreciable change, the substance of the lens generally becomes more nearly homogeneous as age advances, and refracts light less than in youth.

The refractive power of the entire dioptric apparatus of the eye diminishes, necessarily, with that of the crystalline.

B. *Dynamic Refraction.*—If age has a certain influence upon the refraction of the eye at rest, the *static* refraction, that which it exerts upon the *dynamic* refraction, or *accommodation*, is still more powerful.

*The accommodative power of the eye diminishes with advancing age; the* punctum proximum *gradually recedes.* But, while the influence of years, upon static refraction, makes itself felt only at a somewhat advanced age, the diminution in dynamic refraction manifests itself as early as at the age of ten—that is to say, at the earliest age at which it has been possible to make conclusive experiments relative to this matter.

The *punctum proximum* recedes, then, and the refraction which the eye possesses when it brings all its accommodative power into play, diminishes proportionately.

Donders represents, by the curve *p p* of his diagram (Fig. 77), the refraction of the eye when adapted to its *punctum proximum, i.e.,* the maxima of refractive power to which an eye can attain at different periods of life.

The first figure given by the curve is, at the age of ten years, fourteen dioptries. This signifies that a child of ten can, by bringing all his accommodation into play, increase the refractive power of his eye by fourteen dioptries.

From this point on, the curve *p p* falls rapidly. At thirty years, the accommodation, being now only seven dioptries, has already diminished by half.

Between sixty and sixty-five years this curve *p p* reaches the zero line, *i.e.,* the *punctum proximum* is now as distant as the *punctum remotum* formerly was. A person who has been emmetropic until toward the age of fifty, and who has become hyperopic in consequence of the senile change in his crystalline, still barely succeeds, at the age of sixty-seven, in correcting this acquired hyperopia, except only by bringing into play all the accommodation at his command. He can, however, no longer bring his *punctum proximum* within infinity.

From this age on, the curve *p p* descends more and more below the zero-line,—that is to say, the *punctum proximum* is now farther from the eye than the *punctum remotum* was before the age of fifty-three years. The refractive power of the eye, during its maximum effort of accommodation, is now less than it was before, when the eye was at rest. An emmetrope becomes hyperopic by 0·75 dioptry, at the age of sixty-five, and, even when putting forth all the accommodation power at his disposal, he still has remaining 0·25 dioptry of hyperopia.

At seventy-three years the two curves $pp$ and $rr$,—that is to say, the *punctum proximum* and *punctum remotum*, coincide. The refraction is no longer susceptible of any change. There is no accommodation, and, moreover, this point of fusion of the two curves being 1·5 dioptry below the zero-line, the invariable refractive power of the eye is less, by 1·5 dioptry, than it was in youth.

The single curve, formed by the fusion of the curves $pp$ and $rr$, still continues to descend, as we have seen, until the age of eighty years. Beyond that age no conclusive experiments have been made.

The *range of accommodation, i.e.*, the change in refraction of which an eye is susceptible by means of its accommodative power, evidently finds expression in the number of dioptries included between the curve $rr$, of repose, and the curve $pp$ of maximum accommodation $(a=p-r)$.

Thus we obtain, for the range of accommodation, the following table:—

| Years. | Range of Accommodation. |
|---|---|
| 10 | 14 D |
| 15 | 12 ,, |
| 20 | 10 ,, |
| 25 | 8·5 ,, |
| 30 | 7 ,, |
| 35 | 5·5 ,, |
| 40 | 4·5 ,, |
| 45 | 3·5 ,, |
| 50 | 2·5 ,, |
| 55 | 1·75 ,, |
| 60 | 1 ,, |
| 65 | 0·75 ,, |
| 70 | 0·25 ,, |
| 75 | 0 ,, |

We see, then, that the power to accommodate diminishes rapidly and considerably as age advances. There would be nothing surprising about this, were it not that this impairment commences at an age when all other bodily forces and faculties are in a state of progressive development.

One might well wonder to what this early degeneration of so important a function was due. Does the contractility of the ciliary muscle diminish? This is not admissible. The ciliary muscle would, in that case, differ from all the other muscles of the body. Then it must be the elasticity of the crystalline lens which decreases. There is, indeed, no other alternative left, accommodation being due only to the contraction of the ciliary muscle and the change of form which, by virtue of its elasticity, the crystalline lens undergoes.

Therefore, if it be not the first, it must be the latter which is at fault when accommodation diminishes. This is, in fact, the generally admitted and very probably correct explanation.

The crystalline lens commences at this early age to change its physical constitution, and becomes more rigid. However energetic the contraction of the ciliary muscle may be, and however complete the relaxation of the zone of Zinn, the form of the crystalline lens varies less and less under their influence; and, with its convexity, its refraction, during the extreme effort of accommodation, differs less and less from that which it possesses when the eye is at rest. In other words, the range of accommodation diminishes as age advances.[1]

The change in the amplitude of accommodation is the same, whatever be the refractive condition of the eye. Emmetropia and ametropia, without regard to the degree of the latter, are alike subject to the laws governing the range of accommodation at the different periods of life. Donders' diagram is applicable to all refractive conditions. The one that we reproduce corresponds to emmetropia, since the curve *r r* commences at the zero-line (0 dioptry).

For ametropia, we have only to displace the zero-line conformably to the condition of static refraction, as we have done above. Thus, for a myope of 6 D, the curve *r r* would commencè at the number + 6, *i.e.*, six divisions above the zero-line, and would terminate between + 4 and 3, at the right. The curve *p p* would commence $14 + 6 = 20$ divisions above the zero-line (in which case it would be necessary to enlarge the scheme), and would descend as in emmetropia. At sixty-three years it would have reached the line 6, and at seventy-three years it would have become one with the curve *r r*, between five and four dioptries.

But, although the range of accommodation is equal, the position of the *punctum proximum* is evidently not the same for every condition of static refraction of the eye. The latter depends, as we have seen, upon both the refraction and the accommodation. The distance between the *punctum proximum* and the eye equals P, *i.e.*, the focal distance of a lens which represents the refractive power of the eye at a state of maximum accommodation; $P = \frac{1}{p}$, or $p = \frac{1}{P}$. Let us take a few examples:

Where is the *punctum proximum* of an emmetrope at the age of thirty? The diagram and the table tell us that at this age the range of accommodation ($a$) amounts to seven dioptries. In the case of an emmetrope we have $r = 0$, hence $p = a - 0 = a = 7$ D, and P, the focal distance of seven dioptries, equals $\frac{100^{\text{cm.}}}{7} = 14^{\cdot\text{m.}}$

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[1] Donders, *Anomalies, &c.*, p. 205. Schnabel, *Arch. f. Augen- und Ohrenheilk.*, VII., I., p. 118.

If the same man were myopic by 4 D, instead of being emmetropic, where would his *punctum proximum* be? His refraction, in a state of maximum accommodation, would be $p=r+a=4+7=11$ D, and, consequently, P, the distance of the *punctum proximum*, $=\frac{100^{\text{cm.}}}{11}=9{\cdot}9^{\text{cm.}}$

Suppose a person of the same age who is hyperopic by 4 dioptries. He would also have a range of accommodation amounting to 7 D, but he uses 4 of them to correct his hyperopia. He has left only $7-4=3$ D of positive refraction, which gives the distance of his *punctum proximum* as $33{\cdot}3^{\text{cm.}}$

It will be seen that, for the same age and range of accommodation, the situation of the *punctum proximum* is far from being always the same, simply because this depends as much upon the static as upon the dynamic refraction of the eye.

The question may be put in still another way.

Suppose we have before us three persons. The *punctum proximum* in each case is $20^{\text{cm.}}$ distant, but the first is emmetropic, the second has a myopia of 3 dioptries, and the third a hyperopia of the same degree. What is the age of each?

We have only to invert the calculations which we have just been performing;—that is to say, having given $p$ and $r$, we are to find $a$.

P in each of the three cases equals $20^{\text{cm.}}$, hence $p$, the corresponding number of dioptries, equals $\frac{100}{20}=5$ D.

1. For the emmetrope we have $a=p-0=5$ D, in our example, and the table shows us that a range of accommodation amounting to 5 D corresponds to the age of thirty-five and a half years.

2. The myope who, notwithstanding his refractive condition—already positive at a state of rest—does not see at a nearer distance than the emmetrope, must be older than the latter; for if he were of the same age, and consequently had the same range of accommodation, the latter, combined with his myopia, would give him a refractive power ($p$) greater than that of the emmetrope. His *punctum proximum* would then be nearer than that of the latter. If such is not the case, it is a proof that the range of accommodation ($a$) is less, and that, consequently, the person is older. In fact we have, for myopia, the expression $a=p-r$. In our example, $a=5-3=2$ D. Two dioptries of amplitude of accommodation indicate an age of fifty-two years.

3. The hyperope, on the contrary, must possess a greater range of accommodation than the emmetrope, in order to see distinctly at as short a distance as the latter. In the first place, he must correct his ametropia—*i.e.*, bring the point, to which his eye is adapted, from behind the eye to infinity—before he can think of bringing it to a

finite distance. In our example we shall say: The *punctum proximum* being at a distance of $P = \frac{1^{m.}}{5}$, the refractive power presented by the eye is $p = 5$ dioptries. Having been, when at rest, three dioptries below the zero-line—*i.e.*, $- 3$ dioptries—the range of accommodation ($a$) must be $5 + 3 = 8$ dioptries. It will be remembered, in fact, that for hyperopia we had the expression $a = p + r$. To possess eight dioptries of range of accommodation, a person must not be more than twenty-seven years old, as is demonstrated by Donders' diagram.

Finally, the problem may be stated in a third way. We may ask, what is the refractive condition of a person who, at fifty years of age, still sees distinctly at a distance of 11 centimetres?

We know the value of $a$, the range of accommodation. It is 2·5 dioptries at the age of fifty. We know, too, the value of $p$; that is, we can deduce it from P = 11 centimetres: $p = \frac{1}{P} = \frac{1}{0·11} = 9$ D. We have to find the value of $r$, which we get as follows:—The refractive power $p$, represented by the eye when it is adapted to its *punctum proximum*, is greater than the range of accommodation which corresponds to the age given. The surplus can be due only to the static refraction of the eye. In other words, the eye when at rest must have been adapted to a finite distance—*i.e.*, it must have been myopic. And the degree of myopia is given by the difference between the refraction of the eye, when adapted to its *punctum proximum* ($p$), and its range of accommodation ($a$); $r = p - a = 9 - 2·5 = 6·5$ D.

Hence the person in this example has a myopia of 6·5 dioptries. The same methods of calculation are applicable to emmetropia and hyperopia.

## RECAPITULATION.

*The average refractive power of the dioptric apparatus of the eye* may be compared to that of a lens having a focal distance of 20·713 millimetres, or $\frac{1000}{20·713} = 48·27$ dioptries of refractive power.

The purpose of this dioptric apparatus is to form distinct images upon the retina. This is the reason why it is generally considered together with its relations to the retina, and, when speaking of the *refraction of the eye*, we have in mind the direction of luminous rays relatively to the situation of the retina. Thus we have seen that, without any change in the refractive power of the dioptric apparatus, that which we term the *refraction of the eye* varies. The eye is hyperopic, emmetropic, or myopic, according as the retina is near to, or farther from, the dioptric system.

*The measure of the refraction of the eye* cannot be the focal distance of its dioptric apparatus, which distance depends solely upon this apparatus, and has nothing to do with the retina or the length of the eye. The refraction of the eye finds its expression in the *anterior (or first) conjugate focal distance* of the eye, *i.e.*, in the distance at which a point must be situated in order that its image may be formed distinctly upon the retina.

The second, or *posterior conjugate focal distance*, corresponding to this first or anterior one, is evidently the *length of the eye* (or, strictly speaking, the distance between the second principal point and the retina). It could not serve as a measure of refraction; it cannot be directly measured, and of itself has no more practical importance than the refractive power of the dioptric apparatus, taken alone. It has no significance except when taken in its *relation* to that refractive power. This relation finds its expression in the first conjugate focal distance. If we say that the eye sees distinctly an object situated 50 centimetres in front of it, we know that, one of the conjugate foci being 50 centimetres distant, the other is situated upon the retina.

That is what we wish to know in practice; we care less for the distance between the retina and the dioptric apparatus.

The first conjugate focal distance, therefore, gives the measure of the refraction of the eye. It is the *inverse* of it. The nearer the point is situated, the image of which is formed on the retina, the stronger is the refraction of the eye.

To obtain the refraction in dioptries, we measure the first conjugate focal distance with the metre. We say: in order to see clearly an object situated $\frac{1}{2}$ metre away, the eye needs a refraction of $\frac{1}{1^{\text{m.}}/_2} = 2$ dioptries; to see an object X metres in front of it, the eye must have $\frac{1}{\text{X}}$ dioptries of positive refraction.

So long as the object is at a finite distance, we obtain, in this way, for the refraction necessary to see it distinctly, a positive number of dioptries. This number is smaller in proportion as the object is farther removed, *i.e.*, as the rays emanating from it are less divergent.

When the object is at infinity, the first conjugate focal distance is $\infty$, and the refraction $\frac{1}{\infty} = 0$.

When the object is beyond infinity, or behind the eye, *i.e.*, when the rays coming from it are convergent, the first conjugate focal distance and the refraction of the eye are negative.

Thus an eye which focuses, upon its retina, rays which converge toward a point $\frac{1^{\text{m.}}}{2}$ behind it, is said to have two dioptries of negative refraction. In order, for instance, to see, through a convex glass of four dioptries, an object situated at infinity, an eye must have four dioptries of *negative* refraction. Inasmuch as a convex lens No. 4 gives rays, coming from infinity, a convergent direction toward a point $\frac{1^{\text{m.}}}{4}$ behind it, the first conjugate focus, whose image is formed upon the retina, is not, therefore, at infinity, but $\frac{1}{4}$ metre behind the eye, and for that reason the latter's refraction is $-4$

dioptries. In general, to see a (virtual) object, situated X metres behind the eye, $\frac{1}{X}$ dioptries of *negative refraction* are necessary.

When at *rest*, the eye is adapted to its *punctum remotum*. At this moment, therefore, *the punctum remotum is the first conjugate focus* of the eye. This is the way in which we have expressed the refraction of the eye at rest, the *static refraction;* by a fraction whose numerator is 1 metre, and whose denominator is the distance (R), expressed in metres, between the *punctum remotum* and the eye. By performing the division, we obtain the static refraction, expressed in dioptries, which we call $r$.

As long as the *punctum remotum* is at a *finite* distance, the static refraction of the eye is *positive.*

This is what characterises MYOPIA. If the *punctum remotum* be at $\frac{1^{\text{m.}}}{20}$ we have, $R=\frac{1^{\text{m.}}}{20}$ and $r=\frac{1}{1^{\text{m.}}/_{20}}=20$ D; that is to say, the static refraction equals + 20 dioptries. If the *punctum remotum* be 2 metres distant, the static refraction amounts to $\frac{1}{2}$ D.

When the *punctum remotum* is at *infinity*, as in EMMETROPIA ($R=\infty$), the refraction is $\frac{1}{\infty}=0$. That is the reason why emmetropia may be expressed by *zero.*

When the *punctum remotum*, the first conjugate focus of the eye at rest, is *beyond infinity, i.e.*, when it appears behind the eye, its distance (R) from the eye, as well as the refraction of the latter, is *negative ;* $-\frac{1}{R}=-r$.

It is for this reason that HYPEROPIA takes the *minus* sign. An eye at rest, adapted to a point 4 metres behind it, has a negative static refraction of $\frac{1}{4}$ of a dioptry; it has a hyperopia of 0·25 D. When its *punctum remotum* is $\frac{1}{5}$ of a metre behind the eye, the refraction of the latter is $-5$ D, and, if it be in a state of rest that the eye has this refraction, we say that its static refraction equals $-5$ dioptries (hyperopia of 5 D).

The refraction of the eye at rest can be *increased* by a muscular effort, the *accommodation.* To its static refraction the eye adds, in this case, a DYNAMIC REFRACTION $(a)$. At the moment of its maximum of accommodation, the eye evidently presents the greatest amount of refraction of which it is susceptible, since it has added all its dynamic refraction to its static refraction.

Hence, in this condition, in which the refraction of the eye represents the *sum of its static and dynamic refraction*, the eye is adapted to the nearest point that it can see distinctly. This is the PUNCTUM PROXIMUM.

The *punctum proximum is therefore, in this case, the first conjugate focus*, and its distance (P) from the eye gives the measure of the *maximum refraction* (P) of which the eye is susceptible. Each is the inverse of the other: $p=\frac{1}{P}$.

An eye which, on bringing all its accommodation into play, is adapted to $\frac{1}{5}$ of a metre, has $\frac{1}{1^{\text{m}}/_5}=5$ dioptries as its maximum refraction. $P=\frac{1^{\text{m.}}}{5}$;

$p = 5$ D. This maximum of refraction $p$, representing the sum of static refraction ($r$) and dynamic refraction ($a$), is therefore equal to $p = a + r$.

Hence, the total *dynamic refraction*, in short, the *range of accommodation* is, therefore, equal to the maximum refraction of the eye *minus* the static refraction, $a = p - r$. This is the formula for the RANGE OF ACCOMMODATION.

The static refraction is, in turn, equal to the difference between the maximum and the dynamic refraction, $r = p - a$.

The maximum refraction ($p$) of an eye may be either positive, nil, or negative.

So long as $r$ is *positive*, as in *myopia*, $p$ can be only positive. This results directly from the formula $p = a + r$, for $a$, the accommodation, is always either positive or nil, but never negative, since negative accommodation does not exist.

If $r$ be equal to zero, as in emmetropia, $p$ may also be equal to zero, when there is no accommodation, *i.e.*, when $a$ is likewise nil. Otherwise, so long are there is any accommodation, $p$ is always positive. The eye can always be adapted to a point situated at a finite distance, greater or less, according to the value of $a$, the accommodation or dynamic refraction.

When $r$ is *negative*, as in *hyperopia*, $p$ may be positive, if $a$ be greater than $r$, *i.e.*, if the dynamic be greater than the static refraction. A hyperope of 3 D ($r = -3$), whose power of accommodation amounts to 5 D ($a = 5$) will have a $p$ of $5 - 3 = 2$ D. By bringing all his accommodation into play he will see at a distance of $\frac{1}{2}$ metre.

If $a = -r$, $p$ becomes equal to zero. This is the case when the accommodation just suffices to correct the hyperopia. The *punctum proximum* is then at infinity.

When $a$ is less than $-r$, $p$ is negative, which is equivalent to saying that the *punctum proximum* cannot be brought within a finite distance. Suppose a hyperope of 6 D ($r = -6$) has 4 D of range of accommodation ($a = +4$). All his dynamic refraction will not suffice to raise the maximum refraction to zero; $p = 4 - 6 = -2$ D,—that is to say, the *punctum proximum* will be $\frac{1}{2}$ metre behind the eye.

In order to see distinctly at a distance of X metres, an eye must have $\frac{1}{X} = x$ dioptries of positive refraction. For instance, to see an object $\frac{1}{5}$ of a metre in front of the eye ($+x$), 5 D of refraction are necessary.

The eye may already possess this refractive power by its static refraction $r$. This is the case when the object in question is just at the (positive) *punctum remotum*. In our example, the object, requiring a refraction of $+5$ D, will be seen distinctly without effort by an eye whose static refraction ($r$) equals $+5$ D. Such an eye is myopic by 5 D; its *punctum remotum* is $\frac{1}{5}$ of a metre distant.

Sometimes the refraction necessary for distinct vision of the object can be obtained only by adding, to the static, a certain quantity of dynamic refraction. Thus, in our example, a myope of 1 D would need 4 D of dynamic refraction (accommodation) in order to realise the 5 D called for.

When the static and dynamic refraction, together, do not make up the amount necessary for vision at a certain distance, the deficiency must be supplied by a lens of positive refractive power, *i.e.*, by a *convex glass.* This glass must be considered as added to the static refraction; for neither is susceptible of modification. But it really comes to the aid of the dynamic refraction, for it performs a certain amount of work which, without its intervention, would have devolved upon the accommodation.

Let us suppose that our myope of 1 D has at disposal only 2 D of dynamic refraction. He will have then, altogether, $p = 1 + 2 = 3$ D. But he needs 5. Hence he still lacks 2. We will give him, consequently, the convex glass number 2 to make up the requisite number of dioptries. Inversely, if the static refraction is too strong, it is to be diminished by a negative (concave) glass.

## CONVERGENCE.

If vision were effected by means of one eye only, we should know, now, how this vision could be adapted for long as well as for short distance. There would be no point from beyond infinity up to within a very short distance from the eye, to which we could not adjust the dioptric system.

But man generally sees simultaneously with two eyes, and, for him, this *binocular vision* has considerable advantages. Binocular vision means the union, in one single impression, of the retinal images received simultaneously by both eyes. In order that this may be obtained, each eye must, at the same time with its fellow, receive upon its *fovea centralis* a distinct image of the object looked at. Hence it is necessary not only that each eye possess the optical adaptation to the distance of the object, which is indispensable to the *distinctness* of the image; but also that both *lines of fixation* be directed toward the object looked at, which is essential to the union in a single one of the images of the two eyes.

The *direction that the eyes must give to their lines of fixation, in order that they may be simultaneously directed toward the point of fixation, is called* CONVERGENCE.

In order to see, binocularly, an object situated at infinity, the lines of fixation *converge* toward infinity,—that is to say, they do not converge at all in the ordinary acceptance of the term; they are parallel. The nearer the object is, the more the lines of fixation must converge, the more the eyes must be turned inwards.

If the object moves on the *median line*, *i.e.*, in a line perpendicular to the middle of the *base-line* (which unites the centres of rotation of the two eyes), the convergence is always the same for each eye. Hence it is *nil* when the lines of fixation are parallel, and it increases in proportion as the object approaches. It is measured by the angle through which an eye turns when it abandons this parallel direction and fixes the object situated on the median line. This angle we call the *angle of convergence.*

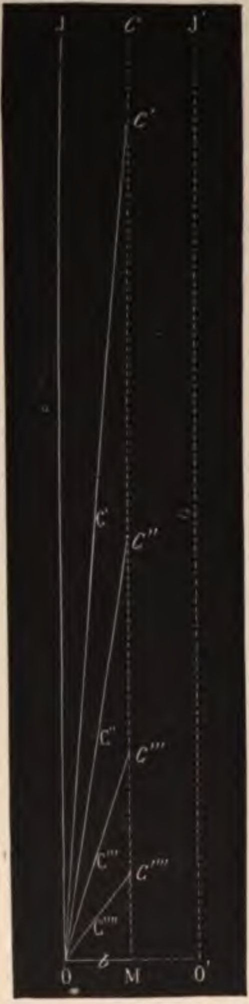

FIG. 78.

Let O and O′ (Fig. 78) represent the centres of rotation of the eyes; O O′ is the base-line, M the middle of it, and M *C* the median line. When the lines of fixation (O J and O′ J′) are parallel, the angle of convergence is *nil* for each of the eyes. When they fix the point *C*′, the angle of convergence, for the left eye, is J O *C*′ or O *C*′ M.

The size of the angle of convergence depends, on the one hand, upon the length of the base-line O O′; on the other hand, upon the distance of the object. The base-line having a constant value for each individual, we may regard the angle of convergence as depending solely upon the distance of the object. In proportion as this distance diminishes, the angle increases; the nearer the object approaches, the more convergence is required in order that it may be seen binocularly. Hence we may say that *the angle of convergence is inversely proportional to the distance between the object fixed and each of the eyes.*

When the eyes are directed straight forward, the convergence of each is $c = \frac{1}{\infty}$; when they are directed toward a point, on the median line, 2 metres distant, the convergence of each is $c = \frac{1}{2}$. For a point 1 metre distant the convergence is $c = \frac{1}{1} = 1$; at $\frac{1^{\text{m.}}}{2}$ it is $c = \frac{1}{\frac{1}{2}} = 2$, and for $\frac{1^{\text{m.}}}{3}$ it is $c = \frac{1}{\frac{1}{3}} = 3$.

In this way we obtain numbers with which to express the effort of convergence. The unit, $c = 1$, is the angle of convergence necessary in order to fix simultaneously, with both eyes, an object situated on the median line, at a distance of 1 metre from each eye. We are indebted to Nagel[1] for the ingenious idea of rendering the mensuration of convergence so simple and, as we shall see, so practical. He calls this unit-angle the "Meterwinkel," *metre-angle.* We shall designate it as $m\,a$.

Convergence is, in this way, easily expressed. If we have to do with an object $\frac{1}{5}$th of a metre distant, we know that the convergence, requisite for binocular vision at this distance, is $\frac{1}{\frac{1}{5}} = 5\ m\,a$; for 5 metres we shall have $c = \frac{1}{5}\ m\,a$, and so forth.

But Nagel's expression has a still greater advantage. Let us recall the fact that the refraction necessary for distinct vision of an object is also the inverse of the distance of that object.

Hence the refraction and convergence must increase and diminish by an equal quantity, the inverse of the distance of the object.

They are both *nil* when the object is at infinity.

If the object $C'$ be 1 metre from the eye ($C' = 1$ metre), the angle of convergence, $O\,C'\,M$, is $c = \frac{1}{1^{m.}} = 1\ m\,a$, and the necessary refraction $x = \frac{1}{1^{m.}} = 1$ dioptry.

When the object is brought to within half a metre of the eye O ($O\,C'' = \frac{1^{m.}}{2}$), the angle of convergence, $O\,C''\,M$, is $\frac{1}{1^{m.}/_2} = 2\ m\,a$, and the refraction $x = \frac{1}{1^{m.}/_2} = 2$ D.

If the object be $\frac{1}{3}$ of a metre distant ($O\,C''' = \frac{1^{\cdot}}{3}$), the convergence is $J\,O\,C'''$ or $O\,C'''\,M = 3\ m\,a$, and the necessary refraction equals 3 D.

If $O\,C''''$ be equal to 25 centimetres = $\frac{1}{4}$ metre, the necessary convergence is $4\ m\,a$ ($J\,O\,C''''$), and the refraction $x = 4$ D.

If the object is 3 metres from the eye, the convergence is $c = \frac{1}{3} = 0{\cdot}333\ m\,a$, and the refraction $x = 0{\cdot}333$ D.

By adopting the same unit, *i.e.*, the metre, as the measure of both refraction and convergence, Nagel succeeds in always giving these functions, whose purposes are analogous, the same expression. The two ought, indeed, always to be equal.

It was of capital importance to establish the equality of the convergence and refraction necessary for binocular vision, and to give them a ready measure and a simple expression.

The absolute value of the angle of convergence is calculated in the following manner:—

We designate O M (Fig. 78), *i.e.*, half the base-line, by $b$, and the distance O $C'$, from the eye to the object, by C′. The angle of convergence in this case is O $C'$ M. It is determined by the expression—

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[1] Nagel in Graefe and Saemisch, *Handbuch*, VI., chap. x., p. 478 *et seq.*

$$\sin \mathrm{O}\, C'\mathrm{M} = \frac{b}{\mathrm{C}'}$$

And if we suppose C′ equal to 1 metre, the *metric angle* will be expressed by—

$$m\,a = arc \sin \frac{b}{1^{\mathrm{m}}}$$[1]

If we designate the different distances, from the object to the eye, by $C'$, C″, C‴, and C⁗ . . . we can construct rectangular triangles O $C'$ M, O $C''$ M, O $C'''$ M, and O $C''''$ M, whose common base is *b*. From these we derive the formulas—

$$b = \mathrm{C}'.\ \sin \mathrm{O}\, C'\mathrm{M} = \mathrm{C}''.\ \sin \mathrm{O}\, C''\mathrm{M} = \mathrm{C}'''.\ \sin \mathrm{O}\, C'''\mathrm{M} = \mathrm{C}''''.\ \sin \mathrm{O}\, \mathrm{C}''''\mathrm{M} \ldots$$

For the small angles with which we are really dealing, the sine may be regarded as equal to the angle, and the equation becomes—

$$b = \mathrm{C}'.\,\mathrm{O}\, C'\mathrm{M} = \mathrm{C}''.\,\mathrm{O}\, C''\mathrm{M} = \mathrm{C}'''.\,\mathrm{O}\,\mathrm{C}'''\mathrm{M} = \mathrm{C}''''.\,\mathrm{O}\,\mathrm{C}''''\mathrm{M} \ldots$$

Letting $b = 1$, we obtain—

$$\text{Angle O } C'\mathrm{M} = \frac{1}{\mathrm{C}}$$

$$\text{,, O } C''\mathrm{M} = \frac{1}{\mathrm{C}''}$$

$$\text{,, O } C'''\mathrm{M} = \frac{1}{\mathrm{C}'''}$$

$$\text{,, O } C''''\mathrm{M} = \frac{1}{\mathrm{C}''''}, \text{ \&c.,}$$

that is to say, the angles of convergence are the inverses of the distances of the object from the eye.

If

$\mathrm{C}' = 1^{\mathrm{m}}$, the corresponding convergence is $c = \frac{1}{1} = 1\ m\ a.$

$\mathrm{C}'' = \frac{1^{\mathrm{m}}}{2}$, ,, ,, $c = \frac{1}{\frac{1}{2}} = 2\ m\ a.$

$\mathrm{C}''' = \frac{1^{\mathrm{m}}}{3}$, ,, ,, $c = \frac{1}{\frac{1}{3}} = 3\ m\ a$

$\mathrm{C}'''' = \frac{1^{\mathrm{m}}}{4}$, ,, ,, $c = \frac{1}{\frac{1}{4}} = 4\ m\ a.$

If $\mathrm{C} = 20^{\mathrm{cm.}} = \frac{20}{100} = \frac{1^{\mathrm{m.}}}{5}$, the convergence is $c = 5\ m\ a.$

If C = 12 metres, the convergence is $c = \frac{1}{12}\ m\ a.$

As we have said, and as follows from the formula for the metre-angle, the absolute size of the latter depends, for each individual, upon the base-line, *i.e.*, upon the distance between the centres of rotation of the two eyes.

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[1] Nagel, *loc. cit.*, p. 479.

Nagel[1] has calculated the metre-angle for the different lengths of the base-line, that are to be met with.

The following are the results of his calculations :—

*Value of the Angle of Convergence at the distance of* 1 *Metre (Metre-Angle) for different lengths of the Base-Line.*

| BASE-LINE in millimetres. 2 *b* (of Fig. 77). | SINE of the metre-angle. *b* (of Fig. 77) in metres. | VALUE OF THE METRE-ANGLE. | |
|---|---|---|---|
| | | In degrees. | In degrees, min., and sec. |
| 50 | 0·025 | 1·430° | 1° 25′ 51″ |
| 51 | 0·0255 | 1·459° | 1° 27′ 35″ |
| 52 | 0·026 | 1·488° | 1° 29′ 18″ |
| 53 | 0·0265 | 1·517° | 1° 31′ 2″ |
| 54 | 0·027 | 1·545° | 1° 32′ 45″ |
| 55 | 0·0275 | 1·574° | 1° 34′ 28″ |
| 56 | 0·028 | 1·603° | 1° 36′ 12″ |
| 57 | 0·0285 | 1·631° | 1° 37′ 56″ |
| 58 | 0·029 | 1·660° | 1° 39′ 39″ |
| 59 | 0·0295 | 1·689° | 1° 41′ 22″ |
| 60 | 0·030 | 1·718° | 1° 43′ 6″ |
| 61 | 0·0305 | 1·746° | 1° 44′ 49″ |
| 62 | 0·031 | 1·775° | 1° 46′ 33″ |
| 63 | 0·0315 | 1·804° | 1° 48′ 16″ |
| 64 | 0·032 | 1·833° | 1° 50′ |
| 65 | 0·0325 | 1·861° | 1° 51′ 43″ |
| 66 | 0·033 | 1·890° | 1° 53′ 26″ |
| 67 | 0·0335 | 1·919° | 1° 55′ 10″ |
| 68 | 0·034 | 1·948° | 1° 56′ 53″ |
| 69 | 0·0345 | 1·976° | 1° 58′ 37″ |
| 70 | 0·035 | 2·005° | 2° 0′ 20″ |
| 71 | 0·0355 | 2·034° | 2° 2′ 4″ |
| 72 | 0·036 | 2·063° | 2° 3′ 47″ |
| 73 | 0·0365 | 2·092° | 2° 5′ 31″ |
| 74 | 0·037 | 2·120° | 2° 7′ 14″ |
| 75 | 0·0375 | 2·149° | 2° 8′ 57″ |

50 and 75 millimetres are the two extreme limits of separation between the two eyes.

Taking 64 millimetres as the average, Nagel finds, for the different distances of the object of fixation, the angles of convergence marked in the following table (page 190)[2].

He calculated these angles in two ways : first considering the angles and their sines as equal (as we have done in the preceding discussion), and again by taking into account their sines. The latter method was important as enabling the computer to detect such errors as might have resulted from the approximation of the other method. It will be seen that such errors might, in reality, be disregarded, inasmuch as the difference of the angles thus calculated exceed 1° only for a distance of 6·66 centimetres from the eye, or 5·85 centimetres from the base-line, *i.e.*, in a case which is very exceptionally met with in practice.

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[1] Nagel, *loc. cit.*, p. 481.

[2] *Ibid.*, *loc. cit.*, p. 482.

*Angles of Convergence calculated in Degrees for different distances of the Object of Fixation, the distance between the Eyes (Base-Line) being 64 Millimetres.*

| DISTANCE FROM THE OBJECT OF FIXATION TO THE EYE. | | VALUE OF THE ANGLE OF CONVERGENCE. | | | Difference between the numbers in the two preceding columns. |
|---|---|---|---|---|---|
| In metres. | In millimetres. | In metre-angles. | In degrees. | In degrees exactly calculated. | |
| 1 | 1000 | 1 | 1° 50′ | 1° 50′ | 0 |
| $\frac{1}{2}$ | 500 | 2 | 3° 40′ | 3° 40′ | 0 |
| $\frac{1}{3}$ | 333·3 | 3 | 5° 30′ | 5° 30′ 41″ | 41″ |
| $\frac{1}{4}$ | 250 | 4 | 7° 20′ | 7° 21′ 23″ | 1′ 23″ |
| $\frac{1}{5}$ | 200 | 5 | 9° 10′ | 9° 12′ 3″ | 2′ 3″ |
| $\frac{1}{6}$ | 166·6 | 6 | 11° | 11° 4′ 17″ | 4′ 17″ |
| $\frac{1}{7}$ | 142·8 | 7 | 12° 50′ | 12° 56′ 47″ | 6′ 47″ |
| $\frac{1}{8}$ | 125 | 8 | 14° 40′ | 14° 50′ | 10′ |
| $\frac{1}{9}$ | 111·1 | 9 | 16° 30′ | 16° 44′ 17″ | 14′ 17″ |
| $\frac{1}{10}$ | 100 | 10 | 18° 20′ | 18° 39′ 38″ | 19′ 38″ |
| $\frac{1}{11}$ | 90·9 | 11 | 20° 10′ | 20° 36′ 40″ | 26′ 40″ |
| $\frac{1}{12}$ | 83·3 | 12 | 22° | 22° 34′ 49″ | 34′ 49″ |
| $\frac{1}{13}$ | 76·9 | 13 | 23° 50′ | 24° 35′ | 45′ |
| $\frac{1}{14}$ | 71·4 | 14 | 25° 40′ | 26° 36′ 55″ | 56′ 55″ |
| $\frac{1}{15}$ | 66·6 | 15 | 27° 30′ | 28° 41′ 1″ | 1° 11′ 1″ |
| $\frac{1}{16}$ | 62·5 | 16 | 29° 20′ | 30° 48′ | 1° 28′ |
| $\frac{1}{17}$ | 58·8 | 17 | 31° 10′ | 32° 57′ 3″ | 1° 47′ 3″ |
| $\frac{1}{18}$ | 55·5 | 18 | 33° | 35° 10′ | 2° 10′ |
| $\frac{1}{19}$ | 52·6 | 19 | 34° 50′ | 37° 26′ 31″ | 2° 36′ 31″ |
| $\frac{1}{20}$ | 50 | 20 | 36° 40′ | 39° 47′ 43″ | 3° 7′ 43″ |

## THE AMPLITUDE OF CONVERGENCE.

$$a^c = \frac{1}{A^c}$$

The whole amount of reciprocal directions, which a person can give to his lines of fixation, constitutes the *amplitude of convergence.*[1]

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[1] Nagel calls the angle of convergence the *angle of fusion*, and gives, to its reciprocal distance, the sign F, so that his formula for the amplitude of fusion is $\frac{1}{F} = \frac{1}{P} - \frac{1}{R}$ We prefer the word *convergence*, because, in the first place, it implies the idea of the direction and movements of the eyes, of which the fusion of retinal images is only a result; in the second place, because the symbols F and *f*, required by the term *fusion*, have already taken another meaning in optics, which might lead to confusion between them and those for focal distances. The letters *c*, C, and A do not possess this disadvantage.

It cannot justly be objected that, when there is divergence of the lines of fixation, our expression *angle of convergence* is unfortunate, for in such cases we give to that angle the *negative* sign, and there is nothing objectionable about the expression *negative convergence*, *i.e.*, the convergence of the backward prolongations of the lines of fixation, or divergence of these actual lines. It is very readily understood, as has been proved to us by experience in our lectures and practical demonstrations. Students are never deceived as to the meaning of the word.

It is necessarily comprised between the *maximum* and the *minimum* of convergence of which a pair of eyes is susceptible.

The *maximum of convergence* is the inverse of the distance between the *nearest point*, which can be fixed binocularly, and each of the eyes.

Let *P* (Fig. 79) be this *punctum proximum of convergence.* If we designate the distance O *P* by P, the maximum of convergence is $\frac{1}{\mathrm{P}}$, or *p* metre-angles if we measure P with the metre.

*The minimum of convergence* is the inverse of the distance between the farthest point, which can be fixed binocularly (*punctum remotum of convergence*), and each of the eyes.

Let *R* (Fig. 79) be this point, and O *R* = R metres the distance which separates it from each eye. The minimum of convergence will be expressed by $\frac{1}{\mathrm{R}}$ or *r* metre-angles.

The difference between these two values is, as we have said, the amplitude of convergence.

We designate it by *a*, and express it in metre-angles by

$$a = p - r.$$

If the distances P and R are not reduced to metre-angles, the maximum and minimum of convergence are expressed by the fractions $\frac{1}{\mathrm{P}}$ and $\frac{1}{\mathrm{R}}$. The amplitude of convergence which results from the subtraction of these two fractions is then, necessarily, also a fraction: $\frac{1}{\mathrm{A}}$, in which A represents a certain number of the same linear measure by which P and R have been measured.

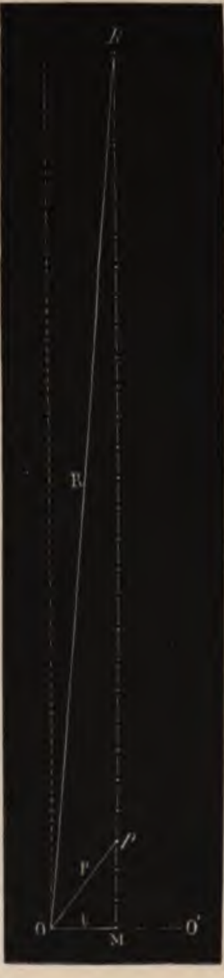

Fig. 79.

It is the inverse of the value *a*, just as P and R are the inverse of *p* and *r*.

It will be seen that one obtains, in this way, for the amplitude of convergence, the same expression as for the amplitude of accommodation.

It cannot be otherwise, since the two functions are estimated according to the same principle. The small letters signify, in both cases, the total amount of the function (*a*), its maximum (*p*), and its minimum ($r$). Only, for convergence they are *metre-angles*, and for accommodation metre-lenses (as Nagel calls them) or *dioptries*. The capital letters express their reciprocals in linear measure: A, the focal distance of the lens which equals the total amount of accommodation, or the inverse of the angle of convergence which represents that of convergence; P and R are the distances of the *punctum proximum* and *punctum remotum* of the two functions.

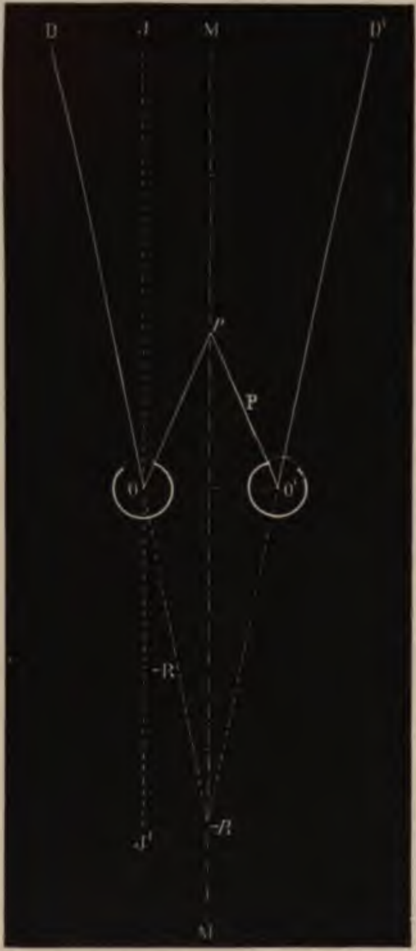

Fig. 80.

In order to be able to distinguish, in a doubtful case, whether accommodation or convergence is referred to, we shall add, to the signs A, P, R and $a, p, r$, a small $a$ for accommodation and a small $c$ for convergence.

Thus $P^a$ will signify the distance of the *punctum proximum* of accommodation, $r^c$ the minimum value of convergence, and so forth.

The *punctum remotum* of convergence is only very seldom (and then in pathological cases) situated at a finite distance as it is shown in the preceding figure.

Sometimes it is situated exactly at infinity, that is to say, the two eyes can be directed parallelly, but cannot diverge. R is then equal to ∞, and the minimum of convergence—

$$r^c = \frac{1}{\mathrm{R}} = \frac{1}{\infty} = 0.$$

But the majority of normal eyes can even *diverge* more or less. In this case, the *punctum remotum* of convergence is no longer situated in front of, but *behind*, the head, at the point $-R$ (Fig. 80) where the lines of fixation, prolonged backward, meet. This convergence of the prolongation of the lines of fixation, which corresponds to a divergence of the lines themselves, is necessarily *negative*. It is therefore necessary to add the sign − to the distance R as well as to the value of metre-angle ($r$) which corresponds to it. *Negative convergence* is therefore equivalent to divergence. In Fig. 80 the minimum of convergence is represented by the angle J O D or $-R$ O J′, and expressed by $-r = -\frac{1}{R}$.

The divergence of the eyes cannot be realised simply with the aid of an object of fixation, since, however far away this may be carried, it cannot pass beyond infinity,—that is to say, it can diminish convergence, at most, to zero, but not below; it cannot render it negative, *i.e.*, change it into divergence.

In order to obtain divergence of the lines of fixation, we place before one or each eye a prism, the edge being directed toward the temple. Such a prism causes luminous rays, coming from distant objects, to deviate toward its base, and when there is one before each eye, the two force the eyes to diverge in order to receive the images upon their *foveæ centrales*. Otherwise the object would be seen in homonymous diplopia. Prisms in this position are called *abducting* prisms.

Let O and O′ (Fig. 81) be the two eyes, and J S and J′ S′ parallel rays given off from a distant object. The prism T will deflect the ray J S toward its base, giving to it the direction S $R$. In order to receive this ray upon its *fovea centralis* F, the eye O must, therefore, diverge in such a way that its line of fixation takes the direction O D or O $R$. So with the eye O′, it will take the direction O′ $R$ under the influence of the prism T′.

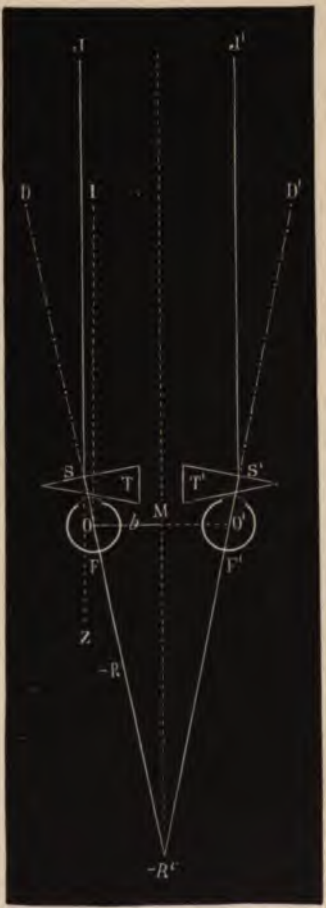

Fig. 81.

The angle of deviation, F S Z, produced by the prism T, is equal to the angle of divergence of the eye J S D or I O D. This angle is also equal to the angle of divergence O $R$ M;—$R$ is in fact the negative *punctum remotum* of convergence, if T and

T′ are the strongest prisms which the person can overcome, without seeing the object of fixation double.

The deviation produced by a glass prism may be regarded as equal to half the angle of the prism.[1]

The value of this angle is generally marked on the prisms in our trial-cases. The prism No. 10 produces, therefore, a deviation of 5 degrees, the No. 18 one of 9 degrees, and so forth.

A person who sees with both eyes, and as single, an object situated at infinity, notwithstanding an abducting prism of 6 degrees placed before each eye, executes with each eye a movement of divergence equal to 3 degrees. If stronger prisms would produce a homonymous diplopia, his minimum angle of convergence equals −3°.

And if, for the distance between the centres of rotation of his eyes (64 millimetres), the metre-angle equals 1° 50′, his minimum of convergence will be—

$$r^c = \frac{3^\circ}{1^\circ\,50'}, = \frac{180}{110} = -1{\cdot}63\ m\ a.$$

If a prism is placed before one eye only, its action extends to both eyes equally. The prism No. X acts like two prisms each of $\frac{X^\circ}{2}$, and produces, for each eye, a deviation of $\frac{X^\circ}{4}$, which can be reduced into metre-angles, as we have just done.

We have found that, in the normal state, the maximum of convergence is, on an average, $p = 9{\cdot}5\ m\ a$; the minimum $r = -1\ m\ a$, therefore the *average amplitude of convergence*—

$$a = 9{\cdot}5 - (-1) = 10{\cdot}5\ m\ a.$$

But it often happens that the maximum rises to 15 $m\ a$ and more. These results correspond with those obtained by other experimenters (Donders, Schuurman, v. Graefe, Noyes, Reich, A. Hoffmann) who have employed different methods.

The state of refraction of the eyes has no great influence either upon the amplitude of convergence or upon the value of its components. Among hyperopes, as well as among moderate myopes, we find the same figures for $p$ $r$ and $a$, as among emmetropes.

It is otherwise with extreme degrees of ametropia. Here the amplitude of convergence is limited at both its extremities, but especially at that of its *punctum remotum.* We shall see, in the clinical chapter, that this phenomenon is explained, sometimes, by a defect of development of the ocular muscles (high degrees of hyperopia), and, in other cases, by the hyperplasia and the peculiar form of the eye-ball (extreme myopia).

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[1] Compare Landolt in De Wecker et Landolt's *Traité Complet d'Ophthalmologie*, vol. i., p. 924.

The same is the case with all causes which diminish the strength of the muscles in general, and that of the muscles of the eyes in particular.

In short, notwithstanding the normal mobility of a single eye, the amplitude of convergence may be limited in consequence of disturbances of innervation, which alter the synergetic functions of both eyes, their co-operation in binocular vision.

## ACCOMMODATION IN ITS RELATIONS WITH CONVERGENCE.

### 1.—*Binocular Range of Accommodation or Amplitude of Binocular Vision.*

As we have said, and it is self-evident, perfect binocular vision requires the correct direction of the eyes, and their perfect optical adaptation. The accomplishment of the first of these two conditions depends, in the great majority of cases, only upon the motor apparatus of the eyes, the function of which, for this purpose, we have designated as "convergence." The optical adaptation is effected in different ways. For the distance of the *punctum remotum* it is given by the static refraction of the eye. For shorter distances the dynamic refraction is added to it, and frequently even convex auxiliary glasses. Myopic eyes, on the contrary, require concave glasses for objects situated beyond their *punctum remotum.*

The extent which a person commands with distinct binocular vision, without auxiliary glasses—that is to say, where he is able to unite, with a given convergence, the same degree of refraction and convergence — this extent has been called by Donders *the region of binocular accommodation.*

Since convergence plays in it a part as important as that of accommodation, it would be more correct to term this extent the *region of binocular vision.* It is evidently limited by the nearest and the farthest points which can be seen binocularly and distinctly—by the *punctum proximum* and *punctum remotum of binocular vision.*

If we express the distance of the first by P, that of the latter by *R*, and if we add to each of them a small *r*, in order to distinguish them from the other analogous expressions, we obtain for the *amplitude of binocular vision*, or, according to Donders, the *binocular range of accommodation*, the formula—

$$\frac{1}{A_2} = \frac{1}{P_2} - \frac{1}{R_2}$$

We can here again express the reciprocal values in terms of metre-angles or dioptries—

$$a_2 = p_2 - r_2.$$

It is easy to understand that the amplitude of binocular vision varies greatly in different individuals.

Even in the normal state, the convergence is not the same in every person. It depends, moreover, upon various circumstances which are liable to strengthen or to weaken the action of the muscles of the eye. Still more important is the influence which is exercised by the static and dynamic refraction upon binocular vision. It is evident that normal binocular vision is possible only when the ranges of convergence and of accommodation at least partially coincide.

Let us take, for instance, the case of a myope of 18 D. If he does not dispose of at least 18 *m a* of convergence, which is very rare, binocular vision would be out of the question. If his maximum of convergence is 7 *m a*, that is to say, less than his maximum of refraction (or accommodation), the two ranges do not even touch each other. The farthest point of refraction is at $\frac{1\text{ m.}}{18}$ (55 millimetres) and the nearest point of convergence at $\frac{1\text{ m.}}{11}$ (91 millimetres), therefore far beyond the first one, and no point can be fixed binocularly.

But even in case accommodation and convergence produce a certain number of equivalent values, even here it may be asked whether these values can be combined with each other, that is to say, whether the individual is able to combine, with a given degree of refraction, the same number of metre-angles of convergence.

The question, of the relation between accommodation and convergence, is of the highest practical interest and we must, therefore, study it more closely.

### *The relative Range, or Amplitude, of Accommodation and Convergence.*

The normal, emmetropic, typical eye is, when in a state of rest, adapted for infinity. Its accommodation is, in this case, *nil.* It is, therefore, exactly the same as the convergence requisite for binocular vision at the same distance.

When the object of fixation approaches, the refraction of the eyes must increase, as we have seen, by the same amount as the convergence.

In emmetropia, this augmentation of refraction is produced only by accommodation. The latter is therefore, in this case, always the same as the convergence. Thus, an emmetrope, for vision at $\frac{1}{5}$ of a metre, unites an effort of accommodation of 5 D with a convergence movement of 5 *m a*. In other words, the zero-point of both functions coinciding in emmetropia, and both being measured by the same principle, both must always be expressed by the same figure, for every distance, in the normal state of the eyes.

This parity of the effort simultaneously required by the two functions in normal eyes has so bound them together that it is difficult to separate them from each other. If one of an emmetrope's eyes be covered, and the other made to fix an object at a given distance, the covered eye turns inward, and makes the same effort of convergence as the uncovered one. Let a concave glass be placed in front of the latter: it is in this case forced to make a greater accommodative effort in order to see the object distinctly. The covered eye not only makes the same effort of accommodation, but accompanies it with an equivalent movement of convergence, and turns inward as if the object had been brought nearer to it, through a distance corresponding to the increase of accommodation.

Inversely, if the whole or a part of the accommodation of the uncovered eye be caused to relax, by means of a convex glass, the covered eye will diverge relatively to the distance of the object.

Another phenomenon, first observed by Donders, is not less adapted to show the relations existing between the two functions.

When we have hitherto spoken of the *punctum proximum* of accommodation, it has always been a question of one eye only. In order to determine it, we covered one eye and brought the object of fixation toward the other, until it reached the nearest point at which it could be seen distinctly.

If, in trying this experiment with a young emmetrope, we suddenly uncover the eye, we see it turning slightly outwards, as if, during the fixation of the other eye, it had squinted inward. This is, in fact, the case. The covered eye was not then directed toward the object, but its line of fixation passed to the nasal side of the latter. This is easy to verify. One has only to cover one eye so that it cannot see the object, but so that it can be watched by the observer.

The convergent strabismus, if we can so term it, of this eye, manifests itself by a homonymous diplopia at the moment when the eye is uncovered. In order to correct this diplopia, the individual is obliged to carry the object farther off—what amounts to the same thing, to draw back his head, which he generally does involuntarily.

It follows from this that the binocular *punctum proximum* is farther away than the monocular one, and that distinct vision at the distance of the latter is obtained only by means of an exaggerated convergence. In other words, if, with a person in a normal condition of sight, accommodation and convergence increase proportionally, he can bring the limit of distinct vision for one eye a little nearer, when, with the other eye, he exerts an exaggerated effort of convergence.

This fact, which is of very great importance, not only for physiology, but also for pathology, proves—

1*st*. That, unquestionably, convergence and accommodation are intimately related to one another, since an increased effort of the former produces also a greater tension of the latter.

2*d*. That, however, this connection is not absolute. In fact, in our example both functions separate from one other, in that the degree of convergence, during the fixation of the *punctum proximum absolutum*, is higher than that of the accommodation.

Accommodation is, therefore, not so absolutely allied with convergence as certain authors,[1] who have attributed to them a common centre of innervation, would have us suppose.

There are, however, other even more valid reasons why this could not be possible. Above all, we must not forget that accommodation is not under the influence of the ciliary muscle only, as convergence under that of the adductor muscles, but that it is also a function of the elasticity of the crystalline lens. Since this elasticity diminishes gradually from childhood on, a given effect of accommodation requires, as age advances, a greater and greater effort of the ciliary muscle, and consequently an increased degree of innervation. The energy of the adductor muscles, on the contrary, not only does not diminish with the accommodation, but constantly increases until the individual has attained his full development, and thereafter remains almost stationary until he has reached an advanced age.

In order that the two functions may act in harmony, man, in his normal state, must be able to gradually devote a greater nervous impulsion to the accommodation than to convergence.

Besides this, emmetropia alone, in the strict sense of the word, always requires the same number of dioptries of accommodation as metre-angles of convergence. But there exists, as we shall see, an important and numerous category of eyes, which, though slightly ametropic, are far from being abnormal. If such eyes always converged to the same degree to which they accommodate, they would be doomed either to indistinct vision or to diplopia.

Let us take, for example, an object situated at $\frac{1}{3}$ of a metre. It requires, in any case, 3 *m a* of convergence, but only from the emmetrope 3 D of accommodation.

A hyperope of 2 D requires 2 D of accommodation in order merely to render himself emmetropic, therefore 5 D in order to see distinctly at $\frac{1^{\text{m.}}}{3}$. If he is not able to combine these five units of accommodation with three units of convergence, but always employs the same amount of each, he will always be exposed to the greatest visual disturbance. If he converged properly, he would be imperfectly adapted—that is to say, with 3 instead of 5 D—and would, therefore, see indistinctly. If, on the contrary, he employed 5 D of accommodation, he would see the object distinctly, but double, since, with the 5 dioptries, 5 metre-angles, *i.e.*, two too many, are associated. This excess of adduction constitutes indeed a convergent strabismus, and is accompanied by homonymous diplopia.

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[1] Porterfield, *A Treatise on the Eye*, vol. i., p. 410, Edinburgh, 1759; and T. Müller, *Vergleichende Physiologie des Gessichtssinnes*, p. 216, 1826.

The reverse would happen to a myope who was unable to dissociate the two functions required for binocular vision. If his myopia amounts to 2 dioptries, he requires only 1 D of accommodation, in order to see distinctly at $\frac{1}{3}$ of a metre. If he employed 3 D—that is to say, as many as units of convergence—he would see the object binocularly but indistinctly, since his refraction would be too strong for that distance. If he accommodated properly with one dioptry and his convergence would be 2 metre-angles below the required degree, the object would appear to him distinctly but in crossed diplopia, since the insufficient convergence corresponds to a divergent strabismus.

It would be putting a slight upon nature to assume that, incapable of creating only emmetropes, she had coupled accommodation and convergence in such a way that, instead of rendering the vision more perfect, they would act only to disturb it. It may, therefore, be taken for granted that, notwithstanding their intimate association, the two functions must be independent from one another to a certain degree.

This is indeed the case. A young emmetrope, for instance, still distinguishes clearly the some far point in spite of concave glasses. These lenses, however, require a more or less considerable effort of accommodation in order that vision may not be rendered indistinct, while the direction of the eyes remains the same, the point not having changed its position.

We can also overcome the effect of concave or tolerate weak convex lenses for reading at the same distance at which we read without glasses—that is to say, we can vary our state of accommodation independently of the convergence of our eyes.

The fact that we can see, distinctly and simultaneously, with both eyes an object situated at an invariable distance, in spite of glasses which necessitate a change in the accommodation, is proof positive that the relation between the latter and convergence is not absolute; and the fact, equally easy to demonstrate, that such glasses can not exceed certain degrees, proves that the independence of the two functions exists only within certain limits.[1]

Donders added to this proof another still more conclusive one. The above experiment proves that, the convergence remaining the same, the accommodation can vary. He has proved that, conversely, the accommodation remaining the same, the convergence may be made to vary.

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[1] Volkmann, *Neue Beitraege zur Physiologie des Gesichtssinnes*, 1836, p. 148. Donders, in *Holländische Beiträge zu den anat. und physiol. Wissenschaften*, by Van Deen, Donders, and Moleschott, 1846, vol. i., p. 379.

For this purpose he used prisms in the following manner. If the same point be fixed, and a prism held before one or each of the eyes, its apex toward the temple, the convergence is diminished, as we have already seen when determining the *punctum remotum* of this function. By turning the apex of the prism toward the nose, the convergence necessary for vision of the same point is, on the contrary, increased. In either case the accommodation does not change, the object not having left its position, and the rays which emanate from it not having undergone any change of direction relatively to each other.

Here, again, there exist for both convergence and divergence limits beyond which the accommodation no longer remains stationary.

By means of both methods Donders has shown that, for nearly all points within the domain of binocular vision, accommodation and convergence are, to a certain degree, independent of each other.

It was important to determine for the whole region of binocular vision—

1*st*. *Convergence remaining the same*, by how many dioptries accommodation can be increased and diminished.

2*nd*. *Accommodation remaining unchanged*, by how many metre-angles convergence can be increased and diminished.

Both these problems were first solved by Donders.

1*st*. He determined, for a series of points situated on the median-line, the strongest concave and the strongest convex glass, which could be overcome without the distinct binocular vision being affected.

In this way, we obtain, for each degree of convergence, a *maximum* and a *minimum* of accommodation.

Donders called the sum of variations which accommodation can undergo for a given degree of convergence the "RELATIVE RANGE OF ACCOMMODATION."

This relative range, or amplitude of accommodation, is, again, necessarily equal to the difference between the refraction which the eye represents during the strongest, and that which it represents during the weakest tension of the ciliary muscle—the difference between the *relative maximum* and the *relative minimum* of accommodation.

We obtain, therefore, for the RELATIVE AMPLITUDE OF ACCOMMODATION, the following formula:—

$$a_1 = p_1 - r_1$$

the members of which we may designate by a small $_1$, in order to distinguish them from the *absolute* ($a$), and the *binocular* ($a_2$) amplitudes of accommodation.

If it seem preferable to employ focal distances (which is not so convenient, but was inevitable at the time when Donders wrote his treatise, the system of dioptries not having yet been introduced into ophthalmology), the distance of the relative *punctum proximum* will be called $P_1$, the distance of the relative *punctum remotum* $R_1$, and $A_1$ will be used to designate the focal distance of the lens, which is equivalent to the difference in the refraction of the eye, during accommodation for the *punctum proximum* or for the *punctum remotum*.

In this case the formula takes the form originally given it by Donders:[1]

$$\frac{1}{A_1} = \frac{1}{P_1} - \frac{1}{R_1}.$$

*2nd.* The notion of a relative range of accommodation necessarily involves that of a *relative range of convergence.* This, too, is equal, for each degree of accommodation, to the difference between the *relative maximum* and the *relative minimum of convergence.* The former may be determined by means of the strongest adducting, and the latter with the aid of the strongest abducting prism, whose influence can, without change in the accommodation, be overcome. Expressing the effect of these prisms in metre-angles, we obtain, for the relative range of convergence, the following formula:—

$$a^c{}_1 = p^c{}_1 - r^c{}_1.$$

Let us now turn our attention to the results of these investigations, which results are most clearly presented by the following diagrams (Figs. 82 to 88).

On the *horizontal* lines (abscissæ) are recorded the degrees of *convergence;* on the *vertical* lines (ordinatæ) the degrees of *accommodation.* The figures along the vertical line, at the left of the scheme, correspond, therefore, to units of convergence—that is to say, to metre-angles—while the figures placed on the horizontal line indicate units of refractive power—*i.e.*, dioptries.[2]

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[1] Donders' *Anomalies, &c.*, p. 110 English edition, p. 98 German edition.

[2] When Donders made his experiments, the dioptry and metre-angle were not yet known. Hence he was forced to choose an arbitrary unit of refraction. As such he took a lens having a focal distance of 24″, and each vertical division of his diagram corresponds to $\frac{1}{24''}$ of refractive power (1·54 dioptry). The angles of convergence are noted directly, so that the figures marking the divisions of the ordinates and abscissæ do not, at first glance, seem to have anything in common. Here, again, the great advantage of the system of dioptries, and of that of metre-angles, becomes apparent.

The diagram (Fig. 82) is that of Donders, but the division is Nagel's. By comparing it with Fig. 57 of Donders' work on the *Anomalies of Refraction and Accommodation*, it will at once be seen that Nagel's is by far the easier to understand.

The diagonal D D, starting from the zero-point, contains all the points for which accommodation and convergence are equal.

Thus the zero-point corresponds to the state of rest, in which the eyes are directed parallelly and relax their accommodation. Hence there is neither convergence nor accommodation. Let us take another point of the diagonal—that at the eighth division, for instance. Here the convergence and accommodation equal 8 units; for at this point the horizontal line (abscissa), which starts from the point 8 of the division at the left, and the ordinate, which starts from the point 8 at the bottom of the figure, intersect. The former line indicates that the lines of fixation have a convergence of 8 metre-angles, while the latter indicates that, at the same time, 8 dioptries of accommodation are in action. In other words, the eyes are directed toward a point which is $\frac{1}{8}$ of a metre distant from each of them, and are exerting a corresponding effort of accommodation—*i.e.*, one of 8 dioptries.

The curved line $p_1$ $p^2$ $p$, in the diagram, indicates the maximum, and $r$ $r_1$ $r_1$ the minimum of accommodation, that can be brought into play for each degree of convergence. The divisions of the vertical lines, included between the curves $p$ and $r$ give, therefore, the number of dioptries of *relative range of accommodation* for the degree of convergence, indicated by the horizontal line, which crosses the diagonal at the same point with this vertical line.

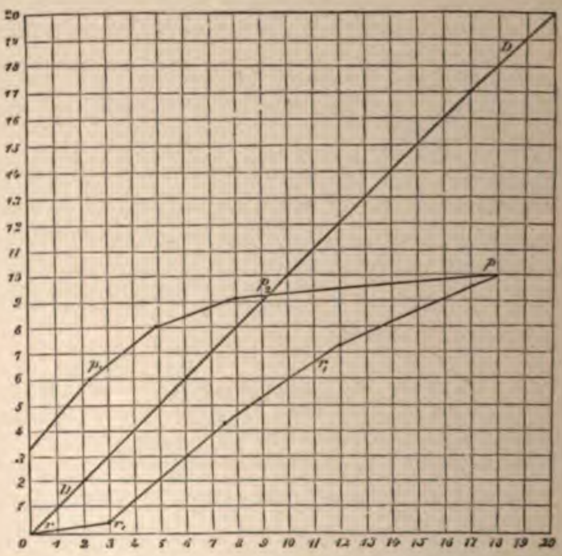

FIG. 82.

Moreover, as was said above, the diagonal contains the points for which convergence and accommodation are, for emmetropia, in harmony.

The vertical line which *ascends* from the diagonal toward the curve $p_1$, indicates the *excess* of accommodation disposed of by the eyes for the given degree of convergence, *i.e.*, the *positive* portion of the relative range of accommodation. It equals the number of dioptries represented by the concave glass which could be overcome in spite of the degree of convergence.

The vertical *descending* from the diagonal toward the curve $r$, corresponds to the *relaxation* of accommodation, of which the eyes are susceptible for the given degree of convergence, *i.e.*, the *negative* portion of the relative range of accommodation. It corresponds with the strongest convex lens that was borne with this degree of convergence.

At the zero-point the curve, in the present case, is between three and four dioptries, which signifies that, with zero of convergence, *i.e.*, the visual lines being parallel, the person under examination can still exert an effort of accommodation equal to 3·35 dioptries. Hence the emmetrope, from whom Donders obtained this diagram, could see at a distance with concave glasses of 3·35 dioptries, without detriment to the distinctness of the retinal images. Or again, although directing his eyes toward a point situated at infinity, he could exert an effort of accommodation equal to that which he would make if fixing a point $\frac{1^{\text{m.}}}{3\cdot35} = 30$ centimetres distant.

The curve $r$, on the contrary, coincides with the zero-point of convergence, which signifies that the emmetrope cannot further relax his accommodation. We were already acquainted with this fact. When an emmetrope looks at a distant object, his accommodation is at rest, and, inasmuch as negative accommodation does not exist, it is not possible to make the accommodation fall below zero. We readily understand, then, what the diagram illustrates, viz., that with zero of convergence the positive portion of the relative amplitude of accommodation amounts to 3·35 dioptries, while the negative portion is *nil.* Hence the entire relative amplitude of accommodation, for a parallel direction of the lines of fixation, is, in this case—

$$a_1 = 3\cdot35 - 0 = 3\cdot35 \text{ dioptries,}$$

conformably with the formula $a_1 = p_1 - r_1$.

When the eyes present a convergence of 3 metre-angles, *i.e.*, when they simultaneously fix an object $\frac{1^{\text{m.}}}{3}$ (333·33 millimetres) from each of them, the *punctum proximum* of the relative accommodation is at $p_1$ (Fig. 82), corresponding to 6·5 dioptries, and the *punctum remotum* at the point $r_1$, *i.e.*, at 0·5 of a dioptry.

The relative amplitude of accommodation for this degree of convergence is, consequently, $a_1 = 6{\cdot}5 - 0{\cdot}5 = 6$ dioptries. Hence the accommodation may vary by 6 dioptries, while the convergence is maintained at 6 metre-angles.

Considering the curves $p_1$ and $r_1$ relatively to the diagonal at the point where the latter crosses the ordinate corresponding to 3 metre-angles, we see that the positive portion of the relative amplitude of accommodation amounts to 3·5 D, and the negative portion to 2·5 D. This signifies that, with 3 *m a* of convergence, the emmetrope in question could bring into action, not only the corresponding 3 dioptries of accommodation, but as many as $3 + 3{\cdot}5 = 6{\cdot}5$ D. He could see the same object distinctly at the same distance when looking through concave glasses of 3·5 D. On the other hand, he could relax his accommodation by 2·5 D, by using convex glasses of 2·5 D, without any change in his convergence.

With a convergence of a little more than 9 *m a* (9·47), the *punctum proximum* of the relative range of accommodation is on the diagonal itself ($p_2$). In other words, the eyes are no longer susceptible of an effort of accommodation greater than is compatible with convergence towards this point, *i.e.*, greater than 9·47 dioptries. Hence the positive portion of the relative amplitude of accommodation is *nil*.

Moreover, the curve $p_1$ does not even follow the diagonal from this point on, but crosses, and remains below it—that is to say that, from this point on, the accommodation is always less than the convergence. Hence the point $p_2$ is the nearest one for which the individual examined was able to bring into play, to the same degree, his convergence and his accommodation—the nearest point that he could see clearly with both eyes; hence $p_2$ is his *binocular punctum proximum*.

The curve $r_1$ is, in this place, at 5·5 dioptries. Hence the relative amplitude of accommodation, here, is $a_1 = 9{\cdot}47 - 5{\cdot}5 = 3{\cdot}97$ D—approximately 4 D. But these 4 D all belong to the *negative* portion of the relative amplitude of accommodation. The emmetrope in question saw clearly with both eyes at a distance of $\frac{1\text{ m.}}{9{\cdot}47}$, without glasses, or with convex glasses of 4 D, but not with concave glasses.

From this moment on, the curve $p$ remains below the diagonal, which is equivalent to saying that the maximum accommodation is always less than the convergence; hence distinct binocular vision no longer exists. Thus, with 12 *m a* of convergence, the eyes present only 9·5 D of accommodation. They can relax this, too, by more than

2 D, but they cannot render it equal to the convergence. In the example chosen, the person examined could cause his visual lines to converge toward a point $\frac{1\text{ m.}}{12}$ distant; he would then have seen binocularly; but, the accommodation amounting to only 9·5 D, the point would have seemed indistinct. Or, on the other hand, he would have seen the point distinctly if it were $\frac{1\text{ m.}}{9\cdot5}$ distant, but would have seen it double, since, in order to realise the requisite 9·5 dioptries of accommodation, he would have been obliged to make an effort of convergence amounting to 12 *m a.* The object is $\frac{1\text{ m.}}{9\cdot5}$ from one of the eyes, but the line of fixation of the other crosses that of the first eye at a distance of $\frac{1\text{ m.}}{12}$ from it. Hence the object is seen distinctly, but double.

The curve $p_1$ reaches its highest point at the horizontal line corresponding to the tenth division of the ordinates; but it has now arrived at the eighteenth degree of the abscissæ. This means that the nearest point that can be seen distinctly is $\frac{1}{10}$th of a metre from the eye—the maximum accommodation of which the person is capable is 10 D. The point $p$ (Fig. 82) is therefore the *absolute punctum proximum.* But this degree of accommodation is produced only by means of an effort of convergence amounting to 18 *m a.* The lines of fixation intersect at $\frac{1\text{ m.}}{18}$ from the eyes, while the eyes are optically adapted to a point $\frac{1\text{ m.}}{10}$ distant.

At the distance of the *absolute punctum proximum* of accommodation, there is, therefore, no longer distinct binocular vision. There is either binocular vision, which is indistinct in consequence of the insufficiency of accommodation, or there is distinct vision, but homonymous diplopia, due to the excess of convergence necessary to produce the requisite accommodation. In the case under consideration, the convergence exceeded the accommodation by $18-12\cdot5=5\cdot5$ *m a.* Donders' emmetrope had a base line of 28½ Paris lines (64 millimetres). The 5·5 metre-angles corresponded, therefore, in his case, to ten degrees for each eye, which constitutes a convergent strabismus of twenty degrees, if the rotation inward be confined to one eye.

Finally, we see that, at the point $p$, the curve for the *punctum proximum* and that for the *punctum remotum* coincide. The relative range of accommodation is therefore reduced to zero. The person examined succeeded in distinguishing, monocularly, a point 10 centimetres distant, by squinting inward forcibly with the other eye, but he could not have seen it with a concave glass, nor even with a convex one, without disturbing his convergence.

So much for the relative amplitude of *accommodation* of the emmetrope.

How does the *relative amplitude of convergence* behave?

I mean: We know, for every degree of convergence, by how many dioptries the accommodation may be increased or diminished. We now ask inversely: *The accommodation remaining unchanged, how much can the convergence be increased and diminished?*

This problem might also be solved experimentally by Donders' method. We should determine, for every distance of the object, *i.e.*, for every degree of accommodation, the strongest adducting and the strongest abducting prism that could be supported without producing diplopia. With its apex toward the nose, the prism would give the *positive* portion of the relative amplitude of convergence, because it necessitates an excess of convergence. With its apex turned toward the temple, it would give the *negative* portion of the relative amplitude of convergence, because it exacts a diminution of convergence,—perhaps even a certain degree of divergence of the lines of fixation. The sum of the angles of deviation of both prisms would give, in degrees, the *relative range of convergence*. From this number we could easily deduce the corresponding number of metre-angles for the base line of the person examined.

But this experiment becomes unnecessary when we once have the diagram of the relative amplitude of accommodation. The latter gives, at the same time, the relative range of convergence. We have only to consider the degrees of accommodation for each point of the diagonal D D, and the corresponding degrees of convergence are found marked on the horizontal line which passes through this point of the diagonal taken as a starting-point for the accommodation.

The *positive* portion of the relative range of accommodation is at the *right*, the *negative* portion to the *left*, so that the curve $r_1$, which corresponds to the course of the *punctum remotum* of *relative accommodation*, gives, at the same time, that of the *punctum proximum* of *relative convergence;* while the curve $p_1$, for the *punctum proximum* of *relative accommodation*, is identical with that for the *punctum remotum of relative convergence.*

For instance, in order to ascertain the limits within which the convergence can vary, while the accommodation is maintained at $\frac{1^{m.}}{4}$, *i.e.*, at four dioptries, we seek the point of the diagonal which is directly above the point 4 of the horizontal division (of dioptries). We find, thus, that the convergence may be increased up to a little more than 7 $m\,a$, that is to say that, when accommodating for $\frac{1^{m.}}{4}$, the eyes can converge toward $\frac{1^{m.}}{7}$, the *punctum proximum* of *relative convergence*.

The latter, on the other hand, can be reduced to 0·5 *m a*, which is equivalent to saying that, notwithstanding the effort of accommodation, amounting to four dioptries, the lines of fixation can cross each other at a point situated $\frac{1^{m.}}{0\cdot5} = 2$ *m a*. This is the *punctum remotum* of relative convergence.

Hence the relative amplitude of convergence, in our example, is—

$$a^c{}_1 = 7 - 0\cdot5 = 6\cdot5 \; m\,a.$$

The *positive* portion of the *relative range of convergence* amounts to 3 *m a* (to the right of the diagonal), the *negative* portion to 3·5 *m a* (to the left of the diagonal). That is to say that the eyes, while maintaining the same accommodation, can overcome the effect of adducting prisms producing a deviation of 3 *m a*, and that of abducting prisms requiring a diminution of convergence equal to 3·5 *m a*.

In Fig. 83 is given a diagram of the relative ranges of accommodation and convergence of another emmetrope. The subject of this examination was Dr. Eperon, who, at the time when it was made, was my clinical assistant, and twenty-six years of age. The dotted curves mark the results of the first examination ; the plain line curves were obtained from an examination made after his eyes had been

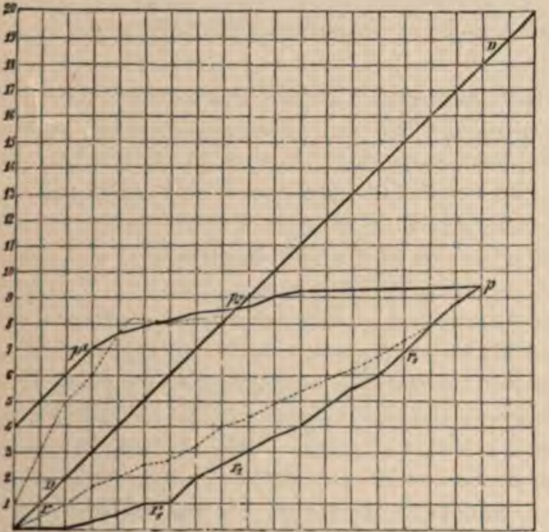

FIG. 83.

trained for some time. The signs used have the same meaning as heretofore: $p_2$ is the binocular, and $p$ the absolute *punctum proximum*. This diagram corresponds very nearly, as will be seen, to that of Donders. But it also shows one very important thing, viz., that the mutual relations of the two functions are easily susceptible of modification.

### *The relative Amplitudes of Accommodation and Convergence in Ametropia.*

We have already remarked above that the relation between accommodation and convergence must necessarily be different in the ametrope, in case he is to see binocularly and distinctly without the aid of auxiliary glasses. Indeed, myopia represents, even in a state of repose, a certain degree of positive refraction, while convergence under such circumstances, is *nil*, and any degree of positive convergence requires a convergence effort. Let us take, for instance, the case of a myope of eight dioptries: when the accommodation is totally relaxed, he sees distinctly at a distance of $\frac{1}{8}$ of a metre. Binocular vision at this distance, however, requires convergence amounting to 8 *m a*. The zero-point of accommodation, therefore, is at $\frac{1^{m.}}{8}$, *i.e.*, at a distance for which the convergence is not zero, but 8 metre-angles.

If the object be brought nearer, the accommodation and convergence will increase; but the accommodation will always be eight units below the convergence. Thus, the object being $\frac{1}{10}$ of a metre distant, 10 metre-angles of convergence and 10 dioptries of accommodation will be required. But eight of these ten dioptries are furnished by the static refraction of the eye, leaving only two to be furnished by the dynamic refraction (accommodation).

For 12 metre-angles of convergence, $12-8=4$ dioptries of accommodation are necessary.

Let us now carry the object beyond the zero-point of accommodation, the *punctum remotum* of the eye, to infinity, for instance. It will then be at the zero of convergence. Hence, in order that vision be clear, the accommodation would have to be eight dioptries below zero. But accommodation cannot become negative. The eye can only nullify it. In this case the eye is adapted to its *punctum remotum*—its zero of accommodation—below which it cannot descend. Our myope will be able to see, binocularly, objects situated between infinity and $\frac{1}{8}$ of a metre from the eyes, if he have a normal range of convergence; but along this whole range he will see indistinctly, because his refraction will always be too strong. For infinity it will be excessive by eight dioptries; for an object 2 metres distant, by $8-\frac{1}{2}=7{\cdot}5$ D; for an object 1 metre away, by $8-1=7$ D; for one at $\frac{1}{2}$ metre, by $8-2=6$ D. Hence the myope must abstain from all effort of accommodation while the convergence increases from zero up to the figure which represents the degree of his myopia. From this moment on, the amount of accommodation brought into play will always be below the value of the convergence, by a number of dioptries equal to that of the myopia.

In *hyperopia* we find a different condition of affairs.

Here the zero of accommodation is below the zero of convergence. When the latter is *nil*, the lines of fixation being parallel, the eyes must have already made an effort of accommodation in order to correct their defect of refraction, *i.e.*, to adapt themselves to infinity. Let us suppose the hyperopia of each eye to amount to two dioptries; the *punctum remotum*, the zero of accommodation, is $\frac{1}{2}$ metre to the rear of the eye, in the negative portion of the diagram, 2 D below zero. In order to see at infinity, the accommodation must make good this refractive defect, while the convergence is still at rest.

If the object be brought nearer, so that the convergence is increased, the accommodation will always be stronger than the convergence, by a number of dioptries equal to the deficit in static refraction, *i.e.*, to the degree of the hyperopia. Thus, the object at a distance of $\frac{1}{8}$ of a metre requires 8 *m a* of convergence and 8 D of positive refractive power. But, in order to furnish these, the eye must first make good the deficiency in its static refraction, which, in our example, equals 2 D. This the eye does by means of its accommodation. After this, only, is it able to give itself the required 8 D of positive refractive power. It borrows these, also, from its dynamic refraction. The latter, therefore, amounts to $2+8=10$ D, while the convergence is one of only 8 *m a.*

It is apparent, from this, that the relation between accommodation and convergence must be quite different in ametropia from what it is in emmetropia.

In myopia the amount of the accommodation remains lower than that of the convergence, while hyperopia calls for more accommodation than convergence.

Hence distinct and binocular vision requires that the relation between the accommodation and convergence be different in ametropia and emmetropia. Is this actually the case? This question is one of primary importance.

We have seen (Fig. 82) that our emmetrope can still relax his accommodation by two dioptries, although maintaining his convergence at $\frac{1}{4}$ of a metre.

This shows that, if he should become myopic by one, two, or even three dioptries, he would, notwithstanding this, be able to converge for an object $\frac{1}{3}$ of a metre distant, and see distinctly. He would make an effort of accommodation one, two, or three dioptries less than that of the emmetrope under like circumstances. On the other hand, he still has at his disposal, for the same degree of convergence, three dioptries of positive, relative accommodation. Hence he could maintain his convergence and, at the same time, correct a hyperopia amounting to three dioptries.

If, in emmetropia, there exist such a latitude in the relations between the two functions, it is to be hoped that the accommodation regulates itself, more or less, according to the convergence, in cases where an ametropia has existed for centuries, or where it has been developed or gradually modified, from generation to generation. We see families, and even peoples that must have been hyperopes from time immemorial. We see others in whose cases hyperopia slowly but regularly changes to emmetropia, and emmetropia to myopia. Why should not the relations of the accommodation and convergence—which depend, it is true, upon the same nerve, though not upon the same muscles—be modified conformably to the exigencies of the case?

According to Iwanoff's investigations, the development of the ciliary muscle appears to be modified in accordance with the refractive condition of the eye. But, even without pronounced anatomical differences, the necessary correspondence between the functions of accommodation and of the motor muscles of the eye might very well exist, provided the innervation were in keeping with the static refraction.

In myopia, for instance, it would be necessary that the innervation of the muscles controlling convergence should not be accompanied, from the first, by an innervation of the muscle of accommodation, and that the former should always be more energetic than the latter.

In hyperopia the nervous impulsion of the ciliary muscle must prevail.

Hence the possibility, of an adaptation of the relation between the accommodation and the convergence to the ametropia, must be admitted, especially if the latter be developed slowly enough in the individual, or rather in the race, to give the normal relation of the two functions time to become modified.

However, we shall not be surprised if we occasionally meet with a lack of harmony between the convergence and the accommodation. We shall see, indeed, in the chapter on the causes of ametropia, that an anomaly of refraction may be developed so rapidly that neither the anatomical disposition of the ciliary muscle, nor the innervation of the intrinsic and extrinsic muscles of the eye could be modified in so short a time.

Moreover, ametropia may reach such degrees that convergence becomes insufficient in myopia, and accommodation in hyperopia.

But let us return from these speculative considerations to the positive domain of experimental research.

Donders and Nagel have carefully examined the relation between the accommodation and the convergence in the case with which we are concerned. Figure 84, for instance, gives us the diagram for a myope of four dioptries.[1]

The subject of the examination was a young man twenty-three years of age, a medical student. The figures in this diagram have the same signification as in that for emmetropia. The zero-point of accommodation, which corresponds to the *punctum remotum* (*r*), is at the point 4 of the horizontal, because the eye already has 4 D of positive refraction when at rest. The diagonal, which contains the points for which the effort of convergence and that of accommodation are equal, takes its origin at this point 4. It ascends toward the right, at an angle of 45 degrees, parallelly with that for emmetropia, but displaced four divisions to the right, which corresponds to four dioptries of myopia.

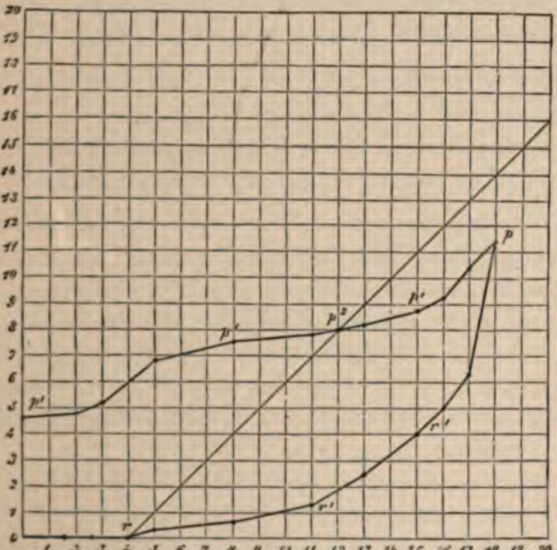

FIG. 84.

*Above* the diagonal we find, as before, the values of the *positive* relative accommodation; *below*, those of *negative* relative accommodation; to the *left*, *negative* relative convergence; and to the *right*, *positive* relative convergence. The curve $r\,r_1\,p$, consequently, indicates the degree of the minimum of accommodation, or of the maximum of convergence. The curve $p_1\,p_2\,p$ corresponds to the maximum of accommodation and to the minimum of convergence.

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[1] Nagel, *loc. cit.*, p. 497, Fig. 47.

The situation of the point $r$ proves that the accommodation is *nil* while the convergence is varying from 0 to 4 metre angles (4 *m a* of negative relative convergence). But, for the same convergence, the subject of the examination can bring into action six dioptries of accommodation (6 D of positive, relative accommodation), as is shown by the curve $p_1$, which crosses the ordinate 4 at a point six divisions above the zero-line.

For zero of convergence, the positive relative accommodation amounts to 4·75 dioptries, as is shown by the point $p_1$ on the zero ordinate. Hence, though directing his eyes parallelly, the myope in question could still call into play 4·75 dioptries of accommodation, which would raise his positive refraction to $4 + 4{\cdot}75 = 8{\cdot}75$ dioptries. He could, for instance, unite the images of a stereoscopic "view" without the aid of either prisms or convex lenses. The view being generally about 166 mm. $(\frac{1\text{ m.}}{6})$ from the glasses in a stereoscope, six dioptries of positive accommodation are required in order to see its pictures distinctly, and the fusion of their images demands a parallelism of the visual lines. Nagel's myope had at his disposal even more positive refraction than he needed in order to see at a distance of $\frac{1^{\text{m}}}{6}$ without changing the parallel direction of his lines of fixation.

From zero to 4 *m a* of convergence there can be no negative relative accommodation, since the accommodation cannot be relaxed so as to become less than zero. But from this point on, the amplitude of accommodation is divided into two parts, just as is the case with the relative amplitude of convergence. They are found in the same way as in emmetropia.

At the point $p_2$ the curve $p_1\,p$ meets the diagonal. This means that, at this point, the maximum of accommodation still corresponds with the convergence, but that, beyond it, the former is less than the latter. Hence the point $p_2$ is the *binocular punctum proximum.* It is $\frac{1}{12}$ metre from each eye (12 *m a*), and the myope in question, in order to see it, must make an effort of accommodation equal to eight dioptries. Indeed, vision of $p_2$ requires 12 D of positive refraction. The myopia furnishes four of them, leaving eight to be supplied by the accommodation. At the same time, the myope could still diminish his accommodative power six dioptries (negative relative accommodation), as is indicated by the curve $r_1$ on the same ordinate. But, with the same effort of accommodation, the myope could converge still more, *i.e.*, up to 17·3 metre-angles. Hence the convergence, in the case under consideration, was greater than the accommodation.

In fact, from the point $p_2$ on, the curves $p_2 p_1 p$ and $r_1 p$ remain below the diagonal. For each degree of convergence, the accommodation could still vary by the amount included between the two curves; but, even with the maximum effort, it could not reach the degree which corresponds to the convergence.

At the point $p$ the two curves unite in one. Relative accommodation no longer exists. This is the *absolute punctum proximum.*

The maximum accommodation obtainable by the myope, in this case, was 11·5 dioptries. This degree of dynamic refraction, combined with his static refraction, gives $11·5 + 4 = 15·5$ dioptries of positive refraction. Hence his absolute *punctum proximum* was $\frac{1^{\text{m.}}}{15·5} = 64·5$ millimetres from each eye. But this maximum of accommodation was obtained only at the expense of an effort of convergence amounting to 18 metre-angles, as is indicated by the diagram. This myope was, therefore, in the same predicament as the emmetrope of the preceding experiment. Fixation of the absolute *punctum proximum* rendered binocular vision impossible, and caused him to squint inward.

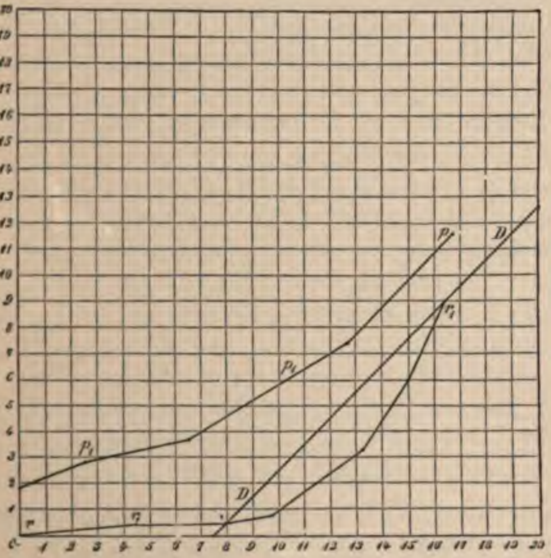

FIG. 85.

Nagel gives a very instructive diagram of another myope. His relative amplitudes of accommodation and convergence are represented by Fig. 85.[1] The origin of the diagonal D D shows that the myopia amounted to 7·5 dioptries. This myope seems to have been unable to harmonise his accommodation and convergence in fixing his absolute *punctum remotum*. The latter is $\frac{1}{7\cdot5}$ of a metre distant, and requires complete relaxation of the ciliary muscle. But we see, by the figure, that the accommodation is at repose ($r$ on the abscissa *zero*) only when the convergence is likewise *nil*, *i.e.*, when the lines of fixation are parallel. When the eyes are directed toward an object $\frac{1}{7\cdot5}$ of a metre distant, the individual makes a useless effort of accommodation, amounting to 0·5 D. Hence the refraction is $7\cdot5+0\cdot5=8$ dioptries, while his convergence amounts to but 7·5 metre-angles. He cannot, therefore, see distinctly at this distance; his absolute *punctum remotum*, $r$, $\frac{1\text{ m.}}{7\cdot5}$ from each eye, is seen clearly only when his lines of fixation are parallel. Hence they *diverge*, relatively to the distance of the *punctum remotum*. This is an important fact to consider in connection with divergent strabismus, which is frequent among myopic people.

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[1] Nagel, *loc. cit.*, p. 127, Fig. 48.

The *binocular punctum remotum* ($r_2$) is $\frac{1}{8}$ of a metre distant, on the diagonal D D. It is only at this distance that the relation between convergence and accommodation is in conformity with the patient's ametropia. From $r_2$ to $r_1$ he possesses a certain relative range of accommodation and convergence, positive as well as negative. But, with 16·5 metre-angles (at the point $r_1$), his convergence has reached its maximum. The curve $r\,r_2\,r_1$, which represents both the minimum of accommodation and the maximum of convergence, crosses the diagonal D D. The accommodation has followed, perfectly, up to that point; hence $r_1$ is the *binocular punctum proximum*. Beyond this point the convergence is exhausted. The curve $p_1\,p$ of the maximum of accommodation, on the contrary, continues to ascend until it reaches the point $p$, where it attains its greatest altitude. The maximum of convergence, in this case, amounted to 16·5 metre-angles. The *punctum proximum* of convergence is at a distance of $\frac{1\text{ m.}}{16\cdot5}=60$ millimetres. The maximum accommodation amounts to 11·5 dioptries; but, the static refraction being equal to 7·5 dioptries, the absolute *punctum proximum* is at a distance of $\frac{1}{11\cdot5+7\cdot5}=\frac{1\text{ m.}}{19}=52$ millimetres. Hence there is *relative divergence* for the absolute *punctum proximum* as well as for the absolute *punctum remotum*, and the harmony between the convergence and the refraction of the eye exists only within the space included between 8 and 16·5 metre-angles.

Let us now consider a case of *hyperopia*. Nagel, again, has given us one of this kind.[1]

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[1] Nagel, *loc. cit.*, p. 498, Fig. 49.

The observation was made upon a subject whose hyperopia amounted to five dioptries (Fig. 86). The starting-point of the diagonal D D, which represents the normal relation between convergence and accommodation, must, therefore, be found at division 5, of the zero-ordinate, which corresponds to five dioptries. Indeed, the convergence (abscissæ) being *nil* (parallelism of the lines of fixation), the hyperope is already obliged to make an effort of accommodation amounting to 5 D, in order to correct his ametropia and see clearly at a distance. One might be tempted to conclude, from this, that, for zero of convergence, there ought to be 5 D of *negative relative* accommodation; this, however, was not the case with Nagel's hyperope. With his eyes directed parallelly, he could relax only 1·5 dioptry of his accommodation (*r m*); he could not wear convex glasses stronger than 1·5 D. This was his *manifest hyperopia*. In order to make him relax all his accommodation and to find his total hyperopia corrected by convex 5, it was necessary to paralyse his ciliary muscle, by means of atropine, or cause his lines of fixation to diverge. Hence total relaxation of the accommodation, the *absolute punctum remotum*, is obtained only at the expense of producing *negative* convergence. The curve *r t p*, which designates the course of the *absolute punctum remotum*, ought, therefore, to have commenced at the left side of the diagram, upon the zero-abscissa.

But, for the case given, only the curve for the *punctum remotum* of the *manifest hyperopia* has any *raison d'être*. This is the one that was experimentally obtained. Adaptation to the absolute *punctum remotum* was realised only by an artificial relaxation of the accommodation.

We have seen, then, that the hyperope in question could diminish his accommodation by 1·5 D, even when looking binocularly, at a distance. On the other hand, he could bring into play 2·75 D of dynamic refraction more than the number required for the correction of his hyperopia. That is to say that he still saw clearly at a distance with concave glasses of 2·75 D, which necessitated an effort of accommodation amounting to $5 + 2{\cdot}75 = 7{\cdot}75$ D, the degree of his hyperopia being given.

Up to about 5 metre-angles, the curve $p_1$ is to the left of the diagonal, *i.e.*, the hyperope had more accommodation than he needed in order to see clearly with both eyes; but, just before reaching 5 *m a*, the curve crosses the diagonal at the point $p_2$. This is the binocular *punctum proximum*. It is therefore, at the utmost, $\frac{1}{5}$ of a metre distant.

From this point on, the curve $p_2\,p$ ascends but little, to the abscissa 10, which it reaches at the point *p*. This is the *absolute punctum* *proximum*. Of the ten dioptries of accommodation that it represents, five are employed solely to correct the hyperopia; only five remain

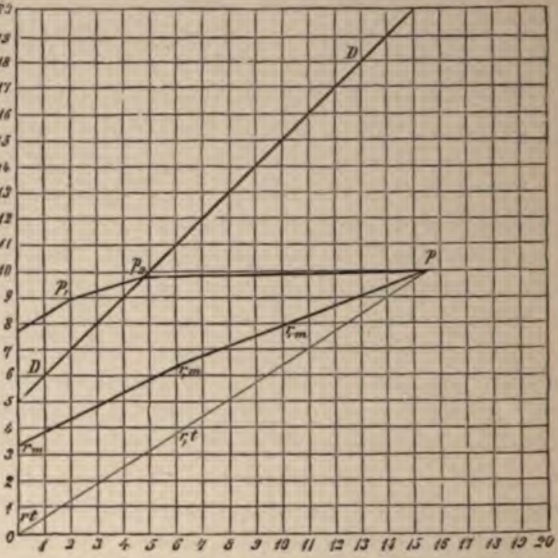

FIG. 86.

with which to bring the point of fixation from infinity to a positive distance. The minimum of the latter is therefore equal to $\frac{1^{m.}}{5} = 20$ centimetres. In order to produce this adaptation of the dioptric apparatus of each of his eyes, the person is obliged to bring into play 15·5 metre-angles of convergence. With this amount of convergence he fixes an object which would require only 5 metre-angles. Hence a convergent strabismus, of 10·5 *m a* for each eye, is produced—equivalent to about 38 degrees of strabismus for a single eye.

The curve ($r\,m$, $r_1\,m$) of the *punctum remotum* of the relative accommodation ascends slowly toward the point *p*. It indicates that the hyperope still had at his disposal a certain degree of negative relative accommodation, which, however, was of no use to him. On the contrary, he concentrated all his power of accommodation in order to correct his defect of refraction. In fact, in spite of all artifices, the attempt to make him relax his ciliary muscle, as much as an emmetrope or a myope would have done under the same conditions, was unsuccessful.

The relative amplitudes of accommodation and convergence vary considerably, not only according to the state of the refraction and the degree of ametropia, but also in different individuals. Thus Fig. 87[1] gives the scheme of a person (Dr. Schleich) having half a dioptry of myopia. It proves to what point the curves, especially that of negative relative accommodation, can be extended by exercise.

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[1] Nagel, *loc. cit.*, p. 492, Fig. 45.

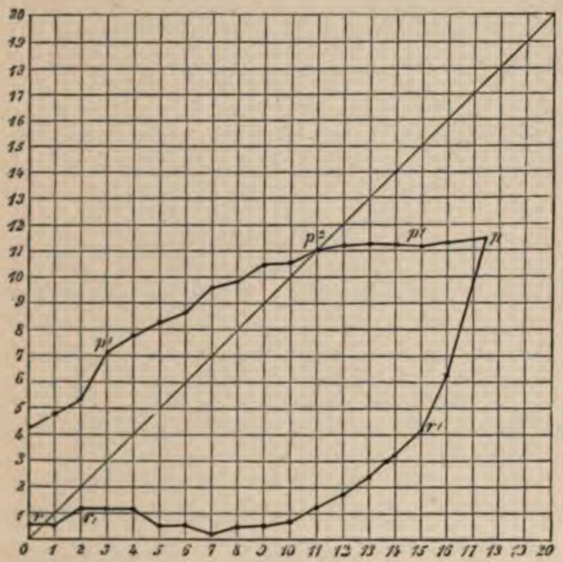

Fig. 87.

Fig. 88,[1] on the contrary, represents the very limited relative ranges of accommodation and convergence of a girl of thirteen years, *slightly* hyperopic.

Considering the above observations, we see that the accommodation has a tendency to modify itself according to the exigencies of the static refraction of the eye.

Thus, in myopia, the same degree of convergence calls for less accommodation in proportion as the myopia is stronger. Comparing the curves of myopes with those of an emmetrope, it is shown that much less accommodation is brought into play in the former than in the latter case.

The hyperope, on the contrary, who exacts much more of his accommodation than does the emmetrope, shows also a higher curve *p* than the latter.

However, this adaptation of the accommodative power has its limits. The diagrams prove this, and would have proved it still more conclusively had the emmetropes and hyperopes examined been older. Indeed, ametropia cannot modify the troublesome influence exerted by age upon the accommodative faculty of the eye.

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[1] Nagel, *loc. cit.*, p. 493, Fig. 46.

Moreover, accommodation and convergence are not in perfect accord in any of the diagrams; otherwise the two curves would always have been equidistant relatively to the diagonal, and the *punctum proximum* of accommodation and that of convergence would have been at the same point with the binocular *punctum proximum* on the diagonal.

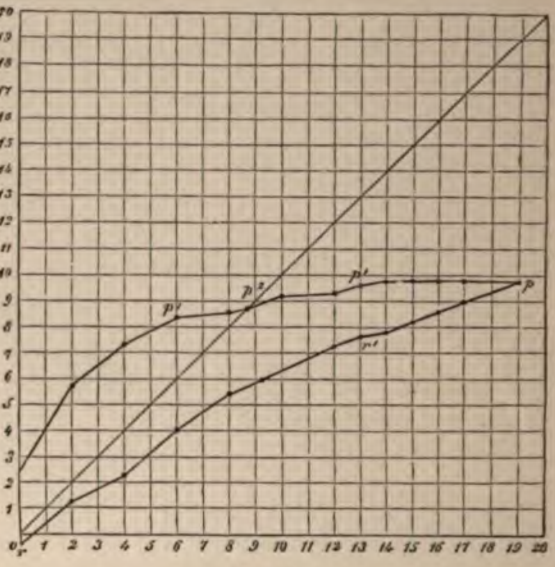

FIG. 88.

In most of the cases cited, the convergence exceeds the accommodation; its *punctum proximum* is nearer than that of accommodation; *p* is to the right of the diagonal. In the observation represented by Fig. 85, on the contrary, *p* is at the left of the diagonal; the accommodation is the stronger of the two, and the convergence is surpassed by it.

After having discussed the general laws governing emmetropia, myopia, hyperopia, accommodation, and convergence, we might hasten to consider an anomaly of the refraction of the eye which, more than any other, deserves the name, since it is due to irregularities of the dioptric system, and is not produced by differences in the length of the eye. This is *astigmatism*.

We think, however, that it will be better not to divide the study of astigmatism and that of the methods of its determination. Either one would thereby lose in interest and clearness. Hence we shall consider astigmatism, altogether, in a separate chapter, and at present discuss the methods of determining the refraction in general.

# CHAPTER III.

## METHODS OF DETERMINATION OF THE REFRACTION AND ACCOMMODATION OF THE EYE.

## DIOPTOMETRY.

### I.—DETERMINATION OF THE STATIC REFRACTION.

We have seen that the refraction of the eye is the inverse of the distance to which it is adapted. If it be adapted to 20 centimetres, *i.e.*, if it see an object distinctly at $\frac{1}{5}$ of a metre, the refraction is $\frac{1}{0{\cdot}20}$ or $\frac{1}{1^{m}/_{5}}$, or, more simply expressed, five dioptries.

When we speak of the refraction of the eye without otherwise defining it, we mean the refraction of the eye *at rest*—the *static refraction*. We know that the eye presents, in this state, its minimum of refractive power. According to what precedes, the *maximum distance*, at which a point can still be seen distinctly, must correspond to the *minimum of refraction*. In short, the eye, when at rest, is adapted to the most distant point that it can distinguish—to its *punctum remotum*.

Hence, to ascertain the static refraction of the eye, we must determine the *punctum remotum*, which is the inverse of what we are seeking. We may then give to the question the following form:—*What methods do we possess by which to determine the situation of the* punctum remotum, *its distance from the eye?*

If a point is seen distinctly, a well-defined image of it must have been formed upon the retina. Now the law of conjugate foci tells us that when an optical system produces, at a distance B, a clear image of an object situated at a distance A, an object placed at the distance B will, inversely, have its image formed at the distance A.

Hence, if the object, placed at the distance of the *punctum remotum*, produces a distinct image upon the retina, objects at the fundus of the eye must, inversely, have their images formed at the distance of the *punctum remotum*.

In order to find the position of the *punctum remotum*, we can therefore proceed in either of *two* ways:—

1. Considering the rays that *enter* the eye, we ask:

*Where must an object be placed in order that a well-defined image of it may be formed upon the retina of the eye at rest*, i.e., *that it may be seen distinctly?*

Or: *What direction must the rays given off from a point have, in order to be focused upon the fundus of the examined eye, without the intervention of accommodation?*

2. Considering the *emergent* rays:

*Where is the image formed, of the fundus of the examined eye, in a state of rest?*

Or: *Toward what point are the rays directed, which come from a point, situated at the fundus of the eye at rest?*

Since the examined eye itself is generally the judge as to the distinctness of the retinal image, we may call *subjective* methods, those methods of dioptometry, which are based upon the *first* principle.

On the contrary, in the case of *emergent* rays, which are generally observed by means of the ophthalmoscope, it is the examiner's eye which judges as to their direction. The *second* principle will therefore furnish us the *objective* methods of dioptometry.

## 1.—SUBJECTIVE METHODS OF DETERMINING THE STATIC REFRACTION OF THE EYE.

### A. *The Method based upon the Acuteness of Vision.*

Before entering upon this subject, let us call attention to a general consideration, viz.: in order to determine the static refraction of the eye, the accommodation must be excluded; otherwise, the dynamic being added to the static refraction, we shall not find the *punctum remotum*, but a nearer point.

The ideal would be, of course, to put the accommodation in a condition of perfect rest while determining the static refraction. This condition is occasionally met with in the case of paralysis of the oculomotorius. But this is rather unusual. A more frequent one is where the accommodation has become very feeble, or even *nil*, on account of age. According to Donders' investigations (p. 175 and Fig. 76), the influence of the ciliary muscle upon the crystalline lens ceases at the age of seventy and, even at the age of fifty-five, it is not strong enough to interfere seriously with a careful examination of the refraction.

But in the majority of cases the accommodation cannot at all be regarded as excluded, and we must try to reduce its influence to its minimum. This is not easily realised, since the accommodation is only exceptionally subject to the will. It requires special training to render one able to relax the ciliary muscle at pleasure.

In subjective dioptometry we secure a certain relaxation of the accommodation best by giving the patient's eyes a parallel direction. The accommodation being, up to a certain point, associated with the convergence, it is nearer a state of rest, when convergence is *nil, i.e.,* when the eyes are directed parallelly, than when they converge. But even then one is often enough deceived, and would make mistakes which might have most annoying consequences if one should blindly conclude, as to the absence of accommodation, from parallelism of the lines of fixation.

To insure a state of rest for the eye, the accommodation must be paralysed by a mydriatic, such as atropine, duboisine or homatropine. But these drugs are not very convenient for use in dioptometry, inasmuch as their effects persist a long time, and patients complain bitterly, and not without reason, if, for the sake of finding a spectacle-glass, they are exposed for several days to the dazzling which results from the dilatation of the pupil, and to the more serious inconvenience of not being able to vary the adaptation of their vision. In certain cases, with which we shall become acquainted, these inconveniences are passed over.

But it must be remarked that none of the substances above mentioned produces total and absolutely certain paralysis of the accommodation in a few minutes. Their action must continue some little time before we can regard the accommodation as entirely paralysed.

Aside from mydriatics, we have no means of entirely excluding the accommodation during a subjective examination of the refraction. But it relaxes during the ophthalmoscopic examination. This permits us to determine the refraction objectively. Hence, whenever it is probable that there is a notable action of accommodation, and when it is not desirable to have recourse to mydriatics, we may check the result of the subjective dioptometry by means of the ophthalmoscope.

Let us now consider the methods which permit us to *determine the situation of the* punctum remotum *by means of the distinctness of the retinal image.*

One might think it possible, *a priori,* to do this by means of a luminous point—a minute hole in a diaphragm, for instance, held between a light and the eye. This point being gradually carried farther from the eye, the *punctum remotum* might be supposed to be situated at the greatest distance at which the minute aperture were seen as a point, and not as a disc—that is to say, in a diffused image. —(See Figs. 53, 54, and 57.)

But this procedure, as will at once be seen, is not practicable. How is one to judge as to the distinctness of the image of a point? It is impossible to tell exactly when it ceases to appear like a very small spot and becomes a distinct point, since, with our always more or less imperfect eyes, we scarcely ever perceive the impression of a mathematical point. Moreover, there would be no way of checking results. Our patients are not always good-natured, and are rarely accurate observers. Some would not, and others could not, describe the impression they receive from a luminous point, and we should be constantly misled.

The type-object must be somewhat complicated, one which can be distinguished in all its details only by means of a well-defined retinal image—a group of points that the patient must count, or a figure or letter that he must distinguish from others. Moreover, the retinal image must be as small as possible. If, then, the patient recognises the object in all its details, we may be sure that the retinal image is distinct, and that the object is at the distance to which the eye is adapted. It is, then, the *acuteness of vision* which will guide us in determining the refraction of the eye.

Acuteness of vision being very different for different eyes, retinal images must be larger for some than for others. We must, therefore, have test-objects, not only of different shapes, but also of different sizes. Hence series of letters and figures of different sizes and forms, black on a white ground, or white on a black ground, are used as test-types for the determination of refraction.

Being in possession of test-types, how shall we proceed, in order to find, with their help, the *punctum remotum* of an eye?

It might be thought that it would be necessary only to take a series of them, and remove it gradually from the eye, until the letters cease to be distinguished. The farthest point, at which the eye could see them distinctly, would be its *punctum remotum*. But this method would be practicable only in the cases in which the *punctum remotum* is at a *finite distance,* that is to say, in *myopia*. Indeed we succeed sometimes, in such cases, in determining the degree of the ametropia, for eyes having a good acuteness of vision, by seeking the greatest distance at which they read very fine print. If this distance be $\frac{1}{3}$ of a metre, the myopia amounts to three dioptries, &c.

But for *emmetropia* this procedure would not be applicable. This is not because we should have to carry the test-types to infinity, where an emmetrope's *punctum remotum* is situated; for we consider in practice that infinity commences at a distance of 5 metres, which is not too great for optometric experiments. But an eye which sees clearly at that distance is not necessarily emmetropic.

It might be hyperopic, and see with the aid of an effort of accommodation. Hence it would be difficult to distinguish, by such a method, between emmetropia and hyperopia, and all the more so to determine the degree of the latter.

Indeed, the *punctum remotum* of hyperopia being situated *beyond infinity*, we would have to carry the test-types beyond infinity, in order to determine this kind of ametropia, and this is, of course, not feasible.

We have seen, however, that a luminous point may be regarded as situated beyond infinity, when the rays emitted by it have been rendered *convergent*. It is in this way that we succeed in diagnosing and measuring *hyperopia*. The test-types are placed at a given distance and the luminous rays emanating from them are rendered convergent by means of convex lenses.

Let us take, for example, a point situated at a great distance. It will emit parallel rays. By interposing a convex lens, the rays will be caused to converge toward the focus of the latter. This convergence will be greater in proportion as the focal distance of the lens is shorter, or, in other words, as the lens is stronger. And upon an eye behind the lens the latter will have the same effect as if we had carried the luminous point beyond infinity. In fact, the expression "a point situated beyond infinity" means simply that the point is formed by the union of convergent rays.

But, just as we can give a convergent direction to rays emitted by a luminous point, we may render them parallel, if they are not so already, or make them diverge. In the first case we cause them to pass through a convex lens, whose focus coincides with their source; in the second case we make use of concave lenses.

If we leave the test-type at such a distance that rays coming from it may be regarded as parallel, and make them pass through a concave lens, we give them the same divergence that they would have if they came from the focus of the concave lens. Suppose the latter to be of two dioptries. Its focal distance is $\frac{1^{\text{m.}}}{2}$, and it will cause parallel rays to diverge as if they came from a point 0·5 metre behind it.

Here, again, the divergence is greater in proportion as the lens is stronger, *i.e.*, as the focal distance of the latter is shorter.

So far, then, as the direction of the rays is concerned, the result is the same whether we place a point 50 centimetres in front of the eye, or, leaving it at infinity, look at it through a concave lens, whose focal distance is 50 centimetres, placed close to the eye.

Hence, what we are forced to do in the case of hyperopia we may also do in that of myopia. Instead of moving the test-types about and seeking the distance at which they are seen most distinctly, we may leave them in one place and vary, by means of lenses, the direction of rays coming from them.

This is what is generally done in practice. The test-types are left at a uniform distance, and the direction of the rays coming from them is changed. We possess innumerable means of doing this, and have the choice of the distance at which we wish to place the test-types.

In the simplest and generally adopted method, which is, as we shall see, at the same time one of the best, the test-types are placed at a distance sufficiently great to be regarded, in practice, as *infinity*.

The adoption of such a distance has, in the first place, the advantage that it disposes the eyes to relax their accommodation, since, in looking at distant objects, the lines of fixation must be parallel and the less convergence there is, the less accommodation is brought into play.

The test-types are a series of black letters, or figures, of different sizes, on a white ground.

This ground should be well lighted up. For this purpose a lamp, with a reflector, brought near the objects, may be employed, or the field may be translucent, and illuminated from behind by a light of unvarying intensity. If the examination be made by daylight, which is much less constant than artificial light, the patient should turn his back to the window, in order not to be dazzled. The card of test-types, on the contrary, should be placed directly opposite the light, and at least five metres away from the patient.

The variation in the direction of the rays coming from the test-types is effected by means of lenses placed before the eye under examination. Such lenses are generally contained in a trial-case, and are in sufficient variety to fulfil all requirements.

During the examination of one eye, the other must be excluded. This is most easily accomplished by means of a bandage or an opaque and non-reflecting diaphragm, placed in a trial-frame in front of the eye. If the patient is allowed to simply close the eye, without its being covered, the examiner will nearly always be misled. The patient will open it more or less, whether voluntarily or involuntarily, especially in cases where the vision of the other eye is not very acute; and, while we are engaged with one eye, the other sees, and so interferes with the examination of its fellow. Or, again, the lids of one eye being perfectly closed, those of the other eye cannot be kept entirely open. The latter then sees through a horizontal slit formed by the edges of the lids. This might mislead the examiner by concealing the presence of astigmatism.

If the patient cover one eye with his hand, there is a risk of his covering it imperfectly, that he will look between his fingers or through a space left free between his hand and his face, or that he may press upon that eye and so affect the sight of it, when we wish to examine it in turn.

*Emmetropia.*—The eye which is not under examination being entirely hidden by, but open behind, the diaphragm, the patient is asked to name the letters or figures (placed at the distance above indicated) that he can see with the free eye, commencing at the largest.

When he reaches the smallest that he can distinguish, and cannot, with however great effort, read further, a weak convex lens (0·5 D, for instance) is placed before the eye. If vision is lessened by this glass, concave glasses are tried, and if these do not enable the patient to see smaller letters than he read at first, we conclude that, weakness of vision being at its maximum without any glass, the *punctum remotum* of the eye is situated at the same distance from it as the test-types, *i.e.*, at infinity; that R[1] is infinite, and that the refraction of the eye, $\frac{1}{R}$, equals $\frac{1}{8} = 0$ – that the eye is *emmetropic.*[2]

*Hyperopia.*—If the eye under examination sees the letters better with a *convex* glass than without a glass, it is evidently *hyperopic.*

We have shown in detail the reasons for this (p. 132 *et seq.*). A person who does not see better, but just as well, at a distance, with a convex glass as with the naked eye, is also hyperopic. The convex glass gives to rays coming from a distance a convergent direction, and the hyperopic eye is the only one that can focus convergent rays upon its retina. One may ask, however, why it sees as well without the convex glass as with it? The answer is easy: the naked eye replaces the action of the correcting-glass by an effort of accommodation, which effort is rendered unnecessary when the eye is furnished with a convex lens. If the accommodation had been paralysed, our patient would not have seen as well without the glass. Perhaps he would have seen none of the letters of the test-types; perhaps only the largest ones—those that he could recognise in spite of the diffuse images of them formed in his retina.

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[1] We designate by R the distance between the *punctum remotum* and the eye. This letter corresponds to the $f'$ in Chapter I.

[2] To be perfectly exact, we ought to say that, the object not being in reality at infinity, but at a distance of 5 metres, R equals 5 metres, and the refraction is + $\frac{1}{5}$ dioptry. Hence an eye which sees better without any glass at this distance, is not perfectly emmetropic; it is not adapted to infinity—that is to say, for parallel rays, but to a finite distance, and for rays diverging from a point 5 metres distant. Hence it is myopic by $\frac{1}{5}$ D. The true emmetrope would be he who, to see clearly at that distance, would need a convex glass, having a focal distance of 5 metres (+ $\frac{1}{5}$ D), which lens would render parallel the rays emanating from an object 5 metres distant. No such glass is found in our trial cases, nor is it known to the trade, for the simple reason that a difference of $\frac{1}{5}$ D in refraction is, practically, almost inappreciable. Vision is not perceptibly changed when the refraction of the eye is increased or diminished by $\frac{1}{5}$ of a dioptry. An eye that sees well at 5 metres sees almost as well at infinity, and *vice versa.* But, in reality, + $\frac{1}{5}$ D should be subtracted from the refraction of an eye adapted to 5 metres (or $\frac{1}{6}$ D for 6 metres) in order to consider it as rigorously emmetropic.

If the accommodation is not paralysed, it nearly always conceals a part, if not all, of the hyperopia. It is therefore advisable to give the patient stronger and stronger convex glasses, and leave them a few moments in front of the eye in order that the ciliary muscle may have time to relax.

When the acuteness of vision, after having attained its maximum, commences to decrease, the entire manifest hyperopia has been corrected; the accommodation cannot be farther relaxed, at least during monocular fixation. So the *strongest convex glass*, with which the patient attains his maximum acuteness of vision, gives the degree of his *manifest* hyperopia.

After having determined the refraction of the other eye and corrected its hyperopia, if there be any, it is well to have the patient look, with both eyes simultaneously, at the distant test-objects. In this way it is often found that the hyperope bears stronger convex glass, *i.e.*, that a greater amount of hyperopia becomes manifest in binocular than in monocular vision.

We have repeatedly spoken of the influence exerted by the relative direction of the eyes (convergence) upon their accommodation. When a person looks at a distant object, one eye being covered, this eye does not necessarily turn toward the same point as the other; that is to say, the lines of fixation are not always parallel. On the contrary, the covered eye sometimes converges. The amount of accommodation, that is naturally associated with such convergence, increases the refraction of the eyes, so that the examined eye may appear less hyperopic, or emmetropic, or even myopic, when it is in reality strongly hyperopic.

A similar construction of the internal recti muscles may increase the apparent degree of a myopia. On the contrary, when the patient is made to use both eyes simultaneously, and especially when their ametropia is corrected and they are thus given their maximum acuteness of vision, their visual lines of fixation are forced to become parallel. Any convergence would produce homonymous diplopia, unless binocular vision had been lost long before.

With parallelism of the lines of fixation, the maximum relaxation of accommodation of which the eyes, in the subjective examination, are capable, without the use of mydriatics, is secured. Hyperopic eyes bear the strongest convex glasses compatible with fixation.

But the degree of hyperopia thus determined, the *manifest hyperopia* ($H^m$) is still very often less than the real degree, the *total hyperopia* ($H^t$). In spite of all precautions, there frequently remains a certain amount of accommodation which conceals part of the refractive defect. This part is called the *latent hyperopia* ($H^l$). It becomes manifest only by the use of mydriatics, or when the refraction is determined with the ophthalmoscope.

It is of the highest practical importance not to forget that

$$H^t = H^m + H^l,$$

*i.e.*, that total hyperopia is made up of a manifest and a latent portion, and that what is found by the subjective examination—the manifest hyperopia—is often only a part of the total.

We shall see, farther on, what relation exists between the parts constituting total hyperopia and the range of accommodation. Let it suffice us, for the present, to bear in mind that *the strongest convex glass which gives the greatest acuteness of vision, for distance, indicates the degree of manifest hyperopia.*

*Myopia.*—When an eye does not see as well, at a distance, with a weak convex glass as without any, we try whether or not a weak *concave* glass increases its acuteness of vision.

It frequently happens that the patient imagines he sees better with this concave glass than without. He cannot, however, make out smaller letters than with the naked eye, yet all of them appear blacker and more distinct. Sometimes he tolerates rather strong concave glasses, without seeing any more than with the naked eye, and yet seems very much pleased with them. Such a person is not myopic. He sees with concave glasses because he exerts an effort of accommodation, and so neutralises the effect of the diverging lens. Here, again, the accommodation makes the refraction of the eye appear stronger than it really is.

In order that the presence of myopia may be regarded as certain, acuteness of vision must be increased by a concave glass; and even then one is often deceived. Just as a spasm of accommodation conceals a part of hyperopia ($H^l$), or even hides all of it, and makes a hyperopic eye appear emmetropic, so an involuntary effort of accommodation often makes myopia seem stronger than it actually is.

Hence we must, first of all, choose, as the expression of the myopia, the *weakest* concave glass that affords the eye its maximum acuteness of vision. This gives the degree of *apparent myopia*. This is generally greater than the *real myopia*, especially in the case of young people who possess strong accommodation. The *real* degree of myopia, like that of hyperopia, is revealed only after forced relaxation of the accommodation, or in the objective (ophthalmoscopic) examination.

It might be supposed that this simple process of determining refraction would, however, take a considerable time; that it would call for an examination prolonged until we had found the glass which, among all those in our trial case, gave the greatest acuteness of vision. Such is not, however, the case, and this examination takes comparatively little time.

We learn to recognise the external signs which characterise the higher degrees at least of ametropia. Hence we shall never commence the examination of a strong myopia with convex or weak concave glasses, but will try strong concave ones from the outset. So, too, for hyperopia.

Again, practice soon enables the examiner to conclude, from the effect of the glasses tried, as to the nature and degree of the ametropia. General reflections, concerning the case in hand, afford us material assistance. If we have before us a young person whose eyes present no apparent anomaly, and the dioptric media are perfectly transparent, but who does not distinguish even the largest characters of the tests, we shall not commence by trying weak glasses. We say: If it be a refractive error that diminishes vision to this extent, it must be one of pretty high degree. If hyperopia be present, a portion of it at least would be corrected by the accommodation. It is unusual to find cases of hyperopia greater in degree than six dioptries, and the amplitude of accommodation continues sufficiently great, until the age of thirty-five years, to correct even the whole of such a hyperopia. After that age it can still correct a good portion of it. On the other hand, a slightly myopic person has generally acquired the faculty of recognising objects which correspond in size to our largest test-types, in spite of the diffuse images of them that are formed on his retina. Hence we try strong concave glasses at the outset, and if the patient does not see much better with them, we may be sure that we have to do with either an affection of the *fundus oculi* or astigmatism.

A multitude of reflections of this sort present themselves to the practitioner, often even when he is unconscious of them. They guide him in the subjective determination of refraction and render it extremely simple.

This method of optometry has, too, besides that of simplicity, other important advantages. Not only do we find, thus, the lens which expresses directly the nature and degree of the ametropia, but, too, the patient is, during the examination, in the same circumstances as those in which he is to use his spectacle-glasses. We have only to give him such glasses as we have selected and placed before his eyes, in order to procure for him the greatest acuteness of distant vision.

Or, if we do not wish to give him the glass, found in this way, which corrects his ametropia, we can watch directly the effect of the glasses that we purpose prescribing for him.

We shall see, in fact, that, in order not to favour or provoke a spasm of accommodation, myopes are never given the glasses that correct the apparent, scarcely such as correct the real myopia.

Another great advantage of this method of determining refraction remains to be pointed out. In order not to complicate the discussion, we have not otherwise defined heretofore the test-objects. We have said that they are objects of different sizes, placed at a distance of at least 5 metres away. Indeed, any series of objects which, in order to be recognised, require distinct retinal images, may serve for the determination of refraction.

But, taking into account the size of the objects and the distance at which they are placed, we may, at the same time, determine not only the refraction, but also the *acuteness of vision* of the eye.[1] The explanation is as follows:—Numerous experiments[2] have taught which is the smallest retinal image that can be distinguished, in its details, by the average normal eye. Starting with this *datum*, Snellen, and others after him, have arranged typographic scales, consisting of letters, numerals, lines or dots of different sizes, and numbers. Each series of these *test-objects* bears a number, which indicates the distance at which they are distinguished by the average eye. The numbers $4^m$, $5^m$, $6^m$, $7^m$, $10^m$, $50^m$, &c., placed above the letters, from the smallest to the largest, signify that they may be removed, the first 4, the second 5, and the last 50 metres, before ceasing to be distinguished by the normal eye. If they be left at a given distance, 5 metres for instance, it is the number 5 that gives the normal acuteness of vision. An eye that distinguishes only the types number 10, which it ought to see at 10 metres, has only $\frac{5}{10}=\frac{1}{2}$ of normal vision. One that sees only the number 50 at 5 metres, possesses $\frac{5}{50}=\frac{1}{10}$ of normal visual acuteness. Another, on the contrary, that distinguishes the number 4, which the average eye is expected to see at 4 metres, has, evidently, an acuteness equal to $\frac{5}{4}$, *i.e.*, above the average.

These tables of type figures, "optotypi," as Snellen calls them, are what we use in optometry.

The glass which enables the eye to see the smallest of these types indicates the refraction; the distance at which they are placed, divided by the number of the types distinguished, gives the *acuteness of vision*. An eye which, with the convex 3, reads type number 6 at 5 metres, has a hyperopia ($H^m$) of 3 D, and an acuteness of vision $V = \frac{5}{6}$.

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[1] Donders, *loc. cit.*, p. 96.

[2] J. Vrœsom de Haan, *Onderzœkingen naar den invlœd van den leeftyd op de gezichtscherpte, &c.*, vol. iii., p. 229, 1862. Compare also Snellen, *Eidoptométrie* in *Traité complet d'ophthalmologie* by de Wecker et Landolt, vol. i., pp. 470-480.

In Monoyer's scale, this division is performed for each series of letters, and for a distance of 5 metres. The sizes of the letters, too, are so chosen that each series corresponds to an acuteness of vision one-tenth greater than that for the preceding series. Hence an eye that distinguishes the third line has a visual acuteness equal to 0·3; the fifth line corresponds to an acuteness of 0·5, the ninth to one of 0·9, and the tenth to one of 1·0, *i.e.*, to the normal.

Excellent and original test-types are those invented by J. Green.[1] What characterises them, besides their careful execution, is that the interval, between each row and the succeeding one, is the same throughout the whole series. It is not, as in Monoyer's tests, the same fraction of 1 (*e.g.*, $\frac{1}{10}$) everywhere. Thus, the proportion between Nos. 0·1 and 0·2 is as 1 : 2, whereas that between 0·9 and 1·0 is only = 9 : 10. But Green's interval is a constant one—$\sqrt[3]{0{\cdot}5} = 0{\cdot}795$. By multiplying the size of the letters of any of his lines by this factor, the size of those in the next line is obtained. So that, for instance, whether the sight increases from the first to the second line, or from the ninth to the tenth, the patient has always gained the same fraction of visual acuteness.

Here an important question suggests itself: the measure of visual acuteness is, as has been said, given by the retinal image, and the visual acuteness of one eye is comparable with that of another only when we know the size of the smallest retinal image that each can still distinguish. If, for example, two persons distinguish the same series of letters, but one with the naked eye and the other with an opera-glass, the visual acuteness of the former is evidently greater than that of the latter, despite the equality of the objects seen, since the former receives smaller retinal images of them than the latter.

We shall be able to draw just conclusions as to acuteness of vision only when we know what change in the size of the image is produced by the glass. Supposing that the enlargement of the retinal image (not of the visual angle) by the glass, in the case just cited, amounts to three diameters, then the second person's acuteness of vision is only one-third of the first's.

Hence test-types can be used as terms of comparison of visual acuteness only when their retinal images are of equal size in the eyes compared. This is the case in typical emmetropic eyes, as in ametropic eyes whose ametropia is of the same kind and degree. The visual acuteness of an axial-myope of 4 D will be to that of another axial-myope whose myopia is of the same degree, in inverse ratio to the size of the test-types that he distinguishes.

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[1] John Green, "On a new Series of Test Letters," &c., *Transactions of the Amer. Ophth. Soc.*, iv. and v., p. 68, 1869.

But will the visual acuteness of an emmetrope, as expressed above, be comparable with that of a myope who needs a concave glass for distant vision, or with that of a hyperope who uses a convex glass for the same distance? The difference in the length of the eyes, or the difference in their dioptric apparatus on the one hand, and the action of correcting-glasses on the other, cannot, evidently, remain without influence upon the size of the retinal images. And if, for instance, the myope, owing to the excessive length of his eye, received larger retinal images than the emmetrope; or if, on the contrary, the size of these images were reduced, by the action of the concave glass, to less than that of the emmetrope's images, the test-types could not serve as terms of comparison of the visual acuteness in the two cases. They would permit a conclusion only as to the *distinctness* of the retinal images, *i.e.*, they would serve in determining the *refraction* of the eye.

Calculations and experiments have proved that *axial-ametropes* receive retinal images equal in size to those of emmetropes, when their eyes are adapted for distant vision by means of glasses placed at their anterior foci.[1]

In the great majority of cases ametropia is due to an abnormal length of the eye; we determine the refraction of the eye when it is adapted for distant vision. The anterior focus is 13 millimetres from the cornea. This is very nearly where we place spectacle-glasses in the method of optometry just described. We find ourselves, therefore, under the most favourable circumstances for obtaining equality of the retinal images, *i.e.*, for determining the visual acuteness at the same time with the refraction.

In curvature-ametropia the correcting-glass must be placed in contact with the cornea, in order that the retinal images may become equal to those of emmetropia.

Let us recapitulate the advantages of this first subjective method of optometry: facility in relaxation of the accommodation, owing to the fact that the examination is made with test-objects situated at a distance; the direct control of the effect of the spectacle-glasses; and the possibility of determining, simultaneously, the refraction and visual acuteness of the eye.

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[1] Giraud-Teulon, *Ann. d'oc.*, Sept.-Oct. 1869, and *La vision, &c.*, p. 198, 1881. Knapp, *The Influence of Spectacles on the Optical Constants and Visual Acuteness of the Eye* (*Arch. of Ophth. and Otol.*, i., 2, 1870). Woinow, *Zur Lehre über den Einfluss von optischen Glæsern auf die Sehschærfe* (*Arch. f. Ophth.*, xix., p. 349, 1872). Mauthner, *Vorlesungen über die opt. Fehler des Auges*, p. 149, 1872, and *Arch. f. Ophth.*, xviii., 2, p. 245. Donders, *Practische opmerkingen over den invlæd van hulplenzen op de Gezichtscherpte.* Landolt, Graefe u. Saemisch, *Handbuch der ges. Augenheilk.*, iii., 1, p. 10, 1874; *Traité complet*, p. 480, and *Onderzœkingen gedaan in het physiolog. Laborat.*, Utrecht, 1872. *Leçons sur le diagnostic des maladies des yeux*, p. 123, Paris, 1877.

If the practitioner has not, for optometry, so great a distance at his disposal, he can nevertheless determine refraction and visual acuteness, provided the correcting-glass be placed at the anterior focus of the eye, and the table of test-types employed be a reduction of the one designed for use at the customary long distance, and calculated for the reduced distance.

Suppose we wish to make these experiments at a distance of only a few centimetres. Placing the test-types at 233 millimetres from the cornea, for instance, only the myope, whose *punctum remotum* is at this distance, will see them distinctly without calling his accommodation into play. The emmetrope will need a convex glass, and if this is at the anterior focus, *i.e.*, 13 millimetres in front of the cornea, it is $233^{\text{mm.}} - 13^{\text{mm.}} = 220^{\text{mm.}} = 22^{\text{cm.}}$ from the test-types. Hence, it must have a focal distance of $\frac{1\text{ m.}}{4{\cdot}5}$, or 4·5 D of refractive power.

Let us leave the test-types and this lens in place. We know that every eye which sees the former distinctly without accommodation, is an emmetropic one. If the addition of spectacle-glasses to this standard lens is required, we apply them to the latter, and they will indicate the nature and degree of the ametropia as well as those found by examination at a greater distance, in the method first discussed.

Or, rather than superimpose two lenses, we may seek the glass which, placed 13 millimetres from the cornea, gives the greatest acuteness of vision, and then subtract 4·5 from its number. Suppose such glass to be the convex 6, giving the greatest acuteness of vision at 22 centimetres. We say: Inasmuch as this eye needs a convex 6 for vision at $\frac{1\text{ m.}}{4{\cdot}5}$, it needs $6 - 4{\cdot}5 = 1{\cdot}5$ with which to see at a distance. This is its correcting-glass, which indicates a hyperopia of 1·5 D.

A hyperope of 0·5 dioptry will require the convex $5\text{ D} = 0{\cdot}5 + 4{\cdot}5\text{ D}$; 0·5 to render himself emmetropic, and 4·5 to make the point of adaptation pass from infinity to 22 centimetres from his anterior focus.

An eye that needs the convex 3 has a myopia of $3 - 4{\cdot}5 = -1{\cdot}5\text{ D}$; that is its correcting-glass for distant vision. A myope of 4 dioptries will require only the convex $4{\cdot}5 - 4 = 0{\cdot}5$ in order to see at our reduced distance, because he already sees near objects without the intervention of a convex lens. As was said above, an eye that sees at this distance, without a glass, will have a myopia of $0 - 4{\cdot}5 = -4{\cdot}5\text{ D}$, and one that even requires a concave glass in order to see so near at hand, will have myopia equal to such glass plus 4·5 dioptries. Thus the concave 2 would indicate a myopia of $-2+(-4{\cdot}5)=-6{\cdot}5$ dioptries.

In this way refraction is found as simply as at the greater distance, and, as has already been said, the visual acuteness may be determined at the same time, since the retinal images of axial-ametropes are, under these conditions, equal in size to those of the emmetrope.

In order that the retinal image of a myope of 4·5 dioptries, or an emmetrope wearing a convex 4·5, may be of the same size as the image received by the myope with his correcting lens or the emmetrope with the naked eye, for distance, the test-type, placed at 233 millimetres, must be 0·32 millimetres in height. For less degrees of acuteness the letters will need to be proportionally larger.

It may be said, as an objection to this method of optometry, that it must render the relaxation of the accommodation more difficult than it is in the examination at the greater distance. The patient has, in fact, during near vision, an involuntary tendency to converge, and, consequently, to bring the ciliary muscle into action, even when one eye is covered and the other adapted, by means of a glass, to the distance of the object fixed. This difficulty may, however, be overcome if the convergence be nullified by means of a stereoscope, which requires parallelism of the visual lines. Javal has made use of this principle in his astigmometer.

The stereoscope is to have the desired length and has no prisms. In front of the eye to be examined, are test-types surrounded by a circle; before the other eye, is a black field surrounded by a circle, either of equal size with, or somewhat larger than, the other. In order that the two circles may be seen united in a single one, or as two concentric circles, the eyes must have a parallel direction. This will dispose them to relax their accommodation, and, in order to see the test-types distinctly, a correcting-glass will be necessary. The latter may be determined for each eye separately, after which we may ascertain if, in binocular vision with the stereoscope, the eyes bear stronger convex or weaker concave glasses.

Besides the little room it requires, this method of optometry would have, over the first, the advantage of facilitating the constancy of illumination. The extremity of the stereoscope bearing the tests could be illuminated by a type-light of unvarying intensity, and the influence of any other luminous source excluded by protecting the patient's eyes by little shades on each side of the eye-piece of the instrument, the examination being made in a dark place.

We have, in the same place, called attention to the fact that, in a similar way (but without a stereoscope), both the amplitude of accommodation and the correcting-glass may be determined for any desired distance. It is only necessary to always subtract the 4·5 dioptries required by the nearness of the test-types.

Thus a practical and exact *optometer* would be obtained. Indeed the name optometer is given to any instrument which serves to determine the refraction of the eye.[1]

### B. *Optometers based upon a single Convex Lens.*

We have said, at the beginning of this chapter, that, besides spectacle-glasses, there are many other methods of changing the direction of luminous rays emanating from an object, and, consequently, of determining the refraction of the eye.

Thus this result may be obtained by means of a *simple convex lens.* If we place an object at the focus of such a lens, the rays coming from it will, after passing through the glass, be parallel. The farther we remove the object from the focus, the more the rays will converge after having passed through the lens.

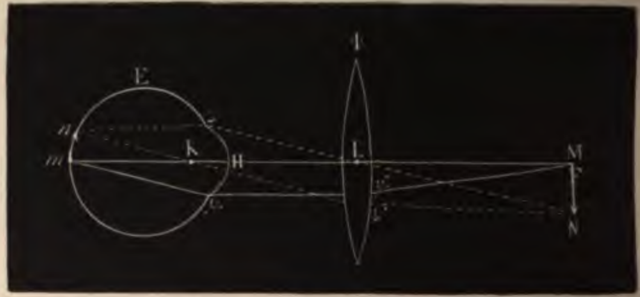

FIG. 89.

Let M (Fig. 89) be the focus of the lens Φ. The rays M L and M $\mu'$, given off from the point M of the object M N, will be parallel (L K and $\mu' \mu$), after having passed through the lens, and the emmetropic eye, E, will unite them, on its retina, at *m*. The same is true of rays coming from the point N; their divergence (N L and N $\nu'$) is changed to parallelism, and their image is formed at *n*.

The farther the object is removed from the focus, the more the rays converge after having passed through the lens. Thus it is that we may obtain the rays that the hyperope requires.

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[1] I have indicated this principle, of optometry at a short distance, in my *Leçons sur le diagnostic des maladies des yeux*, p. 133 *et seq.*, Paris, 1877; and *Manual of Examination of the Eyes*, p. 156 *et seq.*, Philadelphia, 1879.

The *punctum remotum* of the hyperopic eye H (Fig. 90) is at R. On carrying the object M N beyond the focus (F) of the lens, the latter will give, to rays emanating from M, the convergence L K R and $\mu'\mu$ R, and, to rays arising from N, the convergence L $\nu$ X and $\nu'$ K X, so that the former may be focused at *m* and the latter at *n*, on the retina of the hyperopic eye.

If, on the contrary, we bring the object from the focus toward the lens, the rays will become more divergent in proportion as the object is nearer the lens. Thus (Fig. 91), the object being at M N, the rays emanating from the point M will appear to come from R, owing to the divergence ($\mu'\mu$ and L K) left them by the lens.

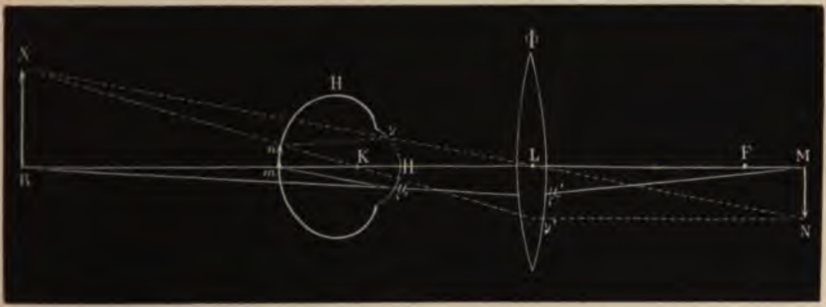

FIG. 90.

The case is the same for rays emanating from N. They will have a divergence (L $\nu$ and $\nu'$ K) as if they came from the point X. A myopic eye, M, whose *punctum remotum* is exactly at R, will therefore be

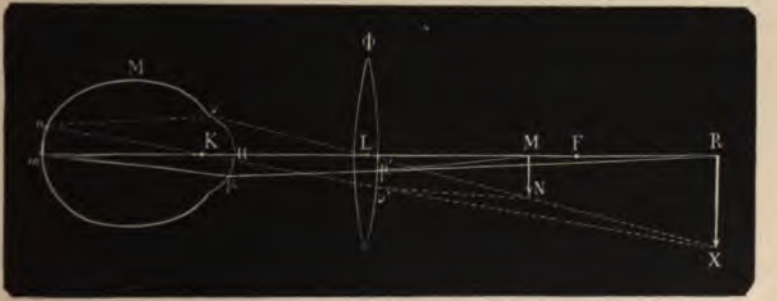

FIG. 91.

able to focus, at *m n*, rays which really come from M N, but the direction of which has been so changed as to adapt it to the refractive condition of this eye.

The single convex lens is, in fact, the simplest means of varying the direction of luminous rays. It has, on this account, been long and frequently employed in optometry.[1]

Coccius,[2] von Hasner,[3] Smee,[4] von Graefe,[5] von Burow,[6] Donders,[7] Laurence,[8] Badal,[9] Burchardt,[10] Sous,[11] Carreras y Arago, Hintzy, and probably others, have made use of this principle.

Some fix the lens on a ruler, upon which are marked metric divisions and along which the test-types are moveable. Others mount it in a tube with or without a stand.

The test-objects consist of threads or lines, or of letters and figures.

A stronger lens permits some authors to give smaller dimensions to their optometers; a weaker lens, on the contrary, increases the excursions of the test-objects, and, consequently, the exactness of the measurements.

All these optometers have, however, the disadvantage that they provoke a certain effort of accommodation, and some of them serve only to determine visual acuteness. According as the convex lens is nearer to, or farther from, the eye, it has a different influence upon the size of retinal images in the different states of refraction. Now, as we have said, an optometer is fitted for the simultaneous determination of refraction and visual acuteness, only on condition that the retinal images, of all the eyes examined, have the same size.

This condition is fulfilled, as we now know, at least for axial ametropia, when the focus of the convex lens is identical with the anterior focus of the eye. If, for instance, the focal distance of the optometer-lens be 100 millimetres, it must be placed 113 millimetres from the cornea, in order that objects seen through this system may furnish retinal images of equal size to the emmetrope and axial-ametrope.

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[1] In Chapter I., pp. 47 and 58, will be found the laws governing this action of the convex lens, as well as the simple formulæ giving the reciprocal distances of the object and its image, and, consequently, the direction of the rays after their passage through the lens. They have been employed in the graduation of all optometers based on the principle of the single convex lens.

[2] Coccius, *Der Augenspiegel*, Leipzig, pp. 148 and 182, 1851.

[3] Von Hasner, *Prager Vierteljahrschrift*, vol. xxxii., p. 166, 1851.

[4] Smee, *The Eye in Health and Disease*, p. 50, 1854.

[5] Von Graefe, *Arch. f. Ophth.*, xi., i., p. 41, 1863.

[6] Von Burow, *Ein neues Optometer*, Berlin, 1863.

[7] Donders, *On the Anomalies of Accommodation and Refraction*, p. 115, 1864.

[8] Laurence, *Optical Defects of the Eye*, London, p. 40, 1865.

[9] Badal, *Ann. d'oculistique*, lxxv. Janv.-Fév. 1876, p. 1, and lxxv., Mars-Avril 1876, p. 1.

[10] Burchardt, "Ueber Bestimmung der Sehweite," *Deutsche med. Wochenschrift*, Nos. 13, 21, 45, 1877.

[11] Sous, *Traité d'optique*, second edition, 1881.

Sous' optometer fulfils this condition; the lens, however, seems a little too strong to give very exact results.[1]

Badal and Burchardt make the focus of the lens correspond to the centre of refraction (nodal point K) of the eye. They obtain, in this way, the equality of the angle under which the test-objects appear to emmetropic and axial-ametropic eyes; but it is easy to see that the same angle determines, upon the retina, an image which is larger in proportion as the retina is farther from the dioptric system—that is to say, as the eye is longer.

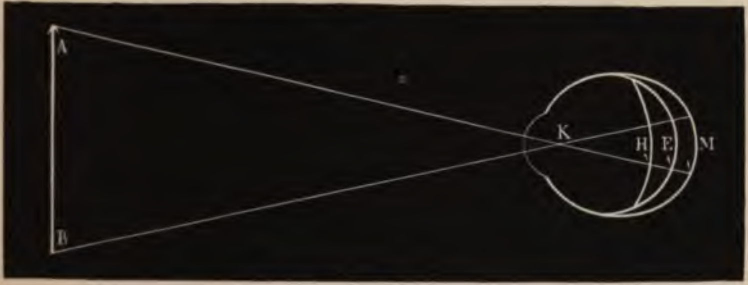

FIG. 92.

The object A B (Fig. 92) is seen by the three eyes, H, E and M, under the same angle, A K B. The retinal images are of unequal size. Hence the longest eye, which receives the largest image, is in more favourable conditions than the shorter eyes, and visual acuteness, whose determination requires equality in size of the retinal images, cannot be found in this way.

Badal's optometer consists of a cylindrical tube about 30 centimetres long, and fixed on a stand, which permits one to regulate the height of the tube, and to give it the desired inclination. A convex lens, having a focal distance of 63 millimetres is placed in the tube at a distance from the eye-hole precisely equal to its focal length. Behind the lens, by means of a rack and pinion, is moved a ground glass plate, on which is, at the left, a photographic reduction of Snellen's metric scale; at the right, pictures of playing cards, for the illiterate, and, between the two, a system of parallel lines with which to measure astigmatism. All of this is seen by transmitted light. The part containing the trial-plate may be removed as easily as a microscope objective, and can be changed with facility. This plate can occupy any position between the lens and the posterior extremity of the tube. According to its position, the refracted luminous rays present, on reaching the eye, all the degrees of convergence or divergence which correspond to the different refractive states that one may have occasion to observe. The graduation of the instrument is marked along the length of the tube in terms of the metric system, and extends from + 15 to − 20 dioptries. For astigmatism, the graduation is made upon the circumference of the posterior opening of the tube.[1]

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[1] Loiseau, *Ann. d'oculistique*, lxxx., p. 26, 1878.

M. Badal writes to the author, that if the eye be withdrawn from his optometer far enough, so that the anterior focus of the eye coincides with the focus of the optometer-lens, the instrument may be used both for optometry and for the determination of visual acuteness. The graduation has the same signification as before. But he prefers to use the optometer as described by him.

### C. *Optometers based upon Galileo's Telescope.*

Galileo's telescope consists of a powerful concave (ocular) and a weaker convex (objective) lens.

By varying the distance between the two lenses, different directions are given to the rays passing through the system. In the first chapter we explained the effect produced upon the direction of luminous rays by such a combination of lenses (p. 67).

If the two glasses are in actual contact with each other, the action of the negative lens, which is the stronger, more than counteracts that of the other; the two together have the effect of a concave lens; rays emanating from a distant object, and being parallel before entering the objective, are divergent after passing through the ocular.

When the convex lens is moved toward the object, the concave one maintaining its position near the eye, the action of the former increases. Hence it destroys, more and more, that of the concave lens. The rays emerging from the telescope are less and less divergent. In one position the lenses perfectly neutralise each other, and the rays are parallel on leaving the system, as they were before entering it. If we continue to move the objective away, the action of the positive glass becomes greater than that of the negative, the emergent rays converge, and that, too, in proportion as the objective is farther from the ocular.

Here, again, we have therefore a very simple means of changing the direction of luminous rays. Every opera-glass may be regarded as an optometer. It is necessary only to graduate the tube of the instrument, and, according as an eye requires more or less shortening or lengthening of the tube in order to see at a distance, its refraction is stronger or weaker.

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[1] *Clinique ophth. du Dr. Badal*, p. 18 : Paris, 1878.

Von Graefe was one of the first to make use of the principle of Galileo's telescope for determining the refraction of the eye.[1]

But his instrument left much to be desired, especially with regard to the size of the retinal images furnished by it. Their size was so variable that the visual acuteness could not be determined, even approximately. Snellen[2] lessened this disadvantage by rendering the enlargement, produced by the glass, as little as possible. The differences in the sizes of retinal images, in eyes of different refractive powers, then become likewise less. To attain this result, he combined two very powerful lenses ( − 40 D and + 20 D) which, with a small excursion, give very different refractive effects. The stronger the lenses in Galileo's telescope are, the less its magnifying power is. In order, so far as possible, to get the patient to relax his accommodation, Snellen recommends that the refraction and visual acuteness be determined for both eyes simultaneously. It is for this reason that he mounted the glasses of his optometer in a spectacle-frame, so that they form, so to say, a double or binocular optometer (Fig. 93).

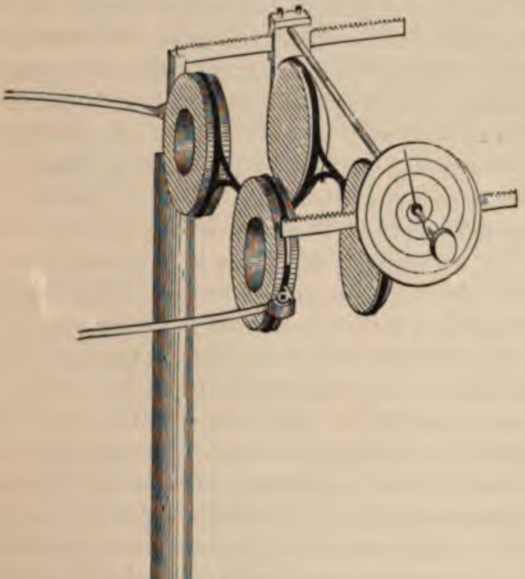

FIG. 93.

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[1] Von Graefe, *Vorzeigung zweckmæssiger Leseglæser und eines binoculæren Refractionsmessers* (*Klin. Monatsbl.*, vol. iii., p. 392); see also De Wecker et Landolt's *Traité complet d'ophthalmologie*, vol. i., p. 682.

[2] *Ibid.*, i., p. 683.

### D. *Optometers based upon the Astronomical Telescope.*

The astronomical telescope has also been used in optometry. It consists essentially of two convex lenses. The objective forms, at its focus, an inverted image of distant objects. This image is observed through the ocular, which magnifies it. The object being very far away, the distance between it and the objective may be considered as constant and infinite. Hence the image remains always at the focus of this lens, and moves with it, when the objective is moved from or toward the ocular. With respect to the ocular, the image plays, therefore, the same part as does an object whose distance is varied. Hence the ocular and this real image from an optometer similar to those, above discussed, which consist of a single convex lens.

If the image is at the focus of the ocular lens, the distance between the lenses being thus equal to the sum of their focal distances, the eye receives parallel rays. If the image lies beyond the focus of the ocular (distance between the lenses greater than the sum of their focal distances), the rays are rendered convergent, and that, too, in proportion as the objective recedes from the ocular. If, on the contrary, the image falls between the ocular and its focus (distance between the lenses less than the sum of their focal distances, but greater than the focal distance of the ocular), the rays emerging from the ocular are divergent. This divergence increases in proportion as the ocular is brought toward the objective, *i.e.*, toward the image produced by it.

This is the principle that Hirschberg[1] has applied in devising an optometer. He combined two convex lenses, the focal distance of one being 40·5 millimetres and that of the other 27 millimetres. These two lenses are mounted in tubes, one of which moves within the other, as in the telescope, and is moved by means of a toothed rack. The author obtains thus a variation of refraction from a myopia of 12·33 to a hyperopia of 12·33, by changing the distance between the lenses from 60·5 millimetres to 82 millimetres—*i.e.*, 21·5 millimetres. This space on the ocular tube is divided into half-millimetres, which permits the determination of refraction to within half a dioptry.

The advantage of the astronomical telescope is that its field is more extended than that of Galileo's telescope. The image furnished by it, being real, can be directly measured, and the focusing controlled by means of a micrometer hair-cross. Moreover, any test-objects may be used in optometry based on this principle, since it is not the object itself which is placed and moved in the tube of the instrument, as is the case with optometers having a single convex lens, but only its image. The object for Hirschberg's optometer may be the real image of ordinary typographic scales, placed at a distance of several metres. We need only take care to hang them upside down, because the image is inverted.

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[1] Hirschberg, *Beitræge zur practischen Augenheilkunde*, i., p. 4, 1877.

The inventor calls attention to another very peculiar advantage of his optometer. On changing ends, so that the objective becomes ocular, and *vice versa*, the instrument remains an optometer, but its optical constants have changed. In order to obtain the same direction of emergent rays, another reciprocal distance must be given to the lenses. Thus, in the preceding example, a myopia of 12·33 D corresponds to a separation, between the two lenses, of 54 millimetres, and a hyperopia of 9·25 to one of 108 millimetres. The emmetrope alone keeps a distance of 67·5 millimetres in both cases.

The instrument has a separate scale for each of the two positions. By making two determinations successively, one by each method, the patient's answers, which ought to agree for both cases, may be compared and checked.

The magnifying power of the instrument varies with the reciprocal distances of the lenses. Accordingly the deviser has taken pains to add to his optometer a reduction-table, which permits the observer to find the real value of the apparent visual acuteness given by his instrument.

All principles of optometry may evidently be used in the construction of *double* optometers. It is only necessary to combine two instruments—one for each eye—and arrange it so that the distance between them may be changed. Hirschberg has put this idea into practice for his optometer, in such a way that it could be applicable to any distance between the eyes.

It seems to us, however, difficult to determine the refraction and visual acuteness of both eyes simultaneously. How are we to know what each eye sees separately, and to what degree the better vision of one comes in to aid the other, whose adaptation is imperfect? The method, according to which parallelism of the visual lines is obtained by means of the stereoscope, is based upon an altogether different principle. In this case the test-objects are presented to the eye under examination only, while the other is directed toward a plain black surface, which has nothing of the typographic scale but the frame. It is the fusion of the outlines of the two fields that maintains the eyes in parallelism, and it is not possible to be deceived as to which eye sees, as one may be with a double optometer.

All single or monocular optometers have the decided disadvantage, frequently mentioned heretofore, of interfering with the result of the examination by provoking an effort of accommodation. The refraction, as determined with these instruments, is almost always much higher than it is in reality.

It is, in fact, a well-known observation that whenever a person not accustomed to experiments (as is the case with the large majority of our patients) looks through an optical instrument, he makes an involuntary effort of accommodation. This is the principal cause of the fatigue produced by the use of opera-glasses, microscopes, and magnifying glasses. There is no kind of work that ought to fatigue the ciliary muscle less than the use of such instruments, since, by their adjustment, they permit of giving to luminous rays any desired direction, especially that which they would have if coming from the *punctum remotum* of the eye. The emmetrope can render the rays coming from his magnifying glass parallel; the myope can cause those coming from infinity to diverge as if they came from his *punctum remotum;* and the hyperope can always obtain the convergent rays required for his state of rest.

All these eyes ought, therefore, to see without any effort of accommodation. Nevertheless, they accommodate, and often very strongly. One proof of this is that persons, not accustomed to the use of optical instruments, *see very small* with these instruments. When we require a pupil in microscopy or ophthalmoscopy—for the dioptric apparatus of the eye is really nothing more or less than a simple microscope—to draw the objects that he sees, giving them the sizes they appear to him to possess, the drawings are generally absurdly small. In my courses of ophthalmoscopy I have seen students draw pictures of the papilla no larger than the head of a pin. This may be explained as follows:[1] the optical instrument (*i.e.*, the examined eye) furnishes the examiner's eye with a retinal image, without indicating the real size of the corresponding object. The size attributed by the observer to the object depends upon the distance to which he refers it, or to which he *projects* his retinal image. The greater such distance is, the larger the object appears to him, and *vice versa.*

This *distance of projection*[1] depends, for the untrained eye, especially upon the state of its accommodation. If it makes an effort of accommodation as if it were fixing an object 15 centimetres distant, the retinal image is projected to 15 centimetres, and the object seems only half as large as if the eye supposed it to be 30 centimetres distant, and adapted itself accordingly. Hence, if an eye sees things very small in a microscope, or with a magnifying glass, or in another eye, we conclude that it is making an exaggerated effort of accommodation.

We have, moreover, the direct proof of the spasm of the ciliary muscle, provoked by the use of optometers, when we compare the refraction, as thus found, with that which we obtain with the ophthalmoscope, or after atropinising the eye. There is nearly always a notable difference between the results of these two methods, and a greater difference than is given by the optometric method in which simple spectacle glasses are used.

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[1] Compare Landolt, *Le grossissement des images ophthalmoscopiques*, Paris, 1874, and "On the Enlargement of Ophthalmoscopic Images," *British Med. Journal*, January 1880.

### E. *Optometers based upon the Measurement of Circles of Diffusion.*

The optometric methods above reviewed are based upon this principle: to seek directly the glass, or optical combination, which renders the ametropic eye emmetropic. This is such a glass as renders distinct the retinal images of objects placed at a distance, or, what amount to the same, objects emitting parallel rays.

Now, from what we have shown in the preceding chapter, concerning ametropia, it results that it ought to be determinable by still other methods. Thus, instead of correcting the ametropia and making the retinal images clear, let us leave them indistinct and determine the diameter of the *circles of diffusion* caused by the ametropia.

We know that the more ametropic an eye is, the larger will be the circle of diffusion formed upon its retina by a luminous point placed at a distance (pp. 121 and 126). If we succeed in measuring the diameter of the circle of diffusion, we shall, therefore, have another method for determining ametropia or the adaptation of the eye generally.

This is possible and has been realised by means of certain apparatus that we may call

### *Optometers based upon Scheiner's Experiments.*

When one looks at a small object placed at a distance for which the eye is not adapted, through minute openings, the distance between which is less than the diameter of the pupil, the object appears multiple, each opening furnishing a separate retinal image. This is known as *Scheiner's experiment.*

The experiment is best performed as follows:—A diaphragm (a visiting card will suffice), through which are pierced several pin-holes, is placed as close as possible to the eye. Take two of these holes, the distance between which is less than the diameter of the pupil, and hold them so that both shall be in front of the pupil. Through these two openings look at the head of a pin placed at a distance for which we have distinct vision,—at reading distance, for example.

The pin will be seen just as it is, only a trifle dimmed, because only a portion of the pupil gives access to the light coming from this object. But if we bring the pin nearer to the eye (or, if myopic, remove it still further) than the distance at which we see distinctly—4 centimetres from the eye, for instance—we see it double. The two images get farther apart in proportion as the object is moved farfher from the point to which the eye is adapted. The phenomenon is easy of explanation.

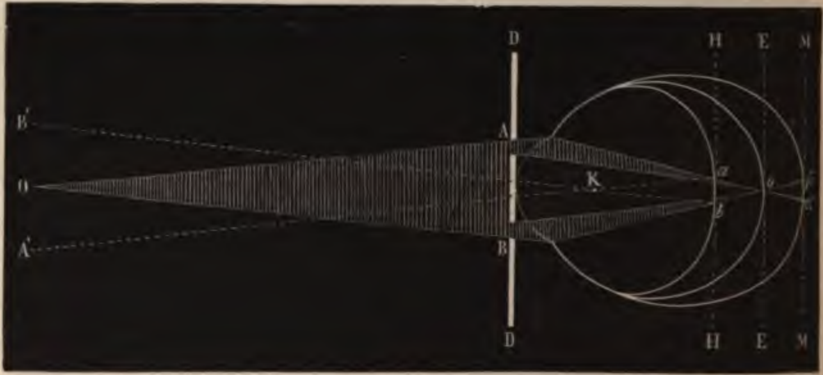

FIG. 94.

Let D D (Fig. 94) be the diaphragm, A and B the openings, and O a point of the object. The latter emits luminous rays in all directions. Without the diaphragm, a luminous cone would have entered the interior of the eye, and have been focused in an image *o*. The diaphragm intercepts the greater part of the light, and permits only rays passing through the openings A and B to enter the eye. Instead of one large luminous cone, whose base would have been the pupillary opening or nearly so, since the meniscus formed by the cornea and the aqueous humor influences the direction of the rays before they reach the pupil, and whose apex would have been the conjugate focus *o*, the eye receives two cones, the bases of which are very small. One of them corresponds to the opening A, the other to the opening B; but the common apex is always at *o*.

If this point *o* be on the retina, *i.e.*, if the eye be adapted to the distance of the point O, it will obtain a distinct image of O, only less luminous, because a smaller number of luminous rays unite to form it than would if the pupil were free.

But if the retina be in front of this point *o*, in the plane H H for instance, the cone corresponding to each opening will form a separate image of diffusion, that for A at *a* and that for B at *b*. Hence the eye will perceive two images of the object O.

These images, although formed by circles of diffusion, are not very indistinct, since the openings to which they are due are so small. We know, indeed, that one sees pretty distinctly through a very small opening, even though the eye is not exactly adapted to the distance of the object,

Hence a person, whose retina is in front of the point of union of rays emitted by the object, will receive two images of it, and, as the figure shows, each image is on the same side with the opening to which it corresponds. If A is the higher, its image is formed on the upper part of the retina, whereas the lower image corresponds to the lower opening. If A is to the right and B to the left, the images *a* and *b* are likewise, respectively, to the right and left.

These images are projected outward *inverted,* as is the case with all retinal images. We receive inverted images, but we project them outward as upright. We obtain the direction, in which the object is seen, by drawing, from different points of the retinal image, straight lines through the nodal point (K) of the eye.

Hence the eye, whose retina occupies the position H H, will project its images *a* and *b* in opposite directions; the upper down and the lower up, the right to the left and the left to the right, as would appear if we should turn the figure so that A were at our right and B at our left. It will project the image *a* at A′ and the image *b* at B′.

Hence we may say that when the retina is *in front* of the focus of rays emitted by the object, the latter is seen with a *monocular diplopia, crossed* relatively to the two stenopaic openings placed before the eye.

The case is reversed when the image of the object is formed *behind* the focus of the optical system of the eye. The luminous cones corresponding to the openings in the diaphragm, after their union at *o*, separate again, and diverge from each other as they continue their course. They meet the retina *beyond* their focus, at $\alpha$ and $\beta$ for instance, if the retina is situated in the plane M M of the figure. In this case the images $\alpha$ and $\beta$ have changed places relatively to the openings to which they correspond. Changed again by their projection, they are finally in the same reciprocal positions as the openings to which they are due.

Hence, if the retina is behind the place where the image of the object is formed, the latter is seen, with a *homonymous monocular diplopia,* through the two stenopaic openings.[1]

It is easy to see that the further the retina is from the focus of the rays emitted by the object, whether in front of or behind it, the wider apart the two images will be from each other.

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[1] The projection of the images $\alpha$ and $\beta$ is not indicated in the figure, because it was thought best not to detract from its clearness.

Hence, the distance of the object, the distance between the openings, and their distance from the eye being given, that between the two images must serve for the determination of the state of adaptation of the eye.

Let us again consider Fig. 94, and suppose the diaphragm to have in it a large circular opening, whose diameter equals A B. This opening would, relatively to the eye, play the part of the pupil, The luminous point O would form, on the retina H H, a circle having a diameter $\beta \alpha$. If we stop up the opening, leaving only the holes A and B, these latter correspond to the two extremities of a diameter of the pupil A B; $a$ and $b$, $\beta$ and $\alpha$, to the two extremities of the diameter of circles of diffusion. Consequently, estimation of the distance between the two retinal images is tantamount to determining the diameter of a circle of diffusion.

This principle served as the basis for several optometers.

The latest and most practical of these is Thomson's.[1] He employs the principle as we have just described it. The diaphragm, placed close to the eye, has two holes or slits in it, each of which is 0·5 millimetres in diameter, they being 4 millimetres apart. One of them is covered by red glass. This makes it easier to ascertain whether the diplopia is crossed or homonymous. The object is a small flame, 5 metres distant. Looking at this through the diaphragm, the *emmetrope* will see it *single*, the *hyperope* will receive two images of it, *crossed* relatively to the openings, while *homonymous diplopia* will indicate *myopia*. The separation of the two images depends, in each case, upon the *degree of ametropia.*

To judge of the distance between the images, Thomson makes use of a second flame, as far from the patient as the first. It is removed from the original flame in a plane perpendicular to the observer's visual line, until one of its images coincides with one of those furnished by the first flame. The person examined, therefore, sees but three images of the two flames under these circumstances. The distance between the flames indicates the distance between the two images.

Knowing (by calculation or by experience) the apparent separation of the two images, for the different degrees of ametropia, we have only to make a table of such distances. The fixed light will be placed at the point *zero*, and the location of the second flame will indicate the degree of ametropia of the eye under examination.

It is well to verify this scale experimentally, *i.e.*, by means of eyes whose refraction has been previously very accurately determined by other subjective and objective methods. The coloured glass placed in front of one of the openings may give rise to errors, if its surfaces are not perfectly plane and parallel, and by the difference in refrangibility between coloured and white light (*vide* p. 249).

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[1] Thomson, W., "An additional Test for the Diagnosis and Correction of the Optical Defects of the Eye," in *Amer. Journ. of Med. Sci.*, pp. 76 and 414, 1870; *Trans. of the Amer. Ophth. Soc.*, p. 93, 1870.

Without having any scale, the refraction may be determined with the aid of this method, by substituting the diaphragm for the mirror in the author's ophthalmoscope. The instrument becomes, in this case, a subjective optometer. The glasses are large enough to cover the largest pupil. We have only to pass these glasses successively before the examined eye, and the one that unites the two images is the correcting-glass for the ametropia, because it renders the eye emmetropic. This method would represent a combination of the first and second principles of optometry.

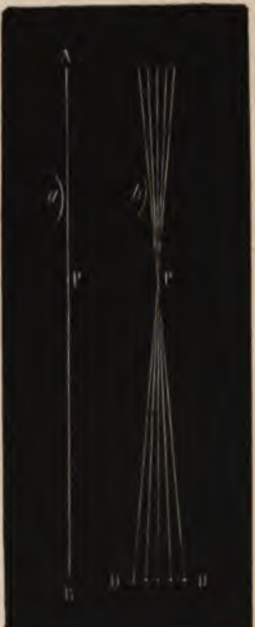

FIG. 95.

The first to use Scheiner's experiment for optometry was Porterfield.[1] His optometer was perfected by Th. Young,[2] and Young's was simplified by Lehôt.[3]

These instruments consist essentially of a small board, on which is drawn a straight line, either white on a black ground or black on a white ground. The board is held horizontally, so that one end of it, corresponding to one end of the line, shall be near the eye to be examined. This eye looks at the line through a row of openings (holes or slits) in a diaphragm held as close as possible to the eye. The line is seen single only at the distance for which the eye is adapted. Everywhere else it is seen multiple, and there appear to be as many lines as there are openings in front of the pupil. These lines diverge more and more on either side of the point for which the eye is adapted, because the circles of diffusion, formed in the eye, increase in size proportionately with the distance of the corresponding objects from the point of adaptation. Thus let A B (Fig. 95, *a*) be the line,

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[1] Porterfield, *On the Eye*, vol. iv., p. 423 (1759). *Edinburgh Medical Essays*, iv., p. 185. Compare De Wecker et Landolt's *Traité complet d'ophthalmologie*, vol. i., p. 676.

[2] Th.. Young, *Phil. Transactions*, p. 34, 1801. Compare, as above, *Traité complet*, vol. i., p. 676.

[3] Lehôt, *Bull. univers. des sciences math.*, p. 417, Nov. 1829. Compare *Traité complet* p. 677.

D D (Fig. 95, *b*) the diaphragm, having four openings in it, and P the point for which the eye is adapted. The line A B will be seen as is shown in the Figure 95, *b*.

The lines seen multiple being very near each other for a short distance on either side the point P, it is difficult to make out the exact spot at which they intersect. This is one great source of error in this method. Moreover, it is applicable, in this form, only to eyes adapted to a not very distant point (the *punctum remotum* of a strong myopia, or a rather near *punctum proximum*). In order to make its employment practicable for all forms of ametropia, it would be necessary to add a strong convex lens to the apparatus.

This is what Stampfer did.[1] He used a tube containing two diaphragms: one of these has in it two slits, each about 0·7 millimetres wide, a little more than a millimetre apart. The tube is furnished with a convex lens of about eight dioptries. The objective diaphragm has also a slit parallel with those in the ocular. It is only one-tenth of a millimetre broad, and is covered by a ground glass. This diaphragm is moved toward or from the eye until the slit is seen single. This is the point for which the eye furnished with the lens is adapted. Subtracting the effect of this lens, we have the adaptation of the eye itself, or rather of that one of its meridians to which the slits correspond. It will be noticed that this method may be

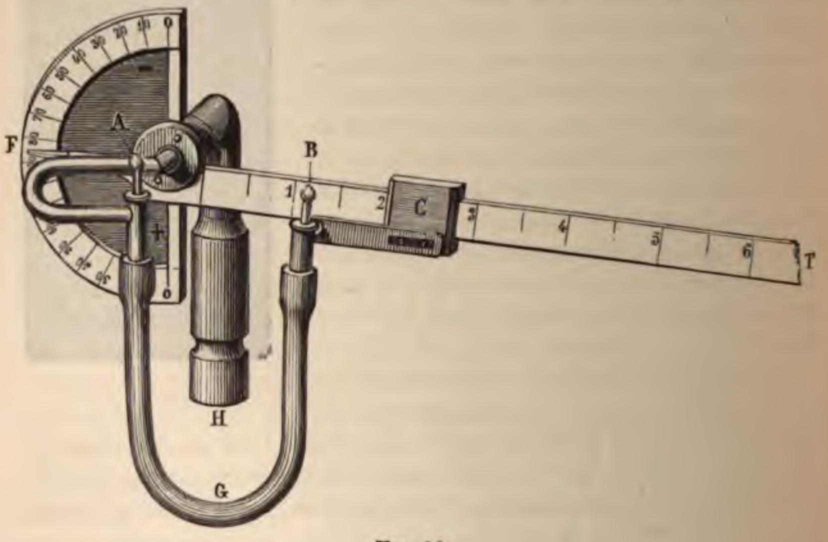

Fig. 96.

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[1] *Vide* De Wecker et Landolt's *Traité complet*, vol. i., p. 676.

advantageously employed for the determination of astigmatism, *i.e.*, for that form of ametropia in which different meridians of the eye have different refracting power.

Dr. Thomson has used still another method of optometry, based upon the measurement of circles of diffusion. His instrument, which he calls an *ametrometer*, is represented by Fig. 96. A and B are small gas flames—the former being stationary, the latter moveable. By moving the slide C along the bar F T, the light B may be brought into actual contact with A or moved 30 centimetres away from it. Moreover, by raising or lowering the end (T) of the bar, B is made to turn about A. The angle, included between the vertical and the line on which the lights are, is given by an index (F) forming one end of the bar, which passes along a graduated semi-circle. The support H is hollow, and may be attached to any gas-pipe.

The apparatus is placed about 5 metres distant from the person to be examined; each of the flames is given a diameter of about 5 millimetres. To an emmetrope they appear like two luminous points. An ametrope, on the contrary, sees them as two discs, whose breadth is proportional to the size of the corresponding circles of diffusion, *i.e.*, to the degree of his ametropia. The examiner now brings the light B toward A until the two discs touch each other. The distance required between the flames, in order that their circles of diffusion may be in contact, is evidently proportional to the diameter of one of these circles—*i.e.*, to the degree of the ametropia.

Hence the bar F T may be so graduated as to indicate directly the degree of ametropia corresponding to each location of B; or a reduction-table may be added to the instrument, which table contains the degree of ametropia corresponding to the centimetres of the graduation. Emmetropia will always be represented by zero, since, in this case, the flames forming no circles of diffusion, they must be brought into actual contact in order that the two images may touch each other.

To ascertain the nature of the ametropia, Dr. Thomson slips a red glass in front of the eye examined, in such a way that half of the pupil becomes gradually covered by it. Then half of each disc of diffusion is coloured red; in *hyperopia* this coloured half is *opposite* the half of the pupil covered by the glass; in *myopia* the coloration appears on *the same* side with the glass. This phenomenon is easily explained, if we bear in mind what has been said (p. 245) about crossed (H) and homonymous (M) monocular diplopia. In the former case the coloured portion of the luminous cone strikes the retina before, in the latter case after, the intersection at the principal focus of the eye, and the retinal image is again inverted when projected outside.

When astigmatism is present, the luminous points A and B appear elongated in the direction of one of the principal meridians. Then B is turned until the two linear images of the flames form portions of the same line. The needle F then indicates the direction of one of the principal meridians of the astigmatic eye. The other meridian is perpendicular to it (L. Purves).

To determine the degree of astigmatism, the experiment is repeated, successively, in the two principal meridians, in the same way as has been described when the apparatus was employed for examining refraction.

### F. *Optometry based upon the Chromatic Aberration of the Eye.*

White light is, as is well known, composed of ethereal vibrations of different wave-lengths. According as these undulations are longer or shorter, they produce in our eyes the impression of different colours. These colours are more or less strongly refracted. This is the reason why a pencil of white light may, by means of a transparent prism, be decomposed into a multicoloured band. If we let a luminous ray enter a dark room, through a round opening, it will form, on the opposite wall, a white, luminous, circular figure; or, if it strikes the wall obliquely, an oval.

If we interpose in the course of this luminous cylinder a strong glass prism with its apex upward, the luminous figure will not only be displaced towards the base of the prism, but the white circle will be transformed into a coloured band,—a *spectrum*. The upper end of this band will be red. In passing toward the other end, we find, in the following order, orange, yellow, green, blue, indigo, and violet, this last terminating the series of colours, or ethereal undulations, perceptible to our visual organ.

The prism has, therefore, deflected toward its base all the colours composing the pencil of white rays, some, however, having been more strongly refracted than others. Thus red is the least deviated from its primitive direction, blue and violet the most. This experiment may be made use of in optometry.

There is a kind of glass which, at first glance, appears dark blue, but which really contains a great deal of red. Let us take a lamp, having an opaque chimney in which is a circular opening about 3 millimetres in diameter. This opening, placed just at the height of the flame, is to be covered by ground glass, and by a piece of the above-mentioned red-blue glass. Thus is formed a luminous point, whose light is composed of red and blue, *i.e.*, of colours far apart from each other in the spectrum.

If rays coming from this point enter the eye, the blue rays, being more strongly reflected than the red, will come to a focus sooner than the latter. The red rays, on the contrary, will be brought to a focus later than the blue ones, while the latter, past their focus, are diverging. Let A B C D (Fig. 97) be the section of a pencil of rays given off from a red-blue point sufficiently distant so that these rays may be regarded as parallel. The focus of the blue is at *b*, that of the red at *r*.

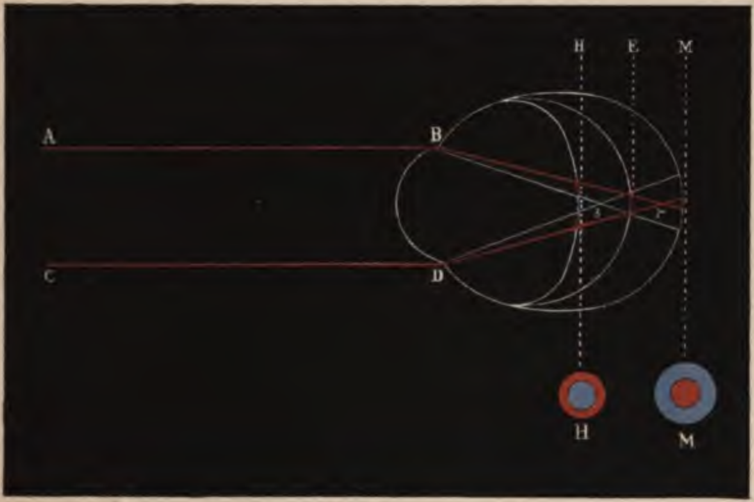

Fig. 97.

An eye is adapted to the distance of the luminous point, when the circle of diffusion, received upon its retina, is at its minimum. This is the case when the sentient layer of the retina lies between the two foci (E, Fig. 97). In this case the point will appear as a small circle, composed of the two colours, that is to say—violet. If the retina be *in front* of this point, at the focus of the *blue* rays, for instance, the eye will perceive a *blue point surrounded by a red circle*, the latter being formed by the periphery of the luminous cone of red rays, which are focused only after having passed the retina. The blue point will become a circle of diffusion, larger in proportion as the retina is nearer the dioptric system, or as the focus for blue is farther behind it. But the blue circle will always be surrounded by a red ring (H, Fig. 97).

If, on the contrary, the retina is *behind the focus for red*, the blue cone will be greater in diameter than the red, and we shall have a *red circle of diffusion*, larger in proportion as the retina is farther from the focus, but always *surrounded by a blue ring* (M, Fig. 97).

If the blue-red point is 5 metres, or more, distant, the *emmetropic* eye will evidently see it more distinctly, *i.e.*, as a small *violet point;*

the *hyperopic* eye, whose retina is situated *in front* of the focus of its dioptric system, will see a *blue circle, surrounded by red;* the *myopic* eye, whose retina is *behind* its focus, will see a *red circle, surrounded by blue.* The size of these circles will be in direct proportion with the amount of ametropia.

To determine this degree of the ametropia, we may use all the above-mentioned optometric methods. The simplest will be to look for the glass in our trial case, which, placed before the eye, will make the point appear uniformly violet, without any margin of another distinct colour. This glass expresses the nature and degree of the ametropia.

In short, the red and blue luminous point may serve as the object in any optometer. It may be useful in doubtful cases, and cases of simulation.

## II.—OBJECTIVE DETERMINATION OF THE STATIC REFRACTION OF THE EYE.

We have said above that, if light *incident* in the eye be used for the *subjective* determination of refraction, the *objective* method makes use of rays *emerging* from the eye examined.

In conformity with the law of conjugate foci, both these kinds of rays follow the same course, but in opposite directions; the *incident* rays come *from* the *punctum remotum,* and go to the retina of the eye, where they are united. The *emergent* rays emanate from the retina, and are directed *toward* the *punctum remotum* of the eye.

Hence we have only to invert what has been described when we were discussing the course of luminous rays in the different subjective methods of optometry, and we shall easily understand the objective methods.

In Figs. 54 to 63 inclusive, we have only to consider N as the object, and $R$ becomes its image. Thus (Fig. 54) parallel rays coming from infinity, *i.e.*, from the *punctum remotum* of the emmetropic eye, are focused at a point $\phi''$ on the retina of this eye. Inversely, if $\phi''$ be a point on the retina of an emmetropic eye, the rays given off from it are parallel after leaving the eye.

The myopic eye focuses in an image (Fig. 58) rays diverging from a luminous point $R$, situated at the distance of its *punctum remotum.* Inversely, a point N on its retina emits rays which converge on leaving the eye, and are focused in a real image $R$, at its *punctum remotum.* The hyperopic eye (Fig. 61) focuses on its retina rays which, before entering it, converge toward its *punctum remotum* $R$, situated behind it. Hence, rays emanating from a point N, on its retina, are *divergent* after leaving the hyperopic eye, and seem to come from a point $R$ situated behind it.

The condition is in no respect altered when the eye is furnished with a lens, a magnifying glass, an opera-glass, a telescope or optometer, &c. If it unites on its retina luminous rays given off from an object, after their passage through whatever optical combination, rays coming from a point on the retina of this eye are focused, by this optical combination, in an image situated where the object was in the former instance. Or, in ordinary terms, if I see an object distinctly, whether with or without an optical instrument, I am sure that the image of my retina will be formed where the object is, provided my retina be illuminated.

It results from this consideration of the course of luminous rays in either direction, that the relations of direction and size are always the same between object and image. I mean that, if the eye receives an inverted retinal image, one-twentieth as large as the object, any part of the *macula* forms, at the distance of the object, an object that is likewise inverted relatively to the direction of the retinal object, and twenty times as large as the latter.

In order to ascertain the course of rays emerging from a retina, the images formed by them must be seen, and, in order to see the images, the object must first be illuminated. The *ophthalmoscope* is made use of to illuminate the retina. Hence the objective determination of refraction is performed with the aid of ophthalmoscopy.

### A. *Determination of Refraction by means of the Erect Image.*

The most accurate and most practical method of objective optometry is in every respect analogous to the first of the subjective methods which we have explained, and to which we have given the preference. The latter consisted, as will be remembered, in finding the glass which permits the eye to focus parallel rays on its retina, which adapts the eye for distant vision, which corrects it or renders it emmetropic.

We *now* seek the glass which renders rays, emanating from the retina, parallel. It will at once be seen that this glass is the same in both cases. This results from the preceding discussions, of which Figs. 59, 62 and 69 furnish a direct explanation. The myopic eye in Fig. 59 needs rays diverging from the point *R*, in order to be able to focus them at N. The concave lens gives to the parallel rays just the desired divergence, and thus adapts the eye to infinity. Inversely, rays given off from the point N, on the retina of this eye, would unite at *R* were it not for the concave lens, which renders them parallel by diminishing their convergence.

For hyperopia the case remains the same (Figs. 62 and 63). In order to focus parallel rays on the retina at N, the convex glass gives them the convergence required by the degree of the hyperopia, *i.e.*, makes them converge toward the *punctum remotum*. This same glass gives a parallel direction to rays emanating from the retina N, and which seem, on leaving the eye, to come from *R*. It is, therefore, the correcting-glass that we are seeking, the one that renders the ametropic eye emmetropic.

How are we to know whether or not rays coming from the eye are parallel? This may be ascertained in a very simple way.

It will be remembered that the emmetropic eye, alone, focuses rays, hitherto parallel, on its retina. Hence the observer has only to render himself emmetropic, if he is not naturally so, by means of his correcting-glass, light up the retina of the eye to be examined, and look into it. If he sees distinctly the retina, the vessels, the optic papilla, &c., the rays emanating from the examined eye are parallel; otherwise he could not have focused them on his retina—could not have seen distinctly. The eye examined is therefore emmetropic. If it is not, the observer sees only a diffuse red light coming from the *fundus oculi*, but no clear detail, because the rays coming from it are not parallel. To make them so, he will interpose, between his own and the examined eye, convex or concave lenses until he finds one that gives him a clear image of the fundus of that eye. This is the glass which renders parallel the rays coming from its retina, the correcting-glass that indicates the nature and degree of the ametropia of the eye examined.

Fig. 98 shows what takes place in such a case. E is the emmetropic examining eye, M the myopic eye examined, and B A an object at the fundus of the latter. Without the lens L L and the examining eye, rays coming from B would have been focused at *R* and those coming from A at A′. These points *R* and A′ are determined by the rays of direction drawn through the nodal point *k*, and the distance of the *punctum remotum* (p. 24).

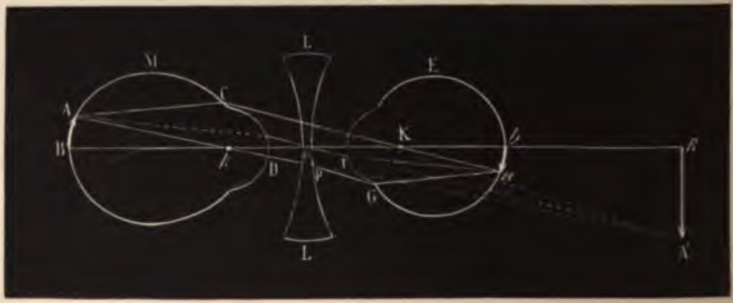

FIG. 98.

The lens L L intercepts these rays, and renders them parallel. In order not to render the figure confusing, we draw only those corresponding to the point A. They strike the eye E as parallel rays. Among them is one, C K, which is directed toward K, the nodal point of E. Hence it is not deviated, and gives, on the retina, the point *a*, at which all rays emanating from A are focused; thus it is with the ray A J, which has passed through the optical centre (O) of the lens without being thereby deviated; and with A D, which, on leaving the eye M, has not been deviated since it passed through the nodal point (*k*) of M, but is deflected by the lens, which renders it parallel (P G) with C K and O J.

The same thing occurs for the point B. Its image is formed at *b*; and, since all the points included between A and B form their images between *a* and *b*, *a b* is the image of A B.

It will be noticed that this image is inverted, relatively to its object. Like all retinal images, it is again inverted by projection. All objects, that we see, form inverted images on our retina, and still we see these objects as erect ones,—that is to say, in their real position. The same is true of the image that we receive, by the aid of the method in question, of objects at the fundus of an eye examined. We project it, outside, upright, and consequently see objects, on the retina that we are observing, in their actual position. This is the reason why this method is called the *examination by the erect image.*

After all that has just been said, it remains for us only to discuss the technique of this method. The principle involved must be thoroughly understood, and is easy to put in practice.

In order that exact results may be obtained, the accommodation of the eye examined, as well as that of the examiner, must, evidently, be in a state of repose.

For the former this desideratum is easily realised. Leaving a prolonged atropinisation out of consideration, the accommodation is never more surely relaxed than when under the influence of an ophthalmoscopic examination, especially when the eye has no object of fixation (Mauthner).

The examiner must also succeed in putting, at will, his accommodation in a condition of repose. This faculty is readily acquired with a little practice. It is important that every ophthalmologist should become master of his own accommodation. The best means of attaining this power is to practise giving a parallel direction to one's eyes. When they have such a direction, the accommodation is relaxed, at least in the cases of the emmetrope and myope. Thus, while looking at a distant object and without changing the direction of the eyes, one interposes in front of them a sheet of white paper on which is a black spot, the latter appears double and indistinct. If the eyes are quite parallel, the distance between the two images is equal to that between the observer's eyes—60 to 64 millimetres. This may easily be ascertained by making the spot the starting-point of a scale, divided into centimetres and millimetres. The practice is repeated until the apparent distance between the two images is equal to 60–64 millimetres. The experiment is most successful when one looks upward; but one soon succeeds in performing it, whatever may be the direction of the eyes or the distance of the object. Little by little the sensation of parallelism of the lines of fixation becomes so familiar, that such parallelism is produced at will. Thus the eyes are directed parallelly, and the accommodation is relaxed, without there being any necessity of previously fixing a distant object.

In this way, one soon acquires skill in determining refraction by examination of the erect image.

For the illumination of the eye, a plane or concave mirror is used, preferably a perforated one, and not one from the centre of which the silvering is simply removed. This opening should be broad. It should be at least 3 millimetres in diameter, else it will act as a stenopaic aperture, which permits distinct vision, even when the eye is not well adapted, because of the consequent diminution of the circles of diffusion. Hence it is not possible to determine the refraction accurately under these circumstances. One must be able to judge, from the distinctness or indistinctness of the ophthalmoscopic image, whether or not the eye is adapted.

As to the correcting-glasses, they are preferably placed, as Helmholtz has shown, behind the opening in the ophthalmoscope. They are held in a disc whose centre of rotation is placed eccentrically with reference to the sight-hole, so that the glasses revolve successively behind the latter. These discs are called, after the inventor, Recoss' discs. The designation of the lenses according to their refractive power has considerably facilitated their combination. Thus, since the introduction of the new system of numbering spectacle-glasses, the construction of Recoss' discs has greatly developed. A modern ophthalmoscope generally contains a sufficient series of correcting-glasses, resulting in part from combinations, for optometric use. It is important, however, to have the glasses large enough; in the first place, so that they may be easy to centre. In small glasses, the axis is often eccentric, and sometimes passes even outside the edge of the glass. Again, the larger glasses are more easily cleaned, which is a point not to be disregarded in optics, and especially in ophthalmoscopy. But the principal reason is still the one urged in favour of the large central opening in the mirror. The glass must be at least as large as this opening, in order that the stempaic effect may not falsify the result of the determination. The best way is to make the ophthalmoscope glasses of greater diameter than the pupil. In this case, the series of lenses may also serve for the subjective determination of refraction. Why, indeed, should not the patient, in the simultaneous investigation of his refraction and visual acuteness, look through the same glass that we use in looking into his eye? In this way the result of the subjective method may be directly controlled by that of the objective method. Hence an ophthalmoscope containing a fairly complete series of sufficiently large glasses may, to a certain degree, replace a trial-case.

The author's ophthalmoscope (Fig. 99) fulfils all these conditions. The lenses are so chosen that, by their combination, all the numbers which are used in ophthalmoscopy may be obtained. They are of 9 millimetres in diameter, therefore easy to clean, and quite large enough for subjective optometry.

Besides the convex lenses 15 and 10 D (X, Fig. 100), necessary for the inverted image, the instrument contains also a concave 10 (V, Fig. 100). The two latter lenses can be added to the series of the Recoss discs for the determination of ametropia, exceeding ten dioptries.

One great advantage of this ophthalmoscope is that the number, resulting from the combination of the two discs, always appears along with the indication of its kind (+ or −), and to the exclusion of all other figures.

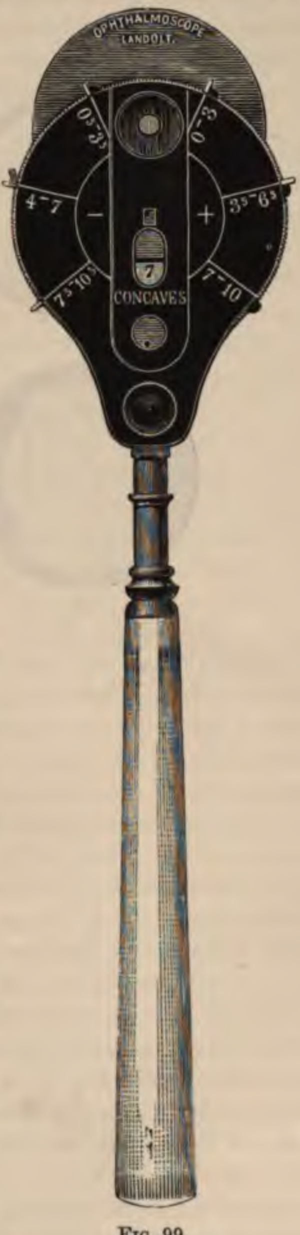

FIG. 99.

The use of the instrument requires therefore neither calculation nor any particular explanation. An astigmatism can also be determined by means of a disc with a slit, which can be rotated in any diameter, the degree of rotation being marked on its circumference (A, Fig. 100).

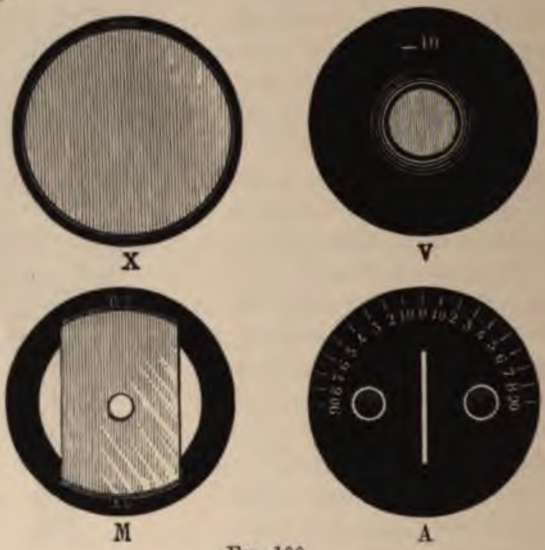

FIG. 100.

The ophthalmoscope may be provided with mirrors parallel to the discs, as well as with a tilting mirror (M, Fig. 100).

The direction of the mirror is indeed of great importance for the examination of the upright image. The inclination of the mirror being necessary in order that the light may be reflected into the eye examined, the correcting-glasses will also be inclined, if they are parallel with the mirror. Their action in the direction of the inclination is then increased relatively to that which they have in the direction of the axis around which the mirror has been revolved. From this results, for an eye looking obliquely through the glass, an effect analogous to that of astigmatism. This difficulty may be avoided by adapting to the ophthalmoscope an oblique mirror, which removes the necessity of inclining the correcting-glasses. Mirrors of this kind have been introduced in practice by Loring, Wadsworth and others.[1]

It is very difficult, under ordinary circumstances, to diagnose degrees of myopia above 8 or 10 D by the erect image. This is due to the diminution of the field of distinct vision, as well as of its illumination, caused by the position of the correcting-glasses beyond the anterior focus of the myopic eye, and by the too great diffusion of the retinal image of the source of light.

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[1] The author's ophthalmoscope is to be had at the opticians, Crétès in Paris, and Meyrowitz in New York.

These two difficulties have been removed in a most advantageous manner by a method due to Dr. Eperon.[1] He interposes, between the eye and the mirror, a divergent meniscus of 13 D, which, as the case may be, converts high myopia into hypermetropia, emmetropia, or low myopia—all of which conditions of refraction are very easily estimated by the erect image.

The employment of a concave glass renders it necessary to use a convergent mirror of very short focus (6 to 5 centimetres), whose surface should be at least 5 millimetres distant from the meniscus.

One important matter to consider is the point of the retina to be chosen in optometry. Different parts of the retina being at different distances from the dioptric apparatus, the latter is not equally adapted to all of them; in other words, the refraction is not the same for all these parts. In practice, the question is nearly always as to what is the refraction corresponding to the *fovea centralis*. Unfortunately this part of the retina is the least suitable for optometric observation. It is not sharply enough outlined so that it can be taken as a criterion of optical adaptation. The light from the mirror falling upon this, the most sensitive portion of the retina, produces a maximum contraction of the pupil; finally, this is just the direction in which reflections from the refractive surfaces (the cornea and surfaces of the crystalline) are most annoying, being formed exactly in the path of the observer's line of vision.

A more appropriate object from which to determine refraction is the papilla, with its clearly defined outlines, sometimes rendered more so by pigment, and, with its vessels, standing out sharply against the lighter background. Moreover, it is not sensitive to light, and the reflections are a little to one side of the observer's visual line, when he is examining the entrance of the optic nerve.

The refraction thus obtained is, however, not always exactly the same as that corresponding to the *fovea centralis*. It is sometimes a little less than the latter, when a notable difference exists between the distances of the papilla and the *fovea centralis* from the dioptric system of the eye, as, for instance, in the case of a deep posterior staphyloma, or a projection of the nerve due to swelling of its tissue, &c. But, in the majority of cases, the refraction corresponding to the external border of the papilla, nearest the macula, does not differ enough from that of the *fovea centralis*, so that we need fear to use this portion of the papilla in optometry.

Another important question is as to where the correcting-glass is to be placed. We know that an infinite number of glasses will correct a given degree of ametropia, according to the distance from the eye, at which the glass is placed. These glasses, each at its proper distance, will give a parallel direction to rays emanating from the eye.

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[1] Eperon, "De la détermination à l'image droite des degrés élevés de miopie" (*Arch. d'opht.*, p. 271, 1884).

In order that the lens resulting from the ophthalmoscopic examination may correspond to that selected by subjective optometry, both must evidently be placed at the same distance from the eye examined. In the subjective examination we placed the lens 13 millimetres from the eye (at its anterior focus). If we bring our ophthalmoscope, with its Recoss disc, to this distance from the eye, the glass obtained by the examination of the erect image will be the one that the patient should wear as spectacle-glass,—in other words, his *correcting-glass.* To deduce from this the distance of the *punctum remotum* from the cornea, or from the first principal point of the eye, the reduction given on pages 127 and 137 may be taken into consideration.

When the observer is not emmetropic, he should render himself so by correcting his own ametropia. Where shall he place his correcting glass? It might be thought that one who wears spectacles or eyeglasses need only keep such correcting-glass while making an ophthalmoscopic examination. But he could not, in this way, get as near the mirror as is desirable, on the one hand, in order to obtain the maximum extent of the field of observation and, again, for the sake of exactness in the determination. The simplest and most practical method is, therefore, to combine, in a single lens, the correcting-glass of the observer's and that of the examined eye. In other words, that glass is sought with which the observer—his accommodation eliminated—sees most distinctly the fundus of the eye examined. From the number of this glass we subtract the number of dioptries necessary for the correction of the examiner's ametropia; the rest belongs to the patient's eye.

It is unnecessary to say that, to be rigorously exact, the distance between this glass and the examining eye should be taken into account. To this end, exactly the same rules as have been given above are to be followed. Indeed, the correcting-glass having parallelised the rays coming from the fundus of the eye examined, the observer finds himself under the same circumstances as when he looks at a distant object. He must give to these parallel rays the same direction as if they came from his *punctum remotum.* Knowing the glass that adapts the examiner's eye to the eye under observation, we shall commence by calculating the part of it belonging to the former and, from the remainder, deduce the exact situation of the *punctum remotum* of the eye examined.

In practice, however, it is not necessary to be so exact. Provided the eyes are deprived of accommodation, and the observer approaches as near as possible to the eye under examination, the distances between the eyes and the correcting-glass may, in the great majority of cases, be disregarded. We need only know the examiner's refraction and subtract the quantity, necessary to correct it, from the glass obtained.

The emmetrope, since he needs no glass to focus parallel rays, finds directly the correcting-glass for the eye whose refraction he wishes to determine.

A hyperope of two dioptries, for instance, will subtract + 2 from the ophthalmoscope-lens. A convex 5 shows him that the patient needs a glass of $5-2=3$ D, *i.e.*, that the latter is hyperopic by three dioptries. From a concave 4 he concludes that the patient is myopic, and that the myopia is one of $-4-2=6$ D.

The case is the same for a myope, only the subtraction of his correcting-glass becomes, in reality, an addition. Thus, a myope of two dioptries, who sees with the convex 5, will know that the patient has a hyperopia of $5-(-2)=5+2=7$ D. If he needs a concave 4 in order to see the fundus of an eye clearly, he concludes that the patient is myopic by $-4-(-2)=-4+2=-2$ D. This patient's myopia must be less than the glass found, since a part of the refractive power of the latter goes to correct the examiner's myopia.

We cannot discuss all the possible cases. The reader will be able to do this for himself after what we have explained.

### B. *Determination of Refraction by means of the Inverted Image.*

Just as the method of optometry by means of the erect image is the analogue of subjective optometry with the correcting-glass, so is that, in which the *inverted image* is used, directly comparable with *optometry by means of a single convex lens.*

It will be remembered that optometers based upon this principle consist essentially of a rather powerful convex lens, placed at a given distance in front of the eye to be examined, and that the refraction of the latter is determined from the distance at which the test-types must be placed, in order to be seen distinctly, *i.e.*, in order that the examined eye may receive well-defined retinal images of them.

We have only to reverse this, and we obtain the objective method of optometry by means of the inverted image. If we illuminate the fundus of the eye, the part of the retina which, in the subjective method, received the retinal image, now becomes the object of which the convex lens forms an image (evidently inverted) at the place where the object was in the former case. One has only to replace the diaphragm that carried the test-types by a ground glass, and the image of the *fundus oculi* will be produced on it.

Just as, in the *subjective* method, the distance of the object from the lens depends upon the refraction of the eye examined, so, in the *objective* method, the distance of the image from the lens is *inversely proportional* to that refraction. The greater the refractive power of the eye, the nearer to the lens is the inverted image produced. The lens focuses the luminous rays which emanate from the eye, and which, without the help of the lens, would be directed toward the *punctum remotum* of the eye. The parallel rays of an emmetrope are united at the focus of the lens; rays from a myopic eye, already convergent before reaching the lens, are focused nearer to it; the divergent rays of the hyperope are united beyond the focus of the lens.

The analogy between these two optometric methods is so close that the figures that have served us when explaining the principle of optometers having a single lens (pp. 234 and 235) are the very ones that I drew, for a previous work,[1] with a view to demonstrate the formation of the inverted opthalmoscopic image.

I reproduce them here. A single glance at them will suffice to show what we have just been discussing. The arrow *m n* represents an object on the retina, Φ the convex lens, M N the inverted image of the object produced by the lens. R X is the image of the retinal object that is formed by the ocular apparatus without the intervention of the lens. It is necessarily at the *punctum remotum* R, and would have actually been formed there in myopia (Fig. 101).

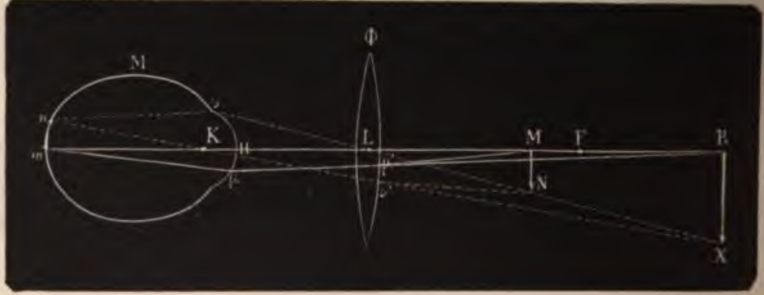

FIG. 101.

In emmetropia (Fig. 102) it is produced at infinity, being composed of parallel rays.

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[1] Landolt, "Ophthalmoscopie" de Wecker et Landolt in *Traité complet d'ophthalmologie*, vol. i., pp. 797 and 798.

In hyperopia (Fig. 103) it is not formed anywhere. It results only from the union (R X) of divergent rays, supposed to be prolonged backward.

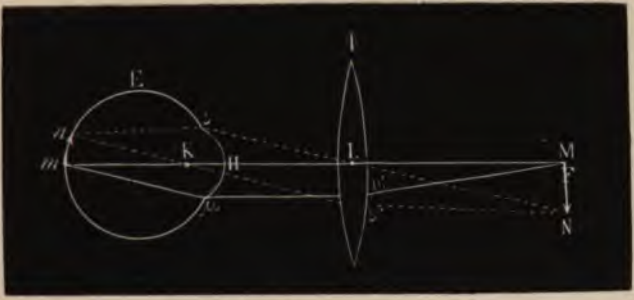

Fig. 102.

The convex lens (Φ) that we use in ophthalmoscopy is strong enough to render rays emanating from the examined eye convergent, whatever their primitive direction may be. Hence it unites them in a real image, which is inverted relatively to the retinal object *m n*. The distance M L between this image and the lens is less (for the same lens) in proportion as the refractive power of the eye is greater.

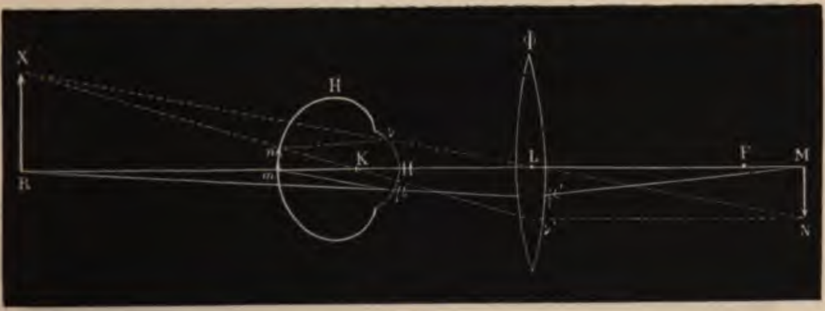

Fig. 103.

If, therefore, we wish to apply this principle to optometry, the very first step will be to determine the *distance* at which the inverted image is formed. This image being real, the first idea that presents itself is to receive it on a screen, which might be inclosed in a tube containing the lens and ophthalmoscopic mirror.

Experiments of this kind have been made for a long time. We may cite, among others, those of Snellen and those made by ourselves.[1] We have even calculated for lenses of 2″, 4″ and 6″ focal-lengths, the distances of the inverted image for different degrees of myopia and hyperopia.[2]

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[1] De Wecker and Landolt, *loc. cit.*, p. 839.

[2] Landolt in Graefe und Saemisch, *Handbuch*, iii., pp. 136-138, 1874.

However demonstrative the reception of the inverted image on a screen may be, we dare not, on account of the difficulties that it presents, recommend this method for the practical determination of refraction. If the mirror be placed at the observer's side behind the screen, the latter receives more light from the mirror than returns to it from the fundus of the examined eye. This interferes with clear perception of the inverted image.

In order to better ascertain its position, we had recourse to a polished glass occupying only a portion of the calibre of the tube, leaving the other free, as Loiseau and Warlomont also did in an instrument to which they gave the name of ophthalmoscoptometer.[1] Moreover, they placed the reflector betwen the lens and the eye to be observed, hoping in this way to avoid the inconveniences of illumination from behind the screen and reflections from the convex lens. But they were obliged to detract considerably from the reflecting power of the mirror, by choosing a transparent one. The illumination of the *fundus oculi* and, consequently, the luminous intensity of the inverted image, suffered from this.

There is another method of using the inverted image for optometry, to the perfecting of which Loiseau has largely contributed, and which seems to us much preferable to the preceding. Schmidt-Rimpler[2] had the ingenious idea of making use, not of the image of objects at the fundus of the eye, but of the one formed there by an object placed at the point, for which the examined eye is adapted by the lens.

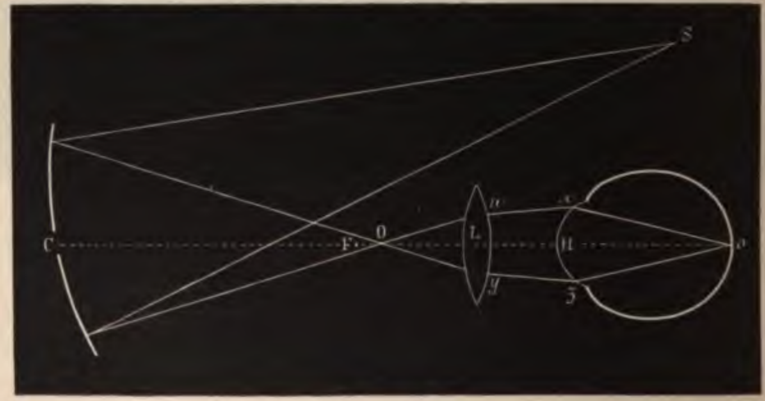

Fig. 104.

If an object *o* forms its image at O, then an object O will form its image at *o*. The object O made use of by Schmidt-Rimpler is simply the real image of the luminous source formed by the ophthalmoscopic mirror.

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[1] Loiseau and Warlomont, *Annales d' oc.*, t. lxxxii., p. 129, 1879.

[2] Schmidt-Rimpler, *Berliner klin. Wochenschrift*, No. 4, 1877.

Let S (Fig. 104) be the source of the light, the mirror C forms a real image of it at O. This image serves to illuminate the fundus of the eye. If the eye be adapted for O, this image in turn forms a clear image at the fundus. The examiner will then see not only the fundus of the eye, but also the image of the luminous source, clearly depicted upon it. Hence he has only to move the mirror toward or away from it, until he sees the image of the lamp distinctly. In this case the image formed by the concave mirror, and the inverted image of the examined eye coincide. This point of coincidence is easy to find directly, by means of a screen, upon which the image of the flame, formed by the mirror, is most distinctly depicted.

The point O being thus known, the refraction of the examined eye might be deducted from it with the aid of a table similar to those that we have calculated for different convex lenses.

The same result is reached, however, in a still simpler way. According to a well-known optical law, viz.:—*When a convex lens is placed at such a distance in front of an eye that its focus coincides with the anterior focus, the nodal point or with the anterior principal point,* in short, *with one of the anterior cardinal points of the eye,* in this case, *similar differences of refraction of the eye* (deprived of accommodation) *require similar displacements of an object in order that it may be seen distinctly.*

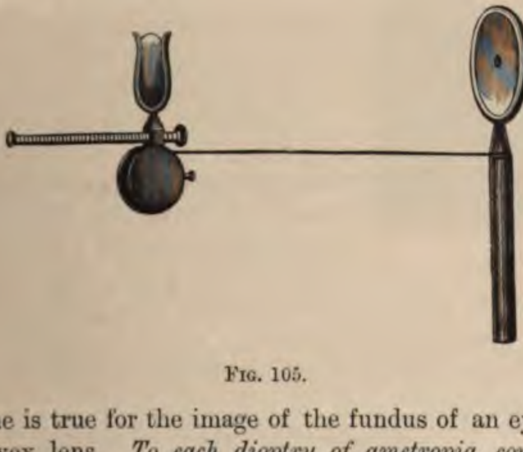

FIG. 105.

The same is true for the image of the fundus of an eye furnished with a convex lens. *To each dioptry of ametropia corresponds the displacement of the point of adaptation of the compound system (eye and lens) by the square of the focal distance of the lens, expressed in terms of the metre.* According as the focus of the latter coincides with one or the other of the cardinal points, the displacement of the clearly seen object, or of the inverted image, gives the degree of the ametropia, reckoned from the cardinal point in question or, what amounts to the same, the number of the correcting-glass placed at this point (Bravais).[1]

With this principle as a basis, Schmidt-Rimpler succeeded in giving his method of determining refraction a very practical form. For the formation of an inverted image he uses a convex lens (L, Fig. 104) of 10 D, the focus of which coincides with the *principal point* (H) of the eye under examination.

If the latter be emmetropic, the inverted image is formed at the other focus (F) of the lens. But should the eye be ametropic, the image will be placed further from the lens (hypermetropia), or nearer to it (myopia). It is evident, from what we have said before, that to each dioptry of ametropia corresponds a change in the position of the the image of $\frac{1}{10} \times \frac{1}{10} = \frac{1\text{ m.}}{100}$, *i.e.*, 1 centimetre, measured from the focus.

In order to be certain of the distance of the lens from the eye, the rim of it is attached to a rod (Fig. 105). If the padded extremity be pressed against the border of the upper jaw, we can ascertain whether or not the focus of the lens coincides with the principal focus of the eye.

For object and source of light (S, Fig. 104), the author uses a figure (Fig. 106) cut in a screen, which is illuminated by a lamp placed behind. With the help of this figure, the adaptation can be very well checked. If the latter is not quite accurate, the sharp outlines of the figure vanish, and the finer bars of the trellis disappear.

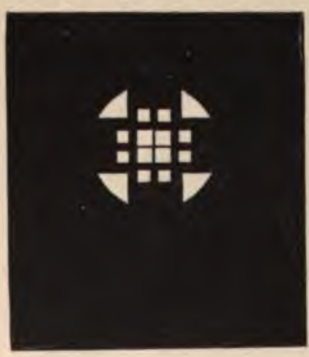

Fig. 106.

The observer, with the aid of a concave mirror (C, Fig. 104) of 15-16 centimetres focus, reflects into the eye under examination the image of this object through the lens L, and gradually approaches the eye of the person, holding the mirror always firmly before his own eye, until he sees with the greatest possible clearness the inverted image of the fundus, and on it the trellis.

In this case, the image of S reflected from the mirror comes to be placed at O, for which point the eye, with the lens L, is adapted, or, in other words, the point where the inverted image of the fundus is formed. In order to ascertain now the refraction of the observed eye, it is necessary only to measure the distance of the image from the lens, or from its focus.

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[1] Nagel, in Graefe und Saemisch, *Handbuch*, vi., p. 319. Loiseau, *Arch. med. belges*, May 1878, and *Ann. d'oc.*, t. lxxx., p. 66, 1878.

For this purpose, a tape which can be rolled up is fastened to the rod bearing the lens, in such a manner that the aperture in its case is placed exactly below the latter. The tape is divided into centimetres. The ophthalmoscope is placed at the zero-point. While the observer rests the other end of the rod on the lower orbital margin, he presses at the same time on the button of the measure. The tape then easily approaches its case by means of the elasticity of the spring which rolls it up. The moment the mirror is in the required position, the observer ceases to press upon the button, and the tape, by means of a special mechanism, remains firmly held in its position.

The distance (C L), of the mirror from the lens, is read off. The distance (O L), of the image from the lens, is then obtained at once, when that of the mirror from the image is known. This is easily found: the end of the rod turned towards the observer terminates in a small metallic plate. Now, without changing the former position, the mirror is approached until the image reflected by it is sharply seen on the plate. Then the motion of the tape is again stopped, and the distance C O read off. The difference between the first (C L) and the second number (C O) evidently gives the required distance O L.

$$OL = CL - CO.$$

If the latter equals 10 centimetres, the observed eye is emmetropic; if the distance is greater than 10 centimetres, the eye is hypermetropic. For example, 13 centimetres correspond to a hyperopia of 3 D. If C O is less than 10 centimetres, we have to do with myopia; 7 centimetres indicate, for instance, a myopia of 3 D.

This principle of determination of refraction by means of the inverted image is certainly calculated to supply some deficiencies in the method based upon the examination of the erect image. It requires only a certain amount of practice, for which the observer is abundantly compensated by the advantages that the method affords.

### C. *Pupilloscopy or Koroscopy* (Κόρη, *the Pupil*).

We have yet to speak of an objective method of determining refraction that has, of late, received considerable attention. We shall designate it *Pupilloscopy*, or more grammatically *Koroscopy*, for reasons which will be easily understood.

This method may be described as follows:—

The person to be examined is seated in a dark room, as for ophthalmoscopic examination. A lamp is placed at his side, or above the head, a little behind the plane of his face, and a screen protects the latter from the direct light of the flame.

The observer, with an ophthalmoscope mirror (let us speak, at first, of a concave one), places himself about a metre from the patient. If, now, the light be thrown at first at one side of the eye, and then, by a slow rotation of the mirror around its vertical axis, the light be made to pass gradually across the pupil, the latter is seen to be lighted up, sometimes first on the side from which the light comes, sometimes on the opposite side, while, at other times, all of it is equally illuminated from the outset. In the first case the pupillary lustre moves in the same direction as the mirror. Thus, when, by the continued rotation of the latter, the reflection commences to leave the eye, the brightness of the pupil seems to pass off in the same direction as the reflection thrown on the patient's face, and the shadow that follows it commences at the opposite side to cover the pupil.

At other times, on the contrary, the light coming from the fundus of the eye moves in a direction opposite to that of the motion of the mirror. If we make the reflection pass from right to left (as seen by the patient), the right side of the pupil is first illuminated, then the whole of it. Finally, the light covering only the right portion, it is the left side of the pupil that is illuminated and toward which the lustre seems to pass, disappearing when the light ceases to enter the pupil.

In still other cases the pupil seems, as we have said, to be lighted up everywhere simultaneously, so that it is difficult to judge whether its lustre follows the movement of the mirror or not.

This variety in the appearance of the pupil, under the influence of ophthalmoscopic illumination, depends principally upon the refractive condition of the eye, and may serve for its determination. But to this end it is necessary that we know exactly how this phenomenon is produced.

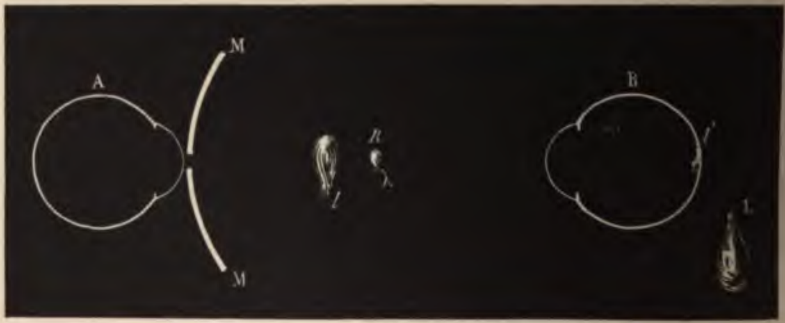

Fig. 107.

Let A (Fig. 107) be the examining eye, B the eye under examination, L the light and M M the concave mirror. Let us suppose this mirror to have a focal distance of 20 centimetres. Being nearly 150 centimetres from the lamp, it produces a real inverted image of its flame at a distance of 23 centimetres, at *l*. This image evidently moves in the same direction as the mirror. If the latter rotates from right to left about its vertical axis, the image *l* passes from the right toward the left; if the former is inclined from above downward, the image descends, and *vice versa*.

This real image *l* plays, so far as the examined eye is concerned, exactly the same part as a luminous object placed 100 − 23 = 77 centimetres from it, if the mirror is 1 metre from that eye.

Hence the observed eye B receives a retinal image *l′* from the flame *l* and, whatever be its refractive condition, the image is always inverted relatively to the object; but it will be more or less distinct according as the eye is better or worse adapted to the distance of the object. Thus, in our example, the image of the flame will be clearly depicted upon the fundus of the examined eye, only when the latter is adapted to 77 centimetres, *i.e.*, if it has a myopia of 1·3 D or makes an effort of accommodation corresponding to this distance.

If the eye be less myopic or emmetropic, or hyperopic, rays emanating from each point of the object will form a circle of diffusion on the retina before coming to a focus, and the image will be diffuse, as well as inverted relatively to the object.

A single glance at Figure 108 will make this clear.

The dioptric system of the eye would unite rays emanating from the point A at the point *α*, situated on the line A K *α*, *i.e.*, on the ray

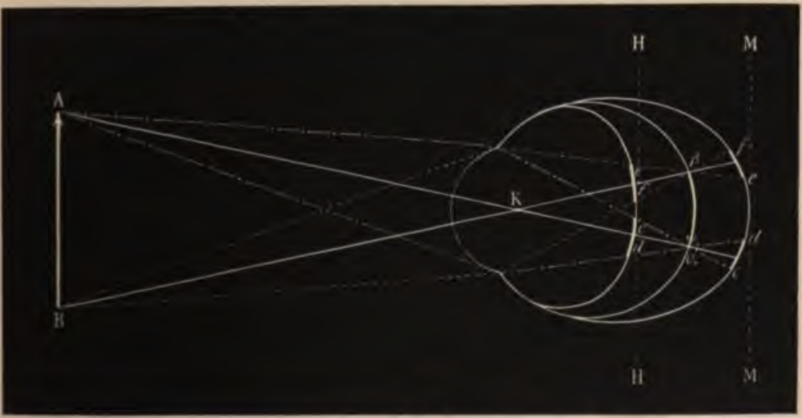

FIG. 108.

which passes through the system without being deviated, because it passes through the nodal point K. After being focused the rays would continue their course, now diverging, toward *d c*. The same would be true for the rays given off from B. They would form their image at $\beta$ on the line B K $\beta$, and thereafter diverge toward *fe*. Hence $\alpha\beta$ is the image of A B, formed by the dioptric media of the eye.

Therefore, if the retina be situated at this place, it receives a distinct and inverted image of the object.

If it be in front of $\alpha\beta$, at H H for example, rays emanating from each point of the object will form circles of diffusion upon it, before having reached their focus. Thus, rays coming from the point A form the circle *cd*, rays coming from B the circle *ef*, and those coming from points between A and B form their circles of diffusion between *d* and *e*. Hence the image is blurred and indistinct, but it is still inverted relatively to the object.

If the retina is beyond $\alpha\beta$, at M M for instance, rays coming from A form the circle of diffusion *dc*, those from B the circle *fe*. Rays corresponding to the same point of the object have exchanged places relatively to their disposition in the pencil before being focused, but the diffuse image is, nevertheless, still inverted, the image of A being below and that of B above.

Hence it is understood and proved that, whether or not the eye be adapted to the object, and whether the retina intercepts the rays in front of or behind their focus, the retinal image, diffuse or distinct, is always inverted.

If the image is inverted, its movements are also inverted relatively to those made by the object. In the case under consideration, this signifies that if, by rotating the mirror from right to left, we make the real image $l$ (Fig. 107), which is the object for the examined eye, pass from right to left, the retinal image $l'$ is displaced from left to right. In short, whatever be the refraction of the examined eye, the image of the flame, that lights up the fundus of this eye, always moves in a direction opposite to that of the motion of the concave ophthalmoscope mirror.

But the illuminated portion of the fundus of the eye, B, becomes in turn the object for the observer's eye, A. If the eye under examination is emmetropic or hyperopic, the rays emanating from it are focused nowhere, unless it be upon the retina of the examining eye. The latter, therefore, sees what occurs at the fundus of the examined eye just as it actually occurs. To use a common expression, the observer sees the fundus of the eye in the *erect image*. The upper part of an object in it appears to be above, the lower part below, and, if this object moves from left to right, on the fundus of the examined eye, the observer sees it move in the same direction, &c. This is exactly what happens in the pupilloscopic examination of an *emmetropic* or *hyperopic* eye: the ocular lustre moves in the same direction with the image of the flame at the fundus of the eye, *i.e., opposite to that of the motion of the mirror.*

But the case is not the same in myopia. Every myopic eye forms at its *punctum remotum* an inverted image of its fundus. Let us suppose the eye B (Fig. 107) to be myopic, and *R* its *punctum remotum.* If we light up this eye with the opthalmoscope, we shall see appear, in the plane of *R*, an inverted image of everything at the fundus of the eye. Among these objects must be counted the image ($l'$) of the flame. Already inverted relatively to its object $l$, it is again inverted by the dioptric media of the eye. Its image $\lambda$ has, therefore, the same direction as $l$. Hence it must also execute the same movements. If $l$ moves with the mirror from right to left, and $l'$ from left to right, then $\lambda$ goes from right to left.

Evidently $l'$ is nothing else than the illuminated portion of the *fundus oculi* that the observer sees in the pupil of the examined eye. The pupil is seen in its real position, while the *fundus oculi* appears inverted. If we make the luminous disc, which illuminates the eye, pass from right to left, the pupil is likewise lighted up from right to left, though the light really passes from left to right at the fundus of the eye examined. Hence it is that, in myopia, the ocular lustre moves in the same direction with the concave ophthalmoscopic mirror.

It will, however, at once be seen that there must be certain restrictions in all this. The observer A (Fig. 107) can see the inverted image of B only when it is at a certain distance in front of him.

If he approaches the eye examined near enough to intercept the rays emanating from it, before they have reached their focus, as happens where A is between *R* and B, the inverted image is not formed in the air in front of the observer, but only in his eye, as in emmetropia and hyperopia, though with less distinctness. This is what happens if the myopia of the examined eye is of low degree, and its *punctum remotum* very distant—beyond the examiner. In this case the pupilloscopic phenomenon is the same in myopia as in emmetropia and hyperopia. Hence it is impossible to diagnose the nature of ametropia at a glance, and from simple pupilloscopic inspection.

If the observer is 1 metre from the patient, the *punctum remotum* of the examined eye must be less than a metre from the latter, in order that the observer may see the inverted image. In other words, in order that the myopia may reveal itself, by the distinctive signs that we have explained, for a distance of 1 metre, it must be of more than a dioptry. Hence it is well, in simply diagnosing the refractive condition of an eye, to station one's self, with the mirror, more than a metre away from it.

This distance between the patient and the examiner also has, however, its disadvantages. The degree of the ametropia may indeed be determined by means of spectacle-glasses placed before the eye to be examined. If the examiner is sufficiently near the patient to be able to hold the glass in his hand, this determination is easily enough achieved. It is more complicated when the distance is greater; in this case the glass must be put in a spectacle frame, resting on the patient's nose, and every time the glass is changed, the examiner is obliged to approach the patient and then return to his point of observation. To avoid this inconvenience, Charnley uses a sort of long forceps with which to hold the glasses.

Between the ametropia for which the pupillary lustre moves in the same direction as the mirror, and that for which it moves manifestly in the opposite direction, there must be intermediate refractive conditions in which these movements become more and more indistinct, and, finally, a determinate degree for which they are imperceptible. This is the case when the *punctum remotum* of the examined eye is near the eye of the observer.

It is by taking this limit, where the movement of the lustre changes direction, as a basis, that we may seek to determine the *degree* of the ametropia.

We know that, if the observer is a little more than a metre distant from the patient, and the lustre of the pupil does not move, but the entire pupil is lighted up at once, we have to do with a myopia of one dioptry. If it be necessary to place a convex glass of one dioptry in front of the examined eye in order to produce this phenomenon, then the eye is emmetropic; if convex 3 D is called for, there is a hyperopia of $3-1=2$ D; if the concave 3 D be necessary, the examined eye has a myopia of $3+1=4$ D. In a word, under the above-mentioned circumstances, one positive dioptry must always be subtracted from the glass, which causes the movements of the light to disappear from the pupil examined, in order to obtain the refraction of that eye. It need scarcely be said that to subtract one positive dioptry from a negative glass, amounts to the same thing as adding a negative dioptry to it, as we saw in the preceding example.

The movement of the image of the flame observed in koroscopy, is of greater range in proportion as the degree of ametropia is less. The intensity of the lustre is inversely proportional to the degree of the ametropia.[1]

The phenomena of pupilloscopy that we have described all refer to the examination with the *concave* mirror, which forms an image of the luminous source in front of the observed eye. They are different for the *plane* mirror. In this case the lustre moves with the mirror in hyperopia, in the opposite direction in myopia, and so forth, always the inverse of the case with the concave mirror.

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[1] Charnley, *On the Theory of the so-called Keratoscopy* (*Ophth. Hosp. Rep.*, vol. x., p. 357).

This is easily understood. The concave mirror produces a *real* image of the flame in front of it—between it and the patient. This image, which is the object for the observed eye, moves in the *same direction* as the reflecting surface of the mirror. The image from the *plane* mirror, on the contrary, is a *virtual* one, situated *behind* the mirror, and moves in the direction opposite to that of the surface turned toward the person being examined.

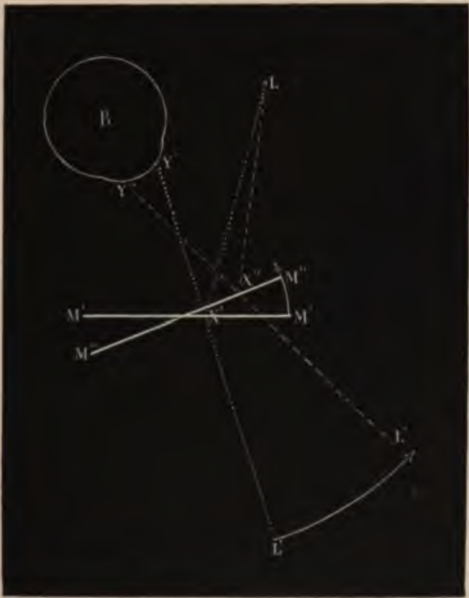

Fig. 109.

Thus, let L (Fig. 109) be the luminous source and B the eye under observation. When the plane mirror has the position M′ M′, it reflects the light along L X′ Y′ and L appears to the eye B as if situated at L′. When, by the rotation shown by the arrow, the mirror is given the direction M″ M″, it reflects the light along L X″ Y″. Hence L appears to be at L″, while, by the rotation of the reflecting surface from right to left, the reflection has also been carried from Y′ to Y″ (which would likewise have been the case with the real image from the concave mirror). The luminous source has, on the contrary, moved from left to right, from L′ to L″. It is this light L′ or L″ that illuminates the fundus of the eye B, of which the latter receives an inverted image, and whose movements, consequently, are reproduced also in a direction the inverse of that of the real movements made by the reflex of L.

When the refraction of the eye varies in different meridians, as in astigmatism, the koroscopic phenomena also vary. It may then happen that the lustre from the fundus of the eye moves in the same direction with the concave mirror in one of the meridians and in the opposite direction in another. This is the case when the former is myopic and the latter hyperopic.

We shall speak of the employment of pupilloscopy in the determination of astigmatism, at the end of the chapter devoted to that form of ametropia.

It is interesting to note that, as long ago as in 1859, Bowman[1] made use of this method for diagnosing commencing keratoconus and astigmatism.[2]

But it remained almost unknown, and employed only by the author, and it may be said that the method of determining refraction, which we have just described under the name of koroscopy, is due to Cuignet[3] (of Lille), who really introduced it into practice, and without being acquainted with Bowman's observation. It is only to be regretted that Cuignet did not better understand the principle involved in this method. Attributing to the cornea the phenomena due to the *fundus oculi*, he called the method *keratoscopy*.

Neither the name, nor the numerous descriptions destined to explain it, and in which the "corneal reflex," the "illuminated side and opposite or shaded side of the cornea," and "phenomena inscribed upon the surface of the cornea," &c. &c., were discussed, were calculated to recommend the method to ophthalmologists. So it remained for a long time confined to the circle of its inventor and his pupils.[4]

It was one of the latter, M. Mengin, who has contributed largely to the propagation of this method, who showed it to us at our clinic.

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[1] Bowman, *Contribution to the General History of Conical Cornea* (*Ophth. Hosp. Reports*, 1859-60, p. 157).

[2] Donders, *Anomalies, &c.*, p. 490 ; and Nagel, *Refractionsanomalien*, 1864, p. 127.

[3] Cuignet, *Kératoscopie* (*Recueil d'ophth.*, 1873, p. 14) ; *Kératoscopie par réflexion* (*Recueil d'ophth.*, 1874, p. 316) ; *Kératoscopie* (*Recueil d'ophth.*, 1877, p. 59) ; *A propos de kératoscopie* (*Recueil, d'ophth.*, June 1880).

[4] One of the most enthusiastic keratoscopists pretended even to determine the refraction with the aid of "the shadow which the cornea projects upon the pupil." (!)

It seems in general that, up to the present day, the shadow plays the principal part for those who avail themselves of this kind of optometry. In their descriptions it is always the appearance, the direction or the movement of the shadow, which comes into question.

I must confess that I never could well understand this way of considering the matter. Certainly every light involves a shadow, but, since I use the ophthalmoscope to throw light in the eye, and not shadow, I find it more natural to fix my attention upon the former than upon the latter.

We gave the first explanation of it, which was reproduced in part, and under our name, by Mengin, in one of his publications.[1]

Parent, who has studied this method with special care, has discussed it in all its details, and has considerably enhanced its value by making use of correcting-glasses, placed in front of the patient's eye. In this way he succeeds in determining the *degree* of the ametropia, which is not possible from simple observation of the reflex.[2] Charnley[3] also gives a perfectly proper explanation of the method.

## II.—DETERMINATION OF THE DYNAMIC REFRACTION OR OF THE AMPLITUDE OF ACCOMMODATION.

The amplitude of accommodation is, as we have seen, the increase in refractive power that the eye can assume by means of the contraction of its ciliary muscle, by its dynamic refraction. In order, therefore, to obtain the amplitude of accommodation, we must first know the refraction of the eye when it is at rest ($r$), and then determine that which it attains under the influence of the maximum curvature of its crystalline lens ($p$). The difference between the two is the amplitude of accommodation ($a$):

$$a = p - r.$$

(See p. 169.)

In the preceding chapters we have given the methods used for determining the static refraction ($r$) of the eye, *i.e.*, its *minimum* refraction. We found that it was given by the distance R of the *punctum remotum*, of which it is the inverse: $r = \frac{1}{R}$.

It remains for us to explain how we succeed in determining the *maximum* refraction of the eye, or that which it presents when adapted to the *nearest* point (*punctum proximum*) that it can clearly distinguish.

This maximum $p$ is, as we know, the inverse of the distance P of the *punctum proximum*: $p = \frac{1}{P}$.

Just as is the case for static refraction, we have two principles, which are really identical for determining dynamic refraction; we either seek the situation of the *punctum proximum*, measure its distance from the eye, and take the inverse of this, or we seek directly the glass whose refractive power equals that of the eye at its maximum of accommodation.

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[1] Mengin, *De la kératoscopie* (*Recueil d'ophth.*, April 1878).

[2] Parent, *De la kératoscopie ; Pratique et théorie* (*Recueil d'ophth.*, Feb. 1880, p. 65). *Kératoscopie* (A M. le professeur Cuignet) (*Recueil d'ophth.*, July 1880, p. 424). *Comment sont réfractés les rayons tombant obliquement sur l'œil* (*Recueil d'ophth.*, April 1882, p. 216). *Diagnostic et détermination de l'astigmatisme* (Paris, 1881, p. 15).

[3] W. Charnley, *loc. cit.* Compare also Forbes (Litton), *On Keratoscopy* (*Oph. Hosp. Rep.*, vol. x., part i.). Loiseau, *Application à l'examen des hommes de guerre du procédé de détermination de la réfraction dit kératoscopie* (*Arch. méd. belges*, July 1882). Morton, *Refraction of the Eye, &c.*, London, 1881, p. 27. Chibret, *Détermination quantitative de la myopie par la kératoscopie* (*fantoscopie rétinienne*) *à l'aide d'un simple miroir plan* (*Ann. d'oc.*, Nov. 1882).

To obtain the *punctum proximum* we use small printed characters, or groups of black dots on a white ground, like those in Burckhardt's optometric scale, or black threads on a white ground stretched across a frame, or small holes in a diaphragm.

The eye not under examination being covered, these objects are brought nearer and nearer, until they commence to get indistinct. They are then at the *punctum proximum* of the eye. We measure the distance of this point from the cornea (or from a point 2·7 millimetres behind the cornea, the anterior principal point). Let us call this distance P.

If it equals $\frac{1^{\text{m.}}}{12}$ the maximum refraction of the eye is $\frac{1}{{}^{1}/_{12}} = 12$ D.

If it equals $0{\cdot}20^{\text{m.}}$ the refraction is $\frac{1^{\text{m.}}}{0{\cdot}20} = 5$ D.

In reality, however, the determination of the *punctum proximum* is not always so simple as in theory. The size of the type-object must, in the first place, bear a certain relation to the visual acuteness of the eye under examination. There are persons who, though possessing a good range of accommodation, could not read fine print at any distance, simply because their visual acuteness is too low. In such cases we must use larger test-types, and those that stand out more clearly from the ground on account of the difference in brightness; very black on a white ground or very bright on a black ground. It is under such circumstances that points, lines or figures cut in a metal diaphragm, and observed against the light, may render great service.

It happens, too, that the *punctum proximum* is so far away that small test-objects, like the threads in the trellis-optometer, are no longer distinguished even by an eye whose visual acuteness is normal. This difficulty is surmounted by placing before the eye a convex lens, which increases the refraction and brings the *punctum proximum* nearer. One has only to subtract the strength of this lens from the refraction corresponding to the *punctum proximum* of the system (eye and lens together) to have that of the eye. Suppose the lens to be + 10, and the point, up to which vision remains distinct, 9 centimetres distant: this point would correspond to $\frac{100}{9} = 11$ D of positive refraction. To obtain the situation of the *punctum proximum*, the strength of the additional lens must be subtracted, which gives $11 - 10 = 1$ D. Hence the *punctum proximum* is 1 metre distant.

The determination of the *punctum proximum* is greatly simplified by the following method, of which we make use in practice.[1]

Take a tape, graduated on one side in centimetres and on the other in focal distances of dioptries, with the numbers of the corresponding dioptries. The zero-end of this tape is attached to the handle of a frame, into which may be introduced either a perforated diaphragm or a paper with any kind of test-objects, as threads or hairs. This frame is brought toward the eye until the objects commence to appear indistinct. At this moment the tape shows on one side the distance (P) of the *punctum proximum* and, on the other, the corresponding dioptries ($p$), the maximum of refractive power of the eye.

For an emmetrope the number of dioptries thus obtained equals the amplitude of accommodation, because, since his *punctum remotum* is situated at infinity, his refraction at rest is $r=0$. Hence the amplitude of accommodation $a=p-r=p-0=p$.

In order to make this same method applicable to ametropia, we leave in front of the eye the glass that corrects its *real* ametropia (total hyperopia or real myopia). Thus it is emmetropic, and the experiment with the tape is the same as for the emmetrope. The divisions give directly the amplitude of accommodation. Or, using the instrument with the naked eye, we introduce, in the formula for $a$, the value of $r$, as we have done above.

We have seen that the principle applied, in optometers, to the determination of static refraction, consists in giving to luminous rays such a direction that they seem to come from the *punctum remotum* of the eye. It is evident that we can, in the same way, give to luminous rays the direction that they would have if they came from the *punctum proximum*. Thus, any optometer may serve for the determination of the *punctum proximum* and, consequently, of the amplitude of accommodation.

A simple lens suffices for determining the refraction of the eye at the moment of its maximum accommodation. Thus, for a person whose *punctum proximum* is situated at a finite distance, it amounts to the same thing whether he looks at an infinitely distant object through a concave lens or, without a glass, at an object situated at the focal distance of the lens before used. In fact, the concave lens gives, to parallel rays coming from a distance, the divergence that they would have if they came from its focus. To see at $\frac{1^{\text{m.}}}{6}$, a refraction of $+\ 6$ D is requisite. In order to see at a distance through a concave (negative) lens of 6 D, an excess of positive refraction amounting to 6 D is evidently necessary, in order to neutralise the negative 6 D of the concave glass. Hence we have only to seek the strongest concave glass through which an eye still sees distinctly at a long distance, and this lens will give the maximum of refraction ($p$) of which the eye is susceptible.

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[1] See Fig. 110.—Landolt's Ophthalmo-Dynamometer.

The focal distance of the lens, approached very close to the cornea, will give the distance (P) of the *punctum proximum*.

For the *emmetrope*, whose eye, when at rest, is adapted to parallel rays, this strongest concave glass expresses directly the amplitude of accommodation. An emmetrope who sees distinctly at a distance with the concave 8 D, but not with a stronger glass, has an amplitude of accommodation amounting to 8 D, because he can neutralise, by the contraction of his ciliary muscle, the negative, divergent action of the concave glass, which diminishes refraction.

For the *myope*, the strongest concave glass with which he sees at a distance still gives the position of the *punctum proximum*, and the refraction corresponding to the maximum accommodation ($p$). But, in order to find the amplitude of accommodation, the refraction at a state of rest ($r$) must be known, *i.e.*, the degree of the myopia. This is given by the *weakest* concave glass that adapts the eye to infinity. Hence the amplitude of accommodation is expressed by the difference between these two glasses.

Let us take the case of a myope of 2 D, for instance. In order to see objects at infinity, *without accommodation*, he needs a concave glass of 2 D. It is only with glasses stronger than $-2$ D that he is obliged to use his accommodation. Hence we must subtract, from the strongest concave glass ($p$) that he can tolerate during distinct distant vision, the one that adapts his eye, when at rest, to this distance, *i.e.*, that requires no accommodative effort. If our myope still sees at a distance with the concave 8, his amplitude of accommodation equals $8-2=6$ D, since two of the eight dioptries are expended in correcting his myopia.

A *hyperope*, who sees at a distance with a concave glass, also has his *punctum proximum* at the focus of the latter and, at this moment, the refraction ($p$) of his eye is likewise equal to the refractive power of this concave glass that it neutralises. But the amplitude of accommodation ($a$) of the hyperope is evidently greater than the strength of the concave lens, inasmuch as, merely to see at a distance without a glass, he has already exerted an amount of accommodation equal to the degree of his ametropia. We know, indeed, that hyperopia may be regarded as an insufficiency of refractive power. To fill up this gap, and make the hyperopic eye emmetropic, either a convex glass—the correcting-glass—or an equivalent effort of accommodation, is necessary. If the degree of the hyperopia ($r$) is 2 D, the convex 2 or an accommodative effort of two dioptries, is requisite to correct it. If this same hyperope still sees at a distance with the convex 8, this proves that his accommodation is not only strong enough to correct his hyperopia ($r=2$ D), but that it can, besides, overcome a concave lens of 8 D. Hence in our example it amounts to $2+8=10$ D.

A case may be found in which the hyperope sees at a distance without a convex glass, but not with even the weakest concave; if so, his *punctum proximum* is at infinity (P being equal to $\infty, \frac{1}{\text{P}}=p=0$), and the amplitude of accommodation is equal to the degree of the hyperopia which this eye just succeeds in correcting by exerting its entire accommodative power. So, too, in the formula $a=p-r, p$ is equal to 0, and $a$ becomes equal to $r: a=0-r=r$.

Again, it may even happen that the maximum effort of accommodation does not suffice to wholly neutralise the hyperopia. The eye remains adapted to convergent rays, though these are less convergent than if the accommodation were passive. Not only is it incapable of overcoming the action of any concave glass, but it still stands in need of a convex one to correct the remainder of its hyperopia and adapt it to infinity. If the strongest convex glass gives the measure of the refraction of the eye when at rest ($r$), the weakest convex glass indicates the maximum refraction obtained with the aid of the accommodation. Hence the difference between the two is still the amplitude of the accommodation. A hyperope of 4 D, who still sees with the convex 1, has an amplitude of accommodation $a=4-1=3$ D.

It may seem astonishing to find the classical formula, $a=p-r$, apparently reversed in this case. In our example we have $a=r-p$. But the difference is one of appearance merely. It will be remembered, that the refraction of the eye being expressed by a positive value as long as the eye is adapted to a finite distance, *i.e.*, to a point situated in front of it, it must necessarily become negative and take the minus sign when the point to which the eye is adapted is behind it,—*i.e*, when it is virtual.

In the example selected, the *punctum remotum*, as well as the *punctum proximum*, is virtual; hence $r$ and $p$ are both negative, and the formula for the amplitude of accommodation becomes $a=-p-(-r)$; that is to say, $a=r-p$.

The same consideration explains also the fact that the *punctum proximum*, in this case, is farther from the eye than the *punctum remotum*; it corresponds to the focus of the weakest convex glass with which the hyperope sees at a distance. This focus is farther away than that of the correcting-glass of the ametropia ($r$),—*i.e.*, of the strongest convex glass that adapts the hyperope to infinity. But both these points are, as we have seen, virtual,—*i.e.*, situated behind the eye.

To recapitulate: the strongest concave or weakest convex glass with which an eye can see at a distance, gives the refraction of that eye, when adapted to its *punctum proximum*. The sign of the glass must, however, be changed; the concave indicates positive and the convex negative refraction.

In emmetropia the number of this (concave) glass directly expresses the amplitude of accommodation.

In myopia we must subtract, from this number, that of the lens which corrects the ametropia.

Finally, in hyperopia the amplitude of accommodation is given by the sum of the number of the strongest concave, and that of the correcting-glass; or, if concave glasses are not borne, by the difference between the weakest convex glass and the degree of the hyperopia.

This method of determining the amplitude of accommodation appears, at first glance, to be very simple. It would seem practical, after having determined the static refraction and acuteness of vision, to ascertain at once the dynamic refraction by the same optometric method. This does, however, not prove entirely satisfactory. The visual acuteness is considerably diminished by the use of concave glasses, which diminish the size of retinal images. Moreover, it is difficult even for experienced observers to judge accurately as to whether their visual acuteness suffers from a lack of clearness, or from the diminution in size of these images.

In the second place, it is difficult to get an eye to put forth its maximum effort of accommodation when looking at a distance. Accommodation is not invited as it is by an object that is brought progressively nearer the eye.

The comparative experiments that we have made in this connection, have proved that, in the majority of cases, the amplitude of accommodation, as obtained by this method, is less than that furnished by a direct determination of *punctum proximum*.

Monocular optometers give, perhaps, better results than a single glass through which the patient is required to look at a distance. Their property of exciting an effort of accommodation, with which we reproached them when discussing the determination of static refraction, may now be urged in their favour.

The use of these instruments for the determination of the amplitude of accommodation needs no further explanation. After having found the static refraction, we vary the focus so as to obtain the maximum refractive power of the examined eye. The difference between the two results gives the amplitude of accommodation.

If the use of glasses alone does not prove satisfactory in the determination of the accommodative power, still they are of service, at times, by rendering the direct examination of the *punctum proximum* more precise. In cases where the latter is very close to the eye, an excursion of only a few centimetres changes the estimate of the refractive power several dioptries and thus alters the result of our examination. Between 7 and 11 centimetres, for instance, the refraction varies four dioptries; from 5 to 10 centimetres it varies even ten dioptries, while from 23 to 50 centimetres it varies only two dioptries. Hence the determination made at this latter distance offers a greater guarantee of accuracy than that made at a very short distance. Nothing is easier than to give to the *punctum proximum* any desired distance from the eye. We have only to provide the examined eye with a concave glass, stronger or weaker, according to the eye's refraction and accommodation, and instead of moving ourselves within the narrow limits of a few centimetres in front of the eye, we can make our examination, with more ease and accuracy, at a greater distance. It is self-evident that the number of dioptries of the concave glass will be added to the refractive power of the eye, as found by the determination of the *punctum proximum*.

Just so, in the case of strong hyperopia and feeble accommodation, the *punctum proximum* may be brought nearer by means of a convex lens, whose number will afterwards be deducted from that represented by the eye when adapted to this, its factitious *punctum proximum*.

Accommodation, being due to a voluntary act, the contraction of a muscle, could never be determined with the same accuracy as the static refraction. The latter corresponds to repose of the eye, a condition that can even be produced by force, with mydriatics. Agents which, like eserine, provoke a forced contraction of the ciliary muscle, do not render an equivalent service for the determination of accommodation that atropine and its substitutes do for that of refraction. If, in practice, we seek the *punctum proximum*, we wish to know the maximum refraction that an eye can assume under the impulsion of the will alone, and not under the influence of a cause foreign to the organism. This will being manifested especially in the interest of near vision, the person examined should always be permitted to be himself master of his desire to see distinctly, and the judge as to the degree in which he succeeds in so doing. In other words, we are dependent, in the determination of accommodation, upon the patient's good nature and intelligence; a fact which often deprives the examination of the desired accuracy.

It is for this reason, too, that there exists no *objective* method (properly so called) of determining the amplitude of accommodation.

After having found the static refraction with the ophthalmoscope, it would be necessary to determine, by the same means, that presented by the eye under the influence of its maximum effort of accommodation, from which to get the amplitude by subtraction. This effort is hard to obtain from the patient, unless we give him an object of fixation. But, by so doing, we return to the subjective method: if the object is beyond or not so far distant as the *punctum proximum*, the patient will not exert the maximum effort of contraction of his ciliary muscle; in the former case, because it is not necessary; in the latter, because he would still be unable to see the object distinctly, if he were to do his best. Hence he dispenses, at least partially, with his accommodation.

In order to know whether or not the object is at the *punctum proximum*, we must again have recourse to the clearness of the visual impression. Hence the method is not independent of either the patient's will or intelligence. On the other hand, if we already know the location of the *punctum proximum*, any further determination of the refraction of the eye, in this condition, becomes useless.

If the question were simply to ascertain, objectively, whether or not any accommodation existed, the relation between this function and convergence might be made use of. An object of fixation being placed at the *punctum proximum* of convergence (whose objective determination, by direct observation of the eyes, is very easy) and the eyes having executed their maximum rotation inward, we should determine with the ophthalmoscope whether or not the refraction were stronger under such circumstances than when the eyes were at rest.

## III.—THE DETERMINATION OF THE AMPLITUDE OF CONVERGENCE.

We remember, from page 191, that the *amplitude of convergence is equivalent to the difference between the maximum and minimum of convergence* which is possible in any individual case—

$$a^c = p^c - r^c$$

Further, that the *maximum of convergence* is inversely proportional to the distance (P) of the nearest point (*P*, Fig. 80), the *punctum proximum of convergence*, which both eyes can fix simultaneously. The distinctness of vision is not taken into account. In fact, in most cases, although binocular vision exists at the nearest point of convergence, distinct vision does not, because the *punctum proximum* of convergence lies much nearer to the eyes than that of refraction or accommodation.

Our *ophthalmo-dynamometer* serves to determine the maximum of convergence. It consists essentially of a metallic cylinder (C, Fig. 110) blackened on the outside, which can be fitted on a candle (B) of ordinary size. The cylinder has a vertical slit, 0·3 millimetres in breadth, covered by a ground glass. This being illuminated by the flame of the candle, forms a luminous vertical line and serves as an object of fixation. Beneath this is attached one end of a tape measure that can be rolled up by means of a spring. This is graduated, on one side, in centimetres and, on the other, in the corresponding numbers of metre-angles (or, what amounts to the same thing, in dioptries).

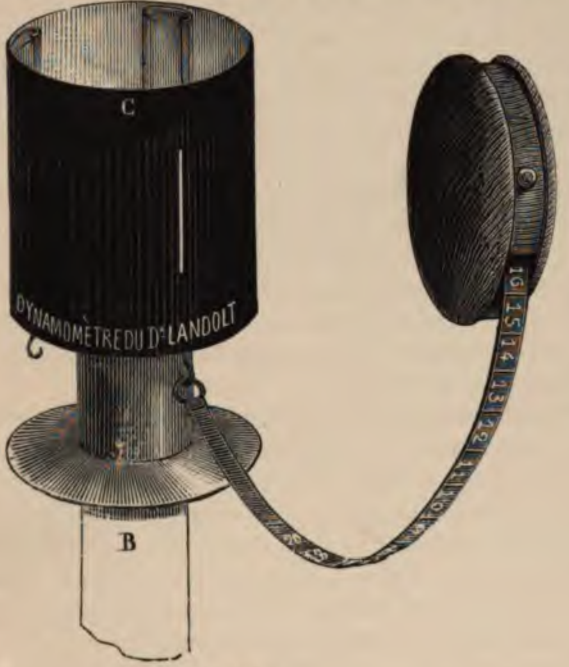

FIG. 110.[1]

If it is desired to find the *punctum proximum* of convergence, the tape measure is drawn out to about 70 centimetres, its case being held beside one of the eyes of the patient, while the object of fixation, turned towards him, is placed in the *median line*. If the patient see the object single, then, by pressing on the knob of the case, the spring is made to roll up the tape, and thus the observer brings the fixation object nearer to the eyes, taking care, however, that it always remains in the median line. The moment the person under observation commences to see double, the nearest point of convergence is attained. In fact, when the eyes have no longer the power which is required in order that they may fix the object simultaneously, there exists a divergence relatively to the position of the latter and, consequently, crossed diplopia.

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[1] The instrument is to be had at the opticians, Roulot in Paris, and Meyrowitz in New York.

At this instant one side of the tape gives, in *centimetres*, the distance of the *punctum proximum* of convergence, and the other side the corresponding *maximum* of convergence in *metre-angles*. For instance, 11 centimetres correspond to 9 *metre-angles*. The first observation suggested by this investigation is, that the motor-apparatus of the eyes does not always perform its functions with as great accuracy as we could desire. It is decidedly inferior to the muscle of accommodation, in the fact that the answers of unpractised persons sometimes vary markedly in different examinations. It is therefore necessary to repeat the experiment several times at one sitting and, especially in pathological cases, on different days.

It is furthermore important, during all the time while the object is being brought nearer, to encourage the patient to fix it energetically. It will also be observed that the convergence near-point, when thus obtained, lies nearer to the eyes than when the object is first placed within this distance and then *withdrawn* from the patient. By the latter method accurate results are not obtained. Hence we would advise that the object of fixation be brought toward the patient from a point for which he can easily converge.

Here, as in all investigations concerning binocular vision, it may happen that the patient is not at first certain whether he sees double or not. In cases, for instance, where one eye is considerably weaker than the other, the patient easily overlooks the retinal image of the former, so that he does not see double, in spite of the faulty direction of the eyes. The same thing can occur when, from other causes, perfect binocular vision has never existed,—as, for instance, in high degrees of myopia, with almost similar eyes, but so myopic that perfect binocular vision is impossible without artificial aid. Even individuals with otherwise healthy eyes, but ill-developed intellectual faculties, are frequently unable to say what or how they see.

In such cases the examination can be rendered much easier by holding a coloured glass (preferably a red one) before one eye. The eye which remains directed toward the fixation-object during the observation should be chosen. The deviating eye is, of course, that one whose retinal image has less value for the individual. Hence it must not be rendered weaker by the obscuration which a coloured glass always produces. We have found that, with proper energy on the part of the observer, the results, with and without glasses, differ only very seldom. Further, in the majority of cases, it is sufficient to have rendered the patient attentive to the second image by means of a coloured glass. We therefore use it only at the commencement of the experiment, and sometimes to verify the patient's answers.

It is always advisable to watch the eyes of the person under examination, and in most cases they will be seen, at first, to follow the object correctly; then, in a certain position, a breach occurs,—one eye seems to stand still as if it hesitated, and, finally, when the object has approached still nearer, it swerves outwards.

At this last moment, however, the *punctum proximum* has already been passed over. The maximum of convergence is attained when the hesitation begins, and, in many cases, even before this. It is important, in those cases, to make the patient attend to the appearance of the diplopia.

In healthy individuals, the *punctum proximum* of convergence often lies not very far from the end of the nose. In such cases, where P can be only approximately determined, the accurate determination of $p = \frac{1}{P}$ becomes difficult, since, as we have already seen in the case of the accommodation, a difference of 1 centimetre may be equivalent to several metre-angles; 7 centimetres = 14 metre-angles; 6 centimetres = 16 metre-angles; 5 centimetres = 20 metre-angles. In these cases we can make use of a method analogous to that used in the determination of the maximum of accommodation—that is to say, by artificially increasing the distance of the *punctum proximum*. In the case of the accommodation we made use, for this purpose, of concave glasses, whose neutralisation was brought about at the expense of a certain portion of the dynamic refraction; here we can, in a similar manner, use adducting prisms, which render convergence more difficult. If we place, for instance, a prism of about $10^\circ{\cdot}5$ before each eye, with the *apex inwards*, they must have a convergence of 3 metre-angles, in order to see a distant object single.

If the latter has approached, binocular fixation always requires 3 metre-angles of convergence in addition to that which corresponds to the distance of the object. If a person possesses a maximum of convergence of 13 metre-angles, this is already reached when the object is at a distance of $\frac{1^{\text{m.}}}{10}$, and not $\frac{1^{\text{m.}}}{13}$, at 10 centimetres instead of 7·5 centimetres, since, in the first case, he makes use, not alone of 10 but of $10 + 3 = 13$ metre-angles of convergence.

It will, however, very rarely be necessary in practice to resort to this stratagem, since, as we have seen above, 10 metre-angles of convergence are sufficient for ordinary work, and it is not of great importance to know accurately whether a person has 13 or 14 metre-angles of converging power. The accurate estimation of the opposite, *i.e., deficient convergence,* is of much greater interest to us. The amount of the deficiency is easily found by means of our dynamometer.

If the maximum of convergence is less than 0·5 metre-angle—that is to say, if the *punctum proximum* is further off than 2 metres, the flame of the candle itself can be used for its determination, since, at this distance, it does not form too large an object of fixation.

The *minimum of convergence* is, as we have seen above, the reciprocal of the distance which separates the farthest point of binocular vision from each eye. If R be the distance of this *punctum remotum of convergence,* the *minimum of convergence* will be $\frac{1}{R} = r$ metre-angle. If this point be situated at a finite distance, the minimum of convergence will be positive, and may be determined with the dynamometer, in a way similar to that with which we determined the maximum. It is only necessary to remove the instrument to such a distance that, convergence having reached its extreme limit, the luminous slit is seen in homonymous diplopia. When this takes place at a greater distance than 2 metres, the flame of the candle becomes a sufficiently small object, and the funnel of the dynamometer may be dispensed with. The reciprocal of its distance from the eye will always give the minimum of convergence in metre-angles ($r$).

However, it seldom occurs, and then only in pathological cases, that the lines of fixation cannot be rendered parallel. In the latter case, the minimum of convergence would be zero, because the *punctum remotum* lies at an infinite distance: $r = \frac{1}{\infty} = o$.

In most persons, however, the eyes can *diverge* somewhat. In such a case the minimum of convergence becomes *negative.* It is always equal to the reciprocal of the distance of the *punctum remotum,* only, when the lines of fixation diverge, the *punctum remotum* is situated, not in front of the head, but behind it, at the point $-R$ (Fig. 80), where these lines of fixation, if continued backward, would meet.

As, in such conditions, the lines of fixation do not meet anywhere in front of the eyes, it is no longer possible, as it was in the preceding case, to measure the distance of the *punctum remotum* and, from it, deduce the angle of convergence. The reverse method of procedure must therefore be adopted. The angle of deviation of the eyes must first be measured, and from it, if required, the distance of the *punctum remotum* can be obtained.

The simplest method of determining this angle consists in placing a prism of varying strength before one eye, with its apex toward the temple. This prism, by deviating the rays of light toward its base, compels the eyes to *diverge* in order to obtain binocular vision of an object placed at a distance.

Such an instrument, the principle of which has already been pointed out by Herschell, can be obtained by superposing too prisms of equal strength, which can be rotated in opposite directions. It may be purchased, mounted on a handle, on which is given the degrees of the prism resulting from the union of the two component glasses. To the above scale, I have added two others, on the rim of the instrument, which give, for a base-line of 64 millimetres, and for one of 58 millimetres, the number of metre-angles necessary for each eye to overcome a prism of given strength.

It is not difficult to show the relation existing between the strength of any prism and the number of metre-angles which expresses the deviation produced. For a base-line (or distance between the centres of rotation of the two eyes) of 58 millimetres, as in children, a metre-angle corresponds to $1^\circ\ 39'\ 39''$—say $100'$.

The deviation produced by a prism may be taken as equal to half its angle of opening, which is marked on each prism in our trial cases, or on the hand of the double prism. Therefore a prism of X° will produce a deviation of $\frac{X^\circ}{2}$ or of $\frac{X \times 60'}{2}$. It is necessary only to divide this value by 100, in order to obtain the corresponding number of metre-angles : $\frac{X \times 60'}{2 \times 100}$.

This formula, reduced to its simplest expression, becomes $\frac{3X}{10}$, that is to say, we have only to multiply the number of the prism by 3, and divide the prism by 10, in order to find, in metre-angles, the deviation for a base-line of 58 millimetres.

When the prism is held before one eye only, as in the determination of the minimum of conveyance by the double prism, its action is divided between the two eyes. The total deviation $\frac{3X}{10}$ gives for each eye $\frac{3X}{20}$. A prism of 6° produces a deviation of $\frac{18}{10} = 1{\cdot}8$ metre-angles. But, if both eyes concur to neutralise its effect, each eye need only change its direction $\frac{18}{20} = 0{\cdot}9$ metre-angle. It is only when a prism of 6° is placed before each eye that the full result of 1·8 metre-angles is obtained ; always, of course, for a base-line of 58 millimetres.

When the base-line is longer, for example 64 millimetres, as in adults, the metre-angle becomes $1^\circ\ 50' = 110'$, and the formula becomes $\frac{3X}{11}$ for the deviation corresponding to the prism of X°, or $\frac{3X}{22}$ for the effect produced on each eye, when the prism is placed before one only.

The mean between these two fractions, $\frac{3X}{20}$ and $\frac{3X}{22}$, is $\frac{3X}{21}$ or $\frac{X}{7}$.

In other words, we may simply divide the number of the prism by 7, in order to obtain *approximately*, in metre-angles, the amount of deviation of each eye, when the prism is placed before one. Two prisms of equal power, one before each eye, necessarily produce a double effect, or $\frac{2X}{7}$.

When, therefore, the *punctum remotum* of convergence is situated at an infinite distance or beyond, the minimum of convergence is determined by the following method. Making the patient fix the flame of a candle at a distance of 6 metres or more, we find the strongest abducting prism, which, placed before one eye, can be overcome without causing homonymous diplopia. We then read off, on my scale attached to the double prism, the minimum of convergence in metre-angles, or we deduce it from the number of the prism, by means of the method just described. Here, too, it may often be necessary to have recourse to a coloured glass, in order to render the patient attentive to his diplopia.

If the *maximum* ($p$) and the *minimum* ($r$) be found, the *amplitude of convergence* ($a$) is obtained, as we have already explained, by subtracting the latter from the former: $a = p - r$.

## IV.—DETERMINATION OF THE RELATION BETWEEN THE CONVERGENCE AND THE OPTICAL ADAPTATION IN BINOCULAR VISION.

Our dynamometer also serves to ascertain whether or not proper *binocular vision* exists for a definite distance. For this purpose its funnel bears a series of small holes arranged in a vertical line. As long as the convergence and optical adaptation of the eyes are correct (with or without glasses), the holes are seen single and distinctly.

If the *refraction* does not correspond to the distance of the object, the points appear indistinct, but the line formed by them still remains single as long as the eyes are normally directed.

If the *convergence* is insufficient, but the optical adaptation correct, the line appears double, while the points are distinct. If, however, both functions are at fault, there will be, at the same time, both diplopia and indistinctness of vision. The line appears double, and its points are blurred, and form discs of diffusion.

## V.—DETERMINATION OF THE RELATIVE RANGE OF ACCOMMODATION AND CONVERGENCE.

We have explained, p. 196 *et seq.*, what is meant by the relative range of accommodation and convergence. The problem of their determination is as follows:—

1*st. The convergence remaining unaltered*, by how many dioptries can the accommodation be increased or diminished?

2*d. The accommodation remaining unaltered*, by how many metre-angles can the convergence be increased or diminished?

These determinations were first undertaken by Donders and MacGillavry,[1] by means of an apparatus consisting of a quadrangular board, 162 centimetres long and 24 centimetres broad, kept in a horizontal position by a standard capable of being lowered or raised at will. One end of the board has a notch for the nose of the person under examination. The position of the eyes is determined by two wooden rods, rounded at the ends and drawn out at pleasure, against which the cheeks rest. From in front of each eye a divided groove extends the whole length of the board, perpendicular to the base-line. In front of each eye is a grooved half-circle for holding lenses, and moveable in an arched groove, whose centre of curvature is situated on an imaginary prolongation of the horizontal groove, within the eye. The eyes should be so placed that their nodal points may, as nearly as possible, be over the line passing through the zero-points of the divisions of the longitudinal groove, and so that their centres of rotation may be considered as coincident with the centre of the above-mentioned arcs. Then the lenses may always be given such a position that their axis will correspond to the lines of fixation. Two small microscopes, attached to the board, indicate, by the crossing-points of their micrometric threads, the position of the corneæ of the eyes under examination, and are useful in watching and controlling the position of the latter. The half-rings, which contain the lenses, can be moved toward, or away from, each other, so that the distance between the lenses may always correspond to the distance between the eyes of the person under examination.

From the middle of the space between the eyes, a third longitudinal groove runs parallelly with the two above mentioned. In it slides a piece of wood destined to carry the object of fixation. The latter consists of a few hairs stretched across a small frame, or of some very small holes in a metallic diaphragm.

Distinct vision of this object demands a greater or less effort of convergence, in proportion as the object is nearer to, or further from, the eyes. In apparatus of this kind, as found in trade, the median groove is marked with two scales, indicating the angles of convergence for two different distances between the centres of rotation,—that is, for two base-lines, one 64 and the other 54 millimetres in length.

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[1] Donders, *Holländische Beiträge, &c.*, vol. i., p. 379, 1846, and MacGillavry, *Onderzækingen over de hægrootheid der Accommodatie:* Inaug. diss. Utrecht, 1858. Donders' *Anomalies, &c.*, p. 97.

To determine the *maximum of relative convergence* (*p*) we place before the eyes *concave* glasses of successively increasing strength, which require an increasing effort of accommodation. To find the *minimum of relative convergence* (*r*), *convex* glasses, on the contrary, are used.

The strongest concave glass that can be borne, without prejudice to binocular and distinct vision, gives the *positive portion*, and the strongest convex glasses the *negative portion* of the amplitude of relative accommodation; and the difference between them constitutes the *relative amplitude of accommodation* ($a_1$).

These experiments are made for different degrees of convergence, and the results noted on diagrams,—i.e., on a system of co-ordinates. On the horizontal lines (*abscissæ*) are recorded the degrees of *convergence*; on the vertical lines (*ordinates*), the degrees of *accommodation*. Hence the figures placed on the vertical line, at the left of the diagram, indicate *metre-angles*; and those at the bottom on the horizontal line, *dioptries*. We have already given and explained a series of such diagrams (pp. [illegible], Figs. [illegible]).

# CHAPTER IV.

# ASTIGMATISM.

## I.—REGULAR ASTIGMATISM.

HERETOFORE we have considered the refractive surfaces of the eye as surfaces of revolution,—*i.e.*, as produced by the rotation of an ellipse, or circle, about the optic axis. A surface of this kind has, necessarily, the same curvature in all its meridians; every plane passed through the axis is limited by the same curve, viz., the curve which, by its rotation, has engendered the surface. From this it follows that light is equally refracted in each of these meridians. Rays emanating from a luminous point are focused on the axis by each meridian, and at a single point of the image. The union of the foci of all the meridians constitutes the focus of the surface.

It is true that we made a certain restriction in the case of a spherical surface (p. 12). We said that the farther the zone, through which the rays pass, is from the axis, the nearer to the surface their focus will be; so that rays passing through a sphere are not all united exactly at one point. But they are, at least, all focused on the axis. The focal line formed by them, in the least favourable case, coincides with this optic axis. Moreover, if the image, of a luminous point, formed by a spherical surface, be received upon a screen held perpendicularly to the axis, this image is everywhere a circle, or a larger or smaller luminous point,—*i.e.*, always similar to its object.

The focusing is more nearly perfect for an ellipsoid of revolution than for a sphere, since, on account of the gradual diminution in curvature of the ellipsoid, the eccentric rays are directed toward the same point as those passing nearer the axis.

The refractive surfaces of a perfectly formed eye are very like an ellipsoid of revolution, called, in this case, ellipsoid with two axes. One of them, the major axis of the ellipse, is, at the same time, the optic axis and that of rotation; the other is perpendicular to it and is equal in all meridians.

When speaking of the form of the dioptric surfaces of the eye, we said that eyes so perfectly constructed are rarely found; that the curvature of the cornea, for instance, is nearly always greater in some

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When speaking of the form of the dioptric surfaces of the eye, we said that eyes so perfectly constructed are rarely found; that the curvature of the cornea, for instance, is nearly always greater in some one meridian than in that which is perpendicular to it. The surface can, then, no longer be regarded as belonging to an ellipsoid of revolution, since the solid figure, of which the former would constitute a part, has not only two axes, but three, and those unequal; the antero-posterior axis, the one corresponding to the meridian of the strongest, and that corresponding to the meridian of the weakest curvature. The two latter are situated in the same plane, and are perpendicular to each other.

Thus, let us suppose that the vertical meridian of a cornea is described with a radius shorter than that of the horizontal one, but that both may be considered as belonging to one ellipse, smaller in the first and larger in the second case. This figure will be an ellipsoid, *with three unequal axes;* the optic axis, the axis of the vertical meridian (shorter), and that of the horizontal meridian (longer).

This irregularity is not, however, always great enough to be susceptible of profitable correction and is disregarded. The great majority of eyes having the same defect, it is endured and disregarded like many other physiological imperfections. We limit ourselves to requiring the average acuteness of vision permitted by the construction of the organ.

But in other cases the difference of curvature in the different meridians of the eye attains such a degree that vision falls far below the average, from which results considerable annoyance for the affected individual and an absolute necessity of remedying the defect. This refractive anomaly is called *regular astigmatism.* We shall explain the etymology of this word farther on.

Without discussing at present the surfaces of the eye that may be the site of this irregularity, let us at once consider the optical phenomena which must result from it.

We have already seen, when discussing the reduced and the artificial eye, that the refraction of the eye may be compared to the action of a single surface, which produces the same effect as all the refractive surfaces of the eye united. Let us suppose this surface to be more convex, described with a shorter radius, in its vertical than in its horizontal meridian.

In Figure 111, V represents a vertical and H a horizontal section of the reduced eye.

To perform the following experiment, which we would highly recommend to the reader as very instructive, an astigmatic system will be most easily procured by means of a convex cylindrical lens, placed in front of the author's artificial eye, or by the combination of a convex spherical lens with a convex cylinder (*e.g.*, sph. + 10 D, and cyl. + 3 D). The latter's axis, in this example, will be placed horizontally.

The meridian of the greatest and that of the least refraction are called *principal meridians*. They are generally perpendicular to each other. In our example they are the vertical and horizontal meridians.

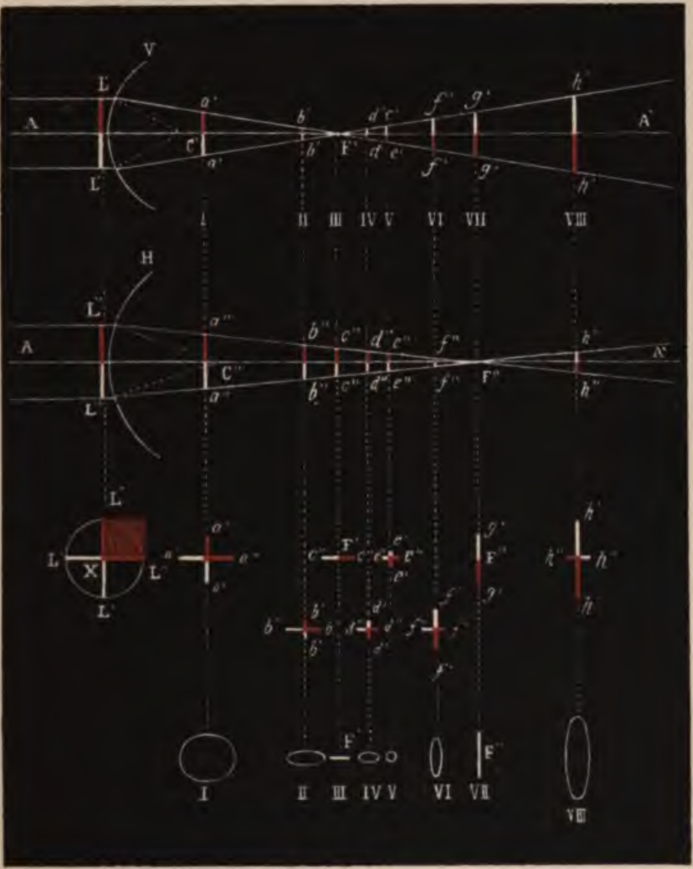

Fig. 111.

A A′, optic axis of the refractive surface; L′ L′ and L″ L″, sections of a pencil of parallel luminous rays; V, vertical meridian; C′, its centre of curvature; H, horizontal meridian; C″, its centre of curvature.

The red lines indicate that in V the upper arm, in H the right arm, of the cross resulting from the combination of the two slits, has been covered with a coloured glass. They show, at the same time, the coloured portions of the images of diffusion.

The figures (I. to VIII.) in the fourth row, whose diameters should exactly correspond to the lines of the fourth, are slightly reduced for the sake of clearness. Had this reduction not been made, some of them would have overlapped others.

Let us suppose a luminous point situated on a prolongation of the optic axis, at infinity. It emits rays parallel to this axis. A pencil of such rays, with the diameter L′ L′ = L″ L″, strikes the surface. In order to take into account only what happens in the vertical meridian, let us place in front of the eye a diaphragm with a narrow vertical slit. This will permit only such rays as are comprised in the vertica plane of the pencil to pass through.

The vertical meridian, having everywhere the same curvature will cause the rays to converge and will focus them at F′, after which they will diverge.

The same will happen for the horizontal meridian, whose action may be isolated by placing the slit horizontally. Only, the horizontal meridian having a less curvature, its focus F″ will be farther from the surface than the focus F′ of the vertical meridian, and back of it.

By using a combination of the vertical and horizontal slits (*i.e.*, a crucial aperture), we may judge of the effect produced by the vertical and horizontal meridians together. If a screen be placed behind the system, successively at the different points from I. to VIII., the figures shown in the third row of the diagram will be obtained, one after another, on the screen.

At I. the rays that have passed through the vertical meridian form a vertical line of diffusion having the length $a' a'$. Those that have passed through the horizontal meridian form the horizontal line $a'' a''$, longer than $a' a'$.

Both together form a cross, whose arms are of unequal length.

At II. the length of both arms has diminished; that of the vertical one, however, more than that of the horizontal. The screen is now near the focus of the vertical meridian.

At F′, the rays refracted by this meridian are focused in a *point*, while those of the horizontal meridian still form a horizontal line of diffusion, $c'' c''$ (III.).

Beyond the focus F′ of the vertical meridian, the vertical line of diffusion commences to be formed anew (IV.), while the horizontal one continues to diminish in length ($d' d'$, $d'' d''$). Hence there ought to be some place where the bars, of the cross formed by these lines of diffusion, are of equal length. This we find at V. ($e' e' = e'' e''$).

From here on the vertical line continues to increase, and the horizontal to diminish in length (VI.), until, at the focus F″ of the horizontal meridian, the horizontal line becomes a point, while the vertical one has attained the length $g' g'$ (VII.).

On continuing the experiment, a cross is again seen to be formed, the length of whose branches constantly increases, the vertical, however, always remaining the longer (VIII.).

The experiment becomes still more conclusive if half the cross (the ascending and right branches, for instance) be covered with a red glass. In front of the first focus, the same parts of the crosses obtained on the screen, and of that formed by the two slits, are coloured.

At IV. the *descending* branch is red, instead of the ascending, while the *right* has remained red.

Beyond the second focus the position of the coloured branches is inverted, relatively to their primitive one; the *descending* and *left* branches are the red ones. Thus we readily know whether the luminous pencils have been intercepted by the screen before or after the crossing of the rays belonging to each of the meridians.

We learn, in this way, what occurs when a pencil of parallel rays passes simultaneously through the two principal meridians isolated. The luminous pencil, whose origin is a point, nowhere forms an image like a point. Its image is sometimes a cross, in which either the vertical or horizontal bar predominates. At the focus of the vertical meridian, it is a horizontal line. At the focus of the horizontal meridian it is a vertical line. Or, again, it is a cross with arms of equal length between the two foci, but nearer to the focus of the more refractive than to that of the less refractive meridian.[1]

The experiments that we have just described, as made with the two principal meridians, do not permit us, however, to draw conclusions as to what occurs for the whole astigmatic system. In order to do so, we must examine the *intermediate meridians* as well.

Let us then place a diaphragm, with an oblique slit, before our astigmatic artificial eye. A strange phenomenon is thereby produced. Placing the screen at I., a rather broad line of diffusion, nearly parallel with the slit, is obtained.

In proportion as the screen moves away from the system, the image of diffusion shortens, becomes more distinct, and inclines toward the horizontal. At the focus F′, the image of the point is a short horizontal line. If we continue to withdraw the screen, the line keeps turning. The right end of it, for instance, gets lower and lower, while its left extremity rises, until, at the focus F″ of the horizontal meridian, the image becomes a vertical line. At the same time the line has gradually lengthened.

Carrying the screen still farther away, we see, finally, that the line approaches its primitive direction. It becomes longer and, at the same time, broader.

Moreover, by covering half of the aperture with a coloured glass, the final image of diffusion is shown to be the inverse of the one obtained at the outset. If we cover the upper half of the aperture with the red glass, the upper half of the image is red at first, while it is the lower half of the image that is red when the screen is at, or beyond, the second focus.

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[1] Let us ask, in passing, what would have happened, under the same conditions, in the case of a surface of revolution—that is to say, with a system which is not astigmatic.

The images of diffusion of the two isolated meridians might have been crosses, but always crosses with equal branches, and never lines. And whatever might have been the form of the aperture in the diaphragm, the rays passing through it would have been focused in a single point, the focus of the surface.

The experiment, of which the foregoing is a description, proves that the luminous rays, emanating from a point and passing through one of the intermediate meridians of an astigmatic eye, are *nowhere* focused. They intersect, but not in the same plane, as do the rays that pass through the principal meridians. Hence they form a sort of helix.

At the focus of either principal meridian they all pass through one line, which is parallel with the meridian to which the focus corresponds. The line is shorter for the more refractive, longer for the less refractive meridian. It is never as long as the line of diffusion formed by the principal meridian, which is not adapted to the distance of the point. The only exceptions are for the place where the images of diffusion, for the principal meridians, are of equal length. Here (V., Fig. 111) the branches of the spiral, formed by the section of the rays that have passed through an intermediate meridian, have the same length as the branches of the cross formed by the principal meridians.

If we unite the images formed by all the meridians—*i.e.*, by the entire astigmatic system—we obtain elliptic figures, the axes of which are nothing else than the images of diffusion from the principal meridians. Hence we have only to describe a curve around any one of the crosses formed by the principal meridians and we shall have, for each plane, the image corresponding to rays emanating from a luminous point.

When one of the axes of the ellipse becomes infinitely short, as is the case at the foci of the principal meridians, the ellipse becomes a line; when the axes are of equal length, the ellipse takes on the form of a circle. Thus, referring to the fourth row in Fig. 111, we see that when one of the axes is vertical, the other horizontal, and the former the more refractive of the two, the image of diffusion is, at first, an ellipse whose major axis is horizontal. Its axes become shorter and shorter, especially the vertical, until, at the focus (F′) of the more refractive meridian, the vertical axis is *nil*, and the image of diffusion is a horizontal line. Beyond this focus the length of the vertical axis commences to increase, while that of the horizontal diminishes. The horizontal ellipse is re-formed (IV.); then its outline becomes circular (V.); and, finally, an ellipse, whose major axis is vertical (VI.). At the focus (F″) of the least refractive meridian, the ellipse has become a line (vertical in this case) perpendicular to the meridian of least curvature. Beyond this focus the vertical axis of the ellipse remains the major one (VIII.); both axes continue to increase in length, but the one that is parallel with the most refractive meridian always remains the longer of the two.

This fact may be shown directly, by observing the diffuse images, of a luminous point, formed on a screen by the astigmatic system, without decomposing the latter into its different meridians, as we have just done with the crucial opening in the diaphragm. Nevertheless, we preferred to make this analysis, since the form, alone, of the diffuse image does not indicate what part each meridian takes in its formation, and it is indispensable to know this if we are to judge correctly, and thoroughly understand the action, of an astigmatic system.[1]

An astigmatic person, then, never sees a point in its real shape,—*i.e.*, as a point; sometimes it appears to be an ellipse, at other times a circle and, at others still, a line. Hence the term *astigmatism*, derived from the Greek *α*, privative, and *στίγμα*, *a point*. It was Whewell who introduced this term into ophthalmology.

We know now how a luminous point appears to the astigmatic eye; it is as an ellipse, or a *line perpendicular to the meridian that is best adapted to the distance of the point*, unless it be as a diffusion-circle between the distances of adaptation of the strongest and weakest meridians. How does an astigmatic person see a *line?*

The way objects generally appear to such a person must result directly from the way he sees a point. We have only to consider the line as composed of a great many points. For each one of them precisely the same phenomenon is produced as for the single isolated point. Thus let us still suppose the *vertical* to be the *most*, and the *horizontal* to be the *least* refractive meridian. We now place a vertical line (I., Fig. 112) at a certain distance from the patient.

If neither of the principal meridians is adapted to the distance of this object, the line will appear slightly elongated, owing to the diffusion-image, of each of the points composing it, produced by the vertical meridian. It will also seem somewhat broadened, on account of the diffusion produced by the horizontal and intermediate meridians (II., Fig. 112).

If the *vertical* meridian is adapted to the distance of this *vertical* line, if we have brought the artificial eye nearer the line, or withdrawn the screen, which represents the retina, enough so that the latter is just at the focus of the vertical meridian, the line appears very diffuse (III., Fig. 112). It does not now seem elongated, because there is no longer any diffusion in the vertical direction, but it appears broadened, because the rays that have passed through the other meridians, particularly those passing through the horizontal one, form horizontal images of diffusion.

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[1] A well-known model, devised by Professor Knapp, who has contributed so largely to our knowledge of the optic apparatus of the eye, and of astigmatism in particular, shows, by means of coloured threads stretched between two boards, the course of luminous rays in the astigmatic eye. A full description of this, with that of another, not less ingenious, method of demonstration, will be found in the *Transactions of the American Medical Association*, 1880, p. 669.

Between the two foci the line is again, apparently, somewhat elongated and still diffuse (IV.), until it becomes *distinct* at the *focus of the horizontal meridian* (V.). This fact is easily explained. Each of the points composing the vertical line forms, at the focus of the horizontal meridian, not a horizontal diffusion-image, but a vertical line of diffusion. All these little diffusion-lines overlap each other, and only give to the line a slightly elongated appearance. It is not, on this account, indistinct; it remains a well-defined line, without breadth, because there is no diffusion in the direction of a perpendicular to the line, but in the direction of its length only. Beyond the focus of the horizontal meridian, the line again appears diffuse in both these directions.

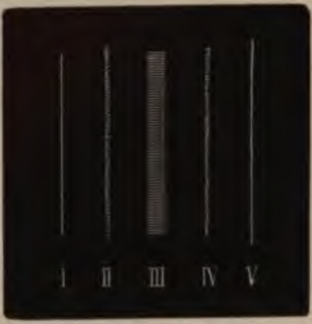

Fig. 112.

Hence, in short: a straight line is seen distinctly by an astigmatic eye only when the meridian, to which it is perpendicular, is adapted to its distance. A *vertical line* is seen *distinctly* when the *horizontal meridian* is adapted to its distance. It appears *indistinct* when its image is formed by the vertical meridian.

When the line is parallel with an intermediate meridian it is not seen distinctly at any distance. The rays emanating from its component points being in this case nowhere focused, the line always forms an image of diffusion. Other things being equal, the diffusion images formed by the intermediate meridians are, however, less extended than those due to the principal meridians. From this it results that, if the line be seen most distinctly when it is perpendicular to one of these two meridians, the image of it that is formed by the other one is more indistinct than that formed by any one of the intermediate meridians.

The way in which an astigmatic person sees points and lines led to the discovery of this irregularity in the refraction of the eye. It still serves for the qualitative and quantitative diagnosis of this special form of ametropia.

## SEAT OF REGULAR ASTIGMATISM.

The inequality of refraction in the different meridians of the eye, which constitutes regular astigmatism, is due to anomalies of curvature in its refractive surfaces, or to eccentricity of these surfaces, or to both these causes existing simultaneously.

It may be congenital or acquired; in the latter case either from irregular development or even irregular action of the eye, or from some external cause, as a traumatism.

We have already spoken of a slight irregularity of curvature, which we may now call astigmatic, from which the cornea is rarely exempt. But, beyond this degree, which may be entirely disregarded, this deformity may assume such proportions that it affects not only the whole dioptric apparatus but also the entire globe of the eye,[1] indeed even half, or the whole, of the head. The eye and the corresponding part of the cranium seem flattened and compressed, in one or another direction.

There are eyes, the vertical diameter of which is less than the horizontal, and in which the vertical meridian, of the cornea and of the surfaces of the crystalline, is more convex than the horizontal. There can be no doubt that in such a case one common cause has produced the irregular development of the cranium and the deformity of the eye. But even the lower degrees of astigmatism are often accompanied by asymmetry of the cranium, which is probably its primary cause.[2]

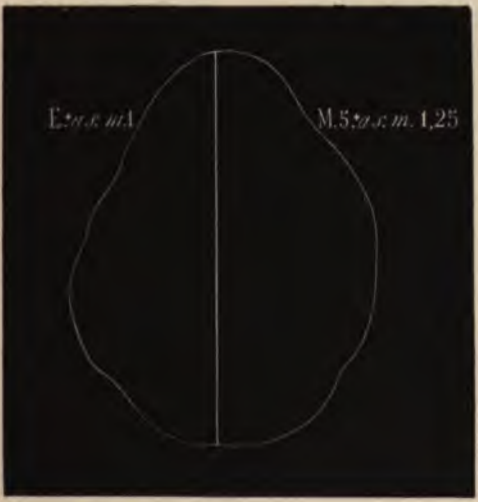

FIG. 113.

Circumference of the cranium viewed from above; the frontal bone above, the occipital below. The left eye presented a simple myopic astigmatism of 1 D, the right eye a myopia of 5 D, with myopic astigmatism of 1·25 D.

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[1] Snellen in Van Haaften, *Het Bepalen van Astigmatisme* (Thesis at Utrecht, 1819).

[2] De Wecker, *Comptés rendus de la Société d'Anthropologie de Paris*, 1868.

Landolt, "Relations between the Conformation of the Cranium and that of the Eye" (*British Medical Journal*, April 1881).

We give, in Fig. 113, a representation of a circumference of a cranium visibly more developed on the right than on the left side, being that of a person both of whose eyes were astigmatic. It is easy to find crania of much greater asymmetry with eyes still more astigmatic.

The vertical meridian, or one near it, is generally the most convex. Some have, on this account, supposed that the pressure exerted by the lids might be looked upon as a cause of astigmatism; that this would tend to produce greater curvature of the cornea, and perhaps of the crystalline (?), in the vertical than in the horizontal direction. Exceptions to this rule are, however, very frequent, and one should not always expect to find the strongest refraction in the vertical, and the weakest in the horizontal meridian. M. Javal has pointed out a fact which is very interesting, both as to the direction of the principal meridians, and as to the heredity of astigmatism. "In my family," he says, "nearly nine persons in ten present, in both eyes, astigmatism that is contrary to the rule. It has seemed to me that, in general, among the Jews, the defect assumed, preferably, this unusual direction."[1]

The cornea is the commonest seat of astigmatism. It becomes so by exaggeration of this irregularity of curvature, which makes this first refractive surface of the eye resemble an ellipsoid with three unequal axes, rather than an ellipsoid of revolution. This congenital malformation of the cornea generally remains stationary through life. It may be considerable in degree and produce a difference of several dioptries between the refraction of the principal meridians.

But the highest degrees of corneal astigmatism are those resulting from operations. The cicatrisation following an incision, whether wholly corneal or partially sclerotical, and made for a simple iridectomy or for the extraction of cataract, always produces such a deformity of this membrane that the meridian parallel with the direction of the wound is more, and the one perpendicular to it less convex. The resulting astigmatism is always modified in the course of time. From being of very high degree for the first four weeks after the healing of the wound, it may diminish considerably, in fact even disappear—this is to say, become so weak that it may be disregarded and not demand special correction.

But very often it persists through life. I recall, among other instances, a case of glaucoma, in which I performed an iridectomy upward, and, after eighteen months, in consequence of a relapse, another downward. The form of the cornea was, at the outset, so modified that its resemblance to a vertical cylinder was apparent to the naked eye, and it required, for compensation, a convex cylinder of 10 D with its axis placed horizontally. Little by little the curvature changed, but, two years later, an astigmatism of 4 D still remained.

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[1] Javal in De Wecker, *Traité, &c.*, second edition, vol. ii., p. 843.

The correction of the regular astigmatism which follows the extraction of cataract often markedly increases visual acuteness and is seldom to be disregarded. Investigations have been made with the purpose of ascertaining which of the operative methods is least likely to produce astigmatism. We have followed nearly all the courses of operative procedure that could reasonably be attempted, in cataract operations, and have obtained cures, with and without noticeable astigmatism, by each of them. At all events, a certain mode of operating being otherwise indicated, the prospect of producing more or less astigmatism should never induce us to abandon it for another. The astigmatism consequent upon a well-performed operation, being regular, is easily corrected.

The crystalline very often partakes of the astigmatism, sometimes in a passive, at other times in an active way. The former case is met with when the globe of the eye, or at least the anterior portion of it, is, as it were, flattened in one meridian. Then the crystalline, compressed, so to say, in a corresponding meridian, approaches the likewise compressed form of the cornea and the *static* crystalline astigmatism adds itself directly to that of the cornea. Sometimes the principal meridians of the astigmatic crystalline do not have the same direction as those of the cornea; at other times they are parallel but in such a way that the maximum curvature of the crystalline corresponds to the minimum curvature of the cornea, and the least convex meridian of the lens has the same direction as the most convex one of the cornea. Thus it is that crystalline astigmatism may partially or wholly compensate that of the cornea, indeed even exceed it.

Basing the conclusion upon numerous measurements and calculations, Donders formulated this important observation as long ago as in 1864, saying that, with a high degree of asymmetry of the cornea, there exists also an asymmetry of the crystalline, which acts in such a direction, that the total astigmatism of the eye is nearly always less than that of the cornea.[1] This astigmatism of the crystalline, the inverse of that of the cornea, is oftenest what might be termed active, or *dynamic*, astigmatism. It is due to an unequal contraction of the ciliary muscle, which causes the lens to become more convex in one direction than in another and for the very purpose of correcting a corneal astigmatism. It was Dobrowolsky who discovered this fact and brought numerous proofs to its support.[2]

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[1] Donders, *loc. cit.*, p. 492.

[2] Dobrowolsky, "Ueber verschiedene Veränderungen des Astigmatismus unter dem Einflusse der Accommodation" (*Arch. für Ophth.*, xix., iii., p. 51, 1868).

In the first place, it may easily be demonstrated that the eye succeeds, with a certain effort, in overcoming the effect of weak cylindrical glasses. By what mechanism would it be possible to neutralise the inequality of refraction produced by the cylinder, if not by the unequal contraction of the ciliary muscle, followed by a corresponding change in the form of the crystalline? This unusual and, for the non-astigmatic eye, unnatural muscular action reveals itself, moreover, by a rather disagreeable sensation, which, from its resemblance to the one experienced during fixation of the *punctum proximum*, leaves no doubt as to its origin. Besides this, Hensen and Voelckers have proved, by experiments performed on animals and by clinical observation, that the lesion of an isolated branch of the ciliary nerves paralyses, while its irritation produces contraction of, an isolated portion of the *sphincter pupillæ* and of the fibres of the ciliary muscle. The contraction of this muscle, limited to a single meridian, must cause unequal relaxation of the crystalline, and make it assume a more convex form in one meridian only.

But the most conclusive proofs of this are furnished by the clinical observations of Dobrowolsky, and of all those who have repeated his experiments.[1]

After paralysing the accommodation by an energetic and prolonged exhibition of atropine, regular astigmatism is sometimes found when no trace of it existed while the accommodation was intact. Since the mydriatic exerts no influence upon the cornea, but only on the form of the crystalline, through the intermediation of the ciliary muscle, the conclusion is inevitable that the astigmatism, which has become manifest by the use of atropine, is due to the cornea, and that, prior to the exhibition of the mydriatic, this astigmatism was concealed, neutralised or corrected by the unequal contraction of the ciliary muscle. The lens was thus rendered astigmatic in a sense opposite to that in which this irregularity exists in the cornea. In other cases an astigmatism, already ascertained when the eye was in its normal condition, increases in degree as a consequence of the paralysis of the accommodation. This proves that the irregularity in the curvature of the cornea was only partially compensated by a similar anomaly in the crystalline. In still other cases, finally, the astigmatism changes direction under the influence of the mydriatic, *i.e.*, the meridian of most powerful refraction, before the instillation of atropine, presents the minimum refraction afterwards, and *vice versâ*. In such a case the corneal astigmatism was over-corrected by the dynamic astigmatism of the crystalline. Without our needing to have recourse to the mydriatic, *latent astigmatism* reveals itself in the ophthalmoscopic examination, inasmuch as the spasm of the ciliary muscle ceases during such examination.

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[1] Mauthner, *Die optischen Fehler, &c.*, p. 734, 1876. Javal and Schiötz, "Un ophthalmomètre pratique" (*Ann. d'oc.*, July 1881, p. 1). Landesberg, "Ueber das Auftreten von regelmässigem Astigmatismus, &c." (*Arch. für Ophth.*, xxvii. ii., p. 89, 1881).

We have said, above, that a difference of refraction in two mutually perpendicular meridians is produced when the eye looks obliquely through a spherical lens or, in other words, when the axis of the lens does not coincide with the axis of the eye—when it is not *centred*, relatively to the latter. It was also said (p. 119) that the centering of the dioptric apparatus of the eye is not, at times, as exact as could be desired. This irregularity, which may be disregarded in most cases, may, in others, attain a sufficiently high degree to produce a manifest astigmatism. Thus Donders found, in his own eye, a regular astigmatism attributable to a slight inclination of the crystalline and a similar one in the case of a young man twenty years of age.[1]

Cases of congenital ectopia or partial luxation of the crystalline, followed by astigmatism, have often been observed.

Having given the fact that the crystalline participates in the production of astigmatism, it is evident that the latter is susceptible of change with age, under the influence of the modifications which the crystalline undergoes. Since the static refraction of the lens diminishes in consequence of the equalisation of the indices of refraction of its different layers, its static astigmatism must also diminish. But it is the dynamic astigmatism that especially varies with the diminution of the accommodative power. Thus it is that the astigmatism of the eye may increase with age, in proportion as the crystalline becomes incapable of correcting it. In those cases, on the contrary, in which the astigmatism is produced or over-corrected by unequal contractions of this muscle, it is seen to diminish as the accommodation becomes weaker.

### REGULAR ASTIGMATISM IN ITS RELATIONS TO THE GENERAL REFRACTION OF THE EYE.

Regular astigmatism has its seat exclusively in the optic apparatus of the eye. It is measured solely by the difference in refraction between the two principal meridians. It has nothing to do with the length of the eye. Wherever the retina may be, the degree of astigmatism is always the same. Thus, in an astigmatism of 4 D, the refraction in one of the meridians is always four dioptries stronger than in the meridian which is perpendicular to it. A concave cylinder number 4, with its axis perpendicular to the meridian of strongest refraction, or a convex cylinder number 4, with its axis perpendicular to the meridian of weakest refraction, equalises the refraction of the two.

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[1] Donders, *loc. cit.*, p. 532.

After having thus studied the properties of the dioptric apparatus of the eye, let us now add to it the retina, as before (p. 120 *et seq.*), and, according as it is nearer to, or farther from, the dioptric apparatus, the foci of the two meridians of the latter will be situated:

(*a.*) one *upon*, and the other *behind*, the retina;
(*b.*) one *upon*, and the other *in front of*, the retina;
(*c.*) *both behind* the retina but *at different distances* from it;
(*d.*) *both in front* of the retina but *at different distances* from it; and, finally—
(*e.*) one *behind*, and the other *in front of*, the retina.

In the first case (*a.*), one of the meridians is emmetropic, the other hyperopic. In the second (*b.*), one is emmetropic, the other myopic. Donders has given, to the form of astigmatism in which one of the meridians is emmetropic, the name *simple* regular astigmatism, and he divides it into *simple hypermetropic* and *simple myopic* astigmatism.

When both the foci are behind the retina, both meridians are hyperopic, and Donders calls this form *compound hypermetropic* astigmatism.

*Compound myopic* astigmatism is, then, that in which the foci of both the principal meridians are *in front of* the retina.

Finally, when the retina is between the two foci of the astigmatic apparatus, evidently one of the meridians is hyperopic and the other myopic. To this form of regular astigmatism Donders applies the qualifying term *mixed.*[1]

## DETERMINATION OF REGULAR ASTIGMATISM.

After all that we have said concerning regular astigmatism, the reader will find no difficulty in understanding the methods of determining this particular form of ametropia.

Let us simply mention again a few characteristic points which it is essential to bear in mind when having to do with the vision of astigmatic persons and with its correction.

In regular astigmatism the refraction, regular in each isolated meridian of the eye, differs from one meridian to the other.

It attains its *maximum* and *minimum* in two meridians, called *principal meridians*, which are perpendicular to each other. The *degree of the astigmatism* is expressed by the difference in refraction that exists between the two principal meridians.

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[1] Donders, *loc. cit.*, pp. 514-531.

If either of the *principal meridians* is adapted to a *luminous point*, the latter forms upon the retina a line of diffusion, or *focal line*, which is *perpendicular to the adapted meridian and parallel with the non-adapted one*.

If the focus of one of the *intermediate meridians* falls upon the retina, the image of the point is an *ellipse*, or a *circle, of diffusion*.

A *line* appears distinct to an astigmatic person when it is *parallel* with the meridian which is *not* adapted to its distance, because, in this case, the elongated images of diffusion, of all the points composing the line, coincide with the image of the line itself.

On the contrary, the line is seen *indistinctly* when it has the *same direction* as the meridian which is adapted to its distance. The diffusion-images of its different points are perpendicular to the line itself and make it appear broadened and diffuse.

## SUBJECTIVE DETERMINATION OF REGULAR ASTIGMATISM.

The presence of astigmatism is generally already revealed during the determination of refraction, especially if this is made simultaneously with the examination of the visual acuteness (see p. 220).

The astigmatic person having reached the limit of his acuteness of vision, or even before this, commits errors into which an eye with regular surfaces would never lead its possessor. He mistakes, for each other, letters that, for the normal eye, do not seem even similar. Among the test-types of the same series he easily distinguishes some, and does not succeed at all in recognising others; while he reads, again, much smaller letters in another series. This peculiarity is sometimes so pronounced that it might lead one to suspect simulation, if it were not explicable by the irregular refraction of the eye.

A little reflection teaches, in fact, that an astigmatic eye, which receives from each point of a letter a *line of diffusion*, obtains from such letters an impression differing from that received from them by an eye that sees them distinctly or as *circles* of diffusion. Thus it is that an O may resemble a Z, if the vertical meridian is not adapted, and the upper and lower lines in the O are broadened by diffusion-images. If, on the contrary, this diffusion is produced in the horizontal direction, the broadening of the lateral, vertical lines of the O may make this letter appear like an H.

Moreover, it is conceivable that a large letter may be less easily recognised than a small one, if the peculiar direction of the lines of diffusion is less favourable to the distinguishing of the shape of the letter in the first case than in the second.

Another characteristic sign of astigmatism is that the successive change of spherical glasses generally exerts less influence upon the acuteness of vision than is the case in non-astigmatic eyes. Cases are met with in which it is difficult to decide between glasses, whose refractive powers differ by several dioptries, as to which gives the greatest acuteness of vision. This is explained by the faculty, that certain astigmatic persons possess, of causing the focus of either of the principal meridians to fall upon the retina. While on this topic it is, however, of importance to note that *an eye sees better with lines, than with ellipses or circles, of diffusion.* This is true to such an extent that myopes are noticed to render themselves artificially astigmatic by the inclination of their spectacle-glasses, when these are too weak. In this way they exchange the diffusion-circles of their insufficiently corrected ametropia for the diffusion-lines of an acquired astigmatism.

The explanation of this fact is as follows:—A spherical lens (concave or convex), by being turned about one of its diameters as an axis, acquires an increased refractive power in the meridian which is perpendicular to such axis. In the meridian that is parallel with this axis, the increase of refractive power is imperceptible. Thus a myope, who inclines his spectacle-glasses around their horizontal axis, obtains the same effect as if he added, to his spectacles, concave cylindrical glasses with their axes horizontal. That is to say, he produces an astigmatism in which the vertical meridian is less refractive than the horizontal. Inversely, when he looks obliquely through his glasses in the horizontal plane he increases their effect and diminishes the refraction of his eyes in this meridian.

It was for a long time believed, by Donders among others, that this stratagem, so common with myopes, was designed to correct an existing astigmatism. But Mauthner calls attention to the fact that it is to be observed in all cases of insufficiently corrected myopia, and he proves that vision is really improved by the production of this regular astigmatism. The same is true in hypermetropia. This experiment, that any one can make, explains the fact, set forth by M. Javal, that astigmatic people try to bring one or the other of their focal lines on the retina,—*i.e.*, they adapt themselves preferably for one or the other of their principal meridians. It is under such circumstances that they see best.

If, now, we wish to determine the astigmatism, we proceed exactly as for the determination of the static refraction by means of the acuteness of vision. After having excluded one eye, we seek the spherical glass that gives to the other the greatest visual acuteness for distant objects. If there be a hesitancy about choosing between several glasses, we select the weakest concave or the strongest convex.

According to what we have said above, this glass, for the very reason that it increases vision to the maximum, must correct one or the other of the principal meridians, by bringing one or the other of the focal lines upon the retina. It only remains now for us to correct the ametropia of the other meridian—reduce to a point the focal line that corresponds to it—and the eye is wholly corrected and emmetropic.

In order to exert an influence upon a special meridian, it is evident that a glass which refracts in one special plane is required. This is the case with *cylindrical* glasses. Since it presents a curvature only in the section perpendicular to, and not in the one that is parallel with, the axis, such a glass exerts its principal influence upon the meridian that is perpendicular to its axis.

Hence we add, to the spherical glass, a cylinder whose curvature is parallel with the already corrected meridian and perpendicular to the uncorrected one. To this end the direction of the principal meridians must be known. This we ascertain by presenting to the eye, at the same distance with the test-types, a figure composed of black radiating lines on a white ground. These lines must be exactly alike, and it is best to have them equidistant (Javal, Green, Snellen).

Each line should bear a number indicating its inclination to the vertical or to some other direction previously agreed upon. I prefer the division that has the vertical for its starting-point. This vertical, then, is numbered 0°, and the lines to the right and left bear corresponding numbers up to 90°, which is horizontal (Snellen).

Thus we do not need the division below the horizontal, for, when a line gets below it, its inclination to the vertical is marked on the other side of the vertical—90° + 10° = 100° to the right of the vertical (Fig. A), corresponds to 90° − 10° = 80° to the left of the vertical (Fig. B).

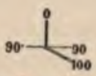

FIG. A.

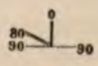

FIG. B.

Javal makes the division begin at the left end of the horizontal meridian, and carries it to the right end, the latter necessarily corresponding to 180°, and 90° being at the summit of the graduated semicircle.

The patient is asked which line he sees best and which one appears to him least distinct and least black. The former is parallel with the meridian that is not properly adapted and the latter to the meridian that is rendered emmetropic.

Let us suppose the spherical glass, that gives the greatest visual acuteness, to be the concave 3 D, and that, with this glass, the vertical lines are those that the patient sees best. We know from this that the horizontal meridian is corrected by the concave 3, that it is myopic by three dioptries, while the vertical meridian is not corrected.

Hence we must add to the spherical glass a cylinder with its axis horizontal. Ought this cylinder to be concave or convex? If the refraction is stronger in the vertical than in the horizontal meridian, a concave cylinder is evidently demanded; if it be weaker, a convex is called for. But we cannot know beforehand which it is. It is true that, in the majority of cases, the vertical meridian, or one near it, is strongest and that one near the horizontal meridian is weakest in refraction.

Hence, in the above supposed case, we will try a concave cylinder, the $-0{\cdot}5$ D or $-1{\cdot}0$ D, for instance, its axis being placed horizontally. If the lines appear less unlike, or if, at least, the difference in distinctness is not more pronounced than without the cylinder, we take a stronger one, and continue doing so until the lines all appear equally distinct. We know then that the vertical meridian in this case is more strongly myopic than the horizontal, by an amount expressed by the number of the cylindrical glass.

This glass evidently represents the difference in refraction between the two principal meridians; hence it expresses the *degree of the astigmatism*. If it be the concave cylinder 2 which corrects the excess of refraction of one of the meridians over the other, we say that the astigmatism amounts to two dioptries. In the example chosen there would have been—in the horizontal meridian, a myopia of 3 D; in the vertical meridian, a myopia of $3+2=5$ D; and astigmatism, $5-3=2$ D.

Suppose the concave cylinder, added to the concave spherical glass, had rendered the horizontal lines more indistinct. From this we should have concluded that, far from being more refractive than the horizontal, the vertical meridian was *less* so, and we should have tried convex cylinders, placing their axes horizontally. If with the convex cylinder number 2 the equality of the radiating lines had been obtained, we should have said, the spherical $-3$ D, which corrects the ametropia of the horizontal meridian, is too strong for that of the vertical. The latter is, therefore, less refractive than the former, since we must, for it, diminish the power of the concave spherical glass by the addition of a convex cylinder, and, if the myopia amounts to 3 D in the horizontal meridian, then it amounts to only $3-2=1$ D in the vertical one. So the degree of astigmatism would still have been 2 D, *i.e.*, equal to the number of the cylindrical glass; only, the vertical meridian would have been the less ametropic of the two.

The myopia amounting to 3 D in the horizontal meridian, and 1 D in the vertical, it can just as well be corrected by a concave spherical 1, combined with a concave cylinder number 2, axis vertical, as with the spherical − 3, and the cylinder + 2, axis horizontal. The optical effect is the same in both cases, but the first combination is more natural than the second.

We should never content ourselves with simply seeking the cylinder that makes all the lines appear equally distinct. We watch with the greatest care the influence of this glass upon the acuteness of vision. The opinion of patients as to the greater or less distinctness of the different lines is generally too uncertain for us to trust to it absolutely. Hence we have the patient look also at the test-letters which we place by the side of the card upon which are the radiating lines. If the cylinder really corrects the astigmatism, the visual acuteness must be better with it than with the spherical lens alone.

This increase in visual power is often considerable, even when due to weak cylinders. It ought to be especially so when high degrees of astigmatism have been corrected.

We consider the examination of astigmatism by means of radiating lines only as a preliminary means of determination, employed with the purpose of ascertaining the direction of the principal meridians. The perfect and final correction is only obtained after checking the former observation with the aid of the visual acuteness.

Thus, having found the spherical and cylindric glasses that equalise the refraction in all the meridians of the eye, in this sense, that the eye no longer sees any difference between the several lines, we introduce these glasses into the grooves of a trial-frame. The part into which the cylinder fits turns in the plane of the glass by means of a rack and pinion and an endless screw. We give to the cylinder the inclination found, from the lines, to be desirable, meantime asking the patient to try if, by slight rotations of the head of the screw, he can find a position of the cylinder that is still more favourable. This being regulated and verified by examination of the visual acuteness, we place, in front of this combination of glasses, weak spherical lenses, to see if the correction cannot be still more improved.

It very often happens, especially in the case of young people, that throughout the duration of the search for the astigmatism, there has been a spasm of the accommodation, which relaxes more or less after the correction of the astigmatism. The patient sees as well, if not better, with weaker concave or stronger convex glasses. The preceding examination has not, on that account, been in vain. The astigmatism remains the same, only we choose weaker correcting glasses. Suppose that, in our first example, the patient does not experience any change in his visual acuteness when we add the convex spherical number 0·5 D to the glasses that appeared to correct his total ametropia. We know from this that the combination was

0·5 D too strong in all the meridians. Instead of the concave spherical 3 D, we now give him the concave spherical $3-0·5=2·5$ D, leaving, of course, the cylinder 2 unchanged.

In the same way we can ascertain, by the addition of weak cylinders placed with their axes parallel with that of the correcting cylinder resulting from the first examination, whether or not the latter has been well chosen, *i.e.*, whether or not it ought to be slightly stronger or weaker.

It is always well to repeat the examination, on another day, before prescribing the correcting-glass.

We very seldom have recourse to atropisation for the determination of astigmatism. It may become necessary, if the patient's answers are very unsatisfactory, if these differ in different examinations, if a marked difference exists between the results of the subjective and of the objective examination of astigmatism, or, finally, if the persistence of asthenopia makes us suspect that there is a spasm of accommodation. This spasm may affect the entire ciliary muscle, or only certain portions of it, producing an effort of accommodation in one meridian only. In either case the artificial paralysis of the accommodation not only makes the asthenopia disappear, but also renders the exact determination of the astigmatism possible.

It is not, however, always certain that the correction thus found will be borne by the eye immediately after the cessation of the action of the mydriatic. Sometimes another combination is more suitable for it. Hence the examination should be repeated without the mydriatic, which is very easy when the real astigmatism of the eye is known. In certain cases, which we shall explain hereafter, it is necessary to accustom the eye to the glass that corrects the astigmatism, as determined during paralysis of the accommodation.

Instead of lines arranged like these on a compass-card, we may make use, in determining the principal meridians, of groups of lines such as O. Becker has suggested. This test-card has on it four rows of groups, each group being composed of four lines. These lines have various directions, and represent, so to say, the compass-card decomposed into its radii. The patient usually indicates with readiness which groups of lines appear to him most distinct, and those which seem to him most diffuse.

The same author has suggested, as a means of examination, a series of concentric circles, which appear like an hour-glass in the meridian that is not adapted to the distance of this test-object. Others, as Pray and Heymann, have made letters composed of parallel lines, having various directions.

Finally, L. Purves has devised a rather ingenious means of determining the principal meridians. We have already had occasion to speak of the principle involved, when discussing Thomson's second optometer. It consists of two luminous points, one of which revolves around the other. Purves obtains them by means of a screen placed in front of a lamp. On the screen is applied a disc which turns about a small round opening as a centre. There is a similar opening near the periphery of the disc. When the latter is turned, the opening moves over a semicircle cut in the screen. In this way the opening is always illuminated through an arc of 180°, and the rest of the semicircular opening in the screen is covered by the disc. The two openings appear to the normal eye, looking at them from a certain distance, as two luminous points. The astigmatic eye sees them as two luminous lines. When they have been so placed, that each line is the prolongation of the other, they indicate the direction of one of the principal meridians. The other is perpendicular to this, and would correspond to the parallel direction of the two lines of diffusion.

Some ophthalmologists have sought to determine astigmatism directly by means of any weak cylindrical glass. It is rotated in front of the eye, which is previously provided with the spherical glass that gives the greatest visual acuteness for distance.

Mauthner[1] still accords the preference to this method. He uses a concave or convex cylinder of about 1 D. This cylinder diminishes the vision of a normal eye, or of one rendered so by its correcting glass, whatever be the direction in which it is placed in front of this eye. If there be astigmatism, however, the influence of the cylinder is not the same in all meridians.

Let us suppose the two principal meridians to be vertical and horizontal, that both are myopic, but the vertical more so than the horizontal, and that the myopia of the latter is corrected by a spherical glass. On rotating a concave cylinder in front of this glass, the visual acuteness will increase in proportion as the axis of the cylinder approaches the horizontal, because it corrects a portion of the excessive refraction of the vertical meridian. If we place the cylinder vertically, vision will be diminished, for, in this case, there is over-correction of the horizontal meridian or, at most, vision would remain the same, if the accommodation neutralises the effect of the cylindrical glass. We therefore place cylinders, of successively increasing strength, with horizontal axes, in front of the eye, until we obtain the maximum acuteness of vision.

It may also happen that the cylinder, thus chosen at hazard, diminishes vision whatever its inclination, but more so in one than in the other of the principal meridians. In this case we would evidently change it for another of opposite effect, convex instead of concave or *vice versâ*, and the examination will be made in the same way as the one above described.

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[1] Mauthner, *Die optischen Fehler des Auges*, p. 739 *et seq.*

But in the higher degrees of astigmatism, the patient can scarcely perceive the change, in his visual acuteness. produced by a weak cylinder. At all events this procedure does not satisfy us, and, were it not recommended by so good an observer as Mauthner, we should not have supposed that it could have a really practical value.

To facilitate the changing of the spherical and cylindrical glasses, Javal has mounted them on two large discs that can be turned one at a time, or simultaneously, in front of the eye examined. The cylindrical glasses can be turned in their settings by means of a rack and pinion, according to a principle that Schœler[1] has made use of for the same purpose in his ophthalmoscope.

It is also possible to measure astigmatism without cylindrical glasses, by determining refraction, with the aid of spherical glasses, successively in one and in the other principal meridian. Thus, after having found the direction of the principal meridians by means of one of the foregoing methods, a diaphragm with a narrow straight slit is placed in front of the eye, the slit being placed in the direction of one of these meridians. The latter is now isolated and its refractive power is determined by means of spherical glasses and the acuteness of vision, following exactly the same principles as for a non-astigmatic eye. Then the other principal meridian is isolated in turn, by giving the slit a direction perpendicular to its former one, and the same examination is repeated.

Suppose that we have, in this way, found a hyperopia of 2 D in the vertical meridian and a hyperopia of 3 D in the horizontal one. We have here an astigmatism of one dioptry, which we shall correct with a spherical convex 2, combined with a convex cylinder of 1 D, its axis placed vertically. This method is easy to put in practice with the author's ophthalmoscope, which, in this case, serves as a subjective optometer; the mirror is exchanged for the diaphragm with a slit, and the latter is given the direction of one of the principal meridians. The finding of the proper correcting-glass is rapidly accomplished, owing to the facility with which the glasses in the Recoss-discs may be changed.

Without isolating the principal meridians by means of a slit, the refraction of each may be separately determined, one after the other, by means of spherical glasses, the patient fixing one of the radiating figures of which we make use in determining the principal meridians.

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[1] *Jahresbericht der Augenklinik von* Dr. Schœler, 1875, p. 55.

We first look for the weakest concave or strongest convex spherical glass, that makes the lines, corresponding to one of the meridians, distinct. Then we determine the glass that clears up the lines perpendicular to the former,—*i.e.*, which corrects the other meridian. Suppose we have found the convex spherical 1 D for the vertical lines, and the concave spherical 2 D for the horizontal ones. We know that the eye examined is hyperopic by one dioptry in the horizontal, and myopic by two dioptries in the vertical meridian. Its astigmatism amounts to $1 + 2 = 3$ D, and may be corrected by a convex cylinder of 1 D, axis vertical, combined with a concave cylinder of 2 D, with its axis horizontal.

It is in accordance with this principle that astigmatism is determined with most of the optometers which, in determining refraction make use of the acuteness of vision. They generally contain a stellate figure and, having no cylindrical glasses, they give the astigmatism by the successive determination of refraction in the principal meridians, by means of the lines that correspond to them.

This method may give good results, especially if we take care to check the final correction by the acuteness of vision obtained by the eye with the combined, spherical and cylindrical, glass. But we still prefer the first process of determination, inasmuch as, in examining the two principal meridians separately, the risk is run that the eye may make different accommodative efforts in the two examinations. In this case—which is not, however, the rule—the degree of astigmatism found is necessarily less than the real degree.

We have mentioned, when discussing them (p. 250), how optometers, not based upon the visual acuteness, may serve for the determination of astigmatism.

An optometer designed specially for the examination of astigmatism, and which serves the purpose very well, is Javal's stereoscope-optometer, described in the *Annales d'Oculistique* (vol. liii., p. 58, 1865), and in De Wecker et Landolt's *Traité complet d'ophthalmologie*, vol. i., p. 689.

## OBJECTIVE DETERMINATION OF REGULAR ASTIGMATISM.

The objective determination of astigmatism, like that of the refraction in general, is best accomplished with the ophthalmoscope. It is only in cases of aphakia that the direct measurement of the curvature of the sole refracting surface may be preferable.

When the refraction differs in two meridians of the eye, this irregularity must influence the retinal images, as we have seen, and likewise the ophthalmoscopic images, as we shall see farther on.

In the first place, an emmetropic observer, or an observer provided with his correcting-glass, will never be able to see distinctly all parts of the ophthalmoscopic image at once, because he is thus adapted to only one or the other meridian at a time, but never for both simultaneously.

Just as in the subjective examination, the adaptation of a meridian will be ascertained by means of a line that is perpendicular to it.

Horizontal lines are seen distinctly when the vertical meridian is properly adapted, and *vice versâ*. Suppose an eye, emmetropic in its horizontal meridian and myopic by 2 D in its vertical meridian. An emmetropic observer is adapted, with the naked eye, only to the horizontal meridian. Hence he will see distinctly the vertical lines at the fundus of the observed eye, retinal vessels which have that direction and the lateral borders of the papilla. If, on the other hand, he provides himself with a concave glass of 2 D, he thereby adapts his eye for the vertical meridian, but forfeits distinct vision for the horizontal one. The horizontal vessels and the upper and lower borders of the papilla will be clear, but the vertical vessels and vertical outlines of the optic papilla will have become indistinct.

At the same time the enlargement of the ophthalmoscopic image is different in the principal meridians. If the eye be examined in the erect image, its fundus, in the meridian whose refractive power is least, is seen with a weaker convex glass than in the meridian that is perpendicular to it. Hence the enlargement is less in the first than in the second case.

In our example, where the most refractive meridian is the *vertical*, objects at the *fundus oculi* will appear specially magnified in the vertical direction. The papilla, if it be round in reality, will assume, in the examination of the upright image, the form of a vertical oval.

The contrary is the case for the *inverted image*. We know, indeed, that the inverted image is smaller in proportion as the refraction of the eye is stronger.[1]

Thus, having its maximum in the highest degree of hyperopia, the size of the inverted image diminishes in direct ratio with this refractive anomaly, passing through emmetropia into myopia, and continues to diminish as the degree of the latter increases.[2]

Consequently the papilla in our example, being really circular, will appear, when examined in the inverted image, elongated horizontally. If we have just been examining it in the erect image, in which case it appeared like an oval with vertical major axis, this change of aspect cannot fail to strike us. The comparison of the forms of the papilla, when examined directly and indirectly, constitutes, indeed, one of the most important diagnostic signs of astigmatism (Schweigger). If the papilla had a constant form—circular, for instance—examination according to one of these methods would suffice for the recognition of astigmatism; but since its shape varies in reality, it is well to make the comparative examination. A papilla that is really oval in shape, and whose major axis is vertical, will appear still longer in the erect image when the vertical meridian is the most refractive. If examined by the indirect method, it will appear, at all events, less elongated, perhaps circular or even longer in its horizontal diameter.

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[1] Landolt, in De Wecker et Landolt's *Traité complet d'ophthalmologie*, vol. i., pp. 800-828.

[2] It is here taken for granted that the convex lens is never to be withdrawn farther than its focal distance from the eye examined. If the distance separating the first principal point from the lens is greater than the focal distance of the latter, the phenomenon becomes the opposite.

The size of the inverted image of an astigmatic eye varies, moreover, unequally in different diameters, according as the convex lens is withdrawn from, or carried toward, the eye (Javal). When this lens is placed very close to an eye whose vertical meridian is the most refractive, the papilla, circular in reality, appears like a horizontal oval. On withdrawing the lens gradually, the form of the papilla approaches more and more that of a circle and becomes quite round when the distance between the lens and the first principal point of the eye (1·75 millimetre behind the cornea) is equal to the focal distance of the former. If the lens is farther withdrawn, the papilla takes on the form of a vertical oval which becomes more and more elongated.

The image of the *fundus oculi* is formed at different distances, according to the refraction of the meridian to which it is due. It is formed *nearer* to, or *farther* from, the eye under examination, according as it corresponds respectively to a *more* or *less* refractive meridian. In our example, the horizontal outlines of the optic papilla, as well as the horizontal vessels, will form images nearer to, and the vertical vessels and outlines farther from, the eye. If one of the meridians is myopic and the other emmetropic or hyperopic, the image due to the former is real, inverted and situated in front of the eye, while that of the latter is virtual, erect and situated behind the eye (Cooper).

Finally, we have yet to consider the symptoms of astigmatism afforded by examination of the retinal reflex, by koroscopy.

The presence of astigmatism is revealed, in pupilloscopy, by the fact that the phenomena of illumination are not the same in all meridians of the eye. On rotating the mirror about its own axis in front of a non-astigmatic eye, the pupillary lustre always moves in the plane of the mirror's rotation, whatever be the inclination of such plane. It seems, when we use the concave mirror, to move in the opposite direction in cases of hyperopia and in the same direction in myopic eyes.

In the astigmatic eye, it may happen that the pupillary reflex seems to move obliquely, relatively to the mirror's plane of rotation. This is the case when the mirror turns in the direction of one of the intermediate meridians. This phenomenon is explainable as follows:—

The difference in refraction between the meridians gives, to the reflex from the *fundus oculi,* the form of an oval directed parallelly with one of the principal meridians. The line of demarcation between the lustre and the shadow has the same direction, and this remains invariable, whatever be the inclination of the axis of the mirror. Since the observer does not see the luminous oval in its entirety, but only one or the other of its edges, he has the illusion that the line, along which the pupillary brightness is displaced, forms a certain angle with the plane in which he rotates the mirror, when this plane does not coincide with either of the principal meridians. The pupil seems to be traversed by a light, the outlines of which are oblique, that is, parallel with one of the principal meridians, and which moves in the same plane as the mirror, but less rapidly than if the latter revolved about an axis parallel with one of these meridians.

The direction of the latter is therefore shown by the line of demarcation of the pupillary lustre, as well as by the inclination that one is obliged to give to the axis of the mirror, in order to obtain the pupilloscopic phenomena of the maximum and minimum refraction.[1]

As to the nature of the refraction of the two principal meridians, it is characterised by the same phenomena as non-astigmatic refraction. Suppose the principal meridians to be vertical and horizontal. If, when the concave mirror is turned about its vertical axis, the lustre appears dull, and moves slowly in a direction opposite to that of the mirror, we conclude that the horizontal meridian is strongly hyperopic. When the lustre, and, consequently, the shadow, are intense and move rapidly, the hyperopia of the horizontal meridian is weak.

If, with the concave mirror, oscillating about a horizontal axis, the pupillary lustre seems to be displaced with great rapidity in the opposite direction, the difference between the light and the shadow being almost imperceptible, this proves that the vertical meridian is nearly, if not quite, emmetropic. If the ocular lustre appears and disappears at the same end of the diameter of the pupil as the reflex from the mirror; if, in short, it moves in just the same direction as the latter, the meridian examined is myopic (Charnley).

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[1] Charnley, *loc. cit.*, pp. 363-366.

It is easy to imagine how the phenomenon will present itself in myopic astigmatism, whether simple or compound, as well as in mixed astigmatism.

Here, again, the phenomena, as we have outlined them, are those for a concave mirror held at a distance greater than that of its focus. With the plane mirror, the ocular lustre moves in the opposite direction.

All these symptoms have their value for showing the presence of astigmatism, and they are so easily produced that one does well to have recourse to all of them when in doubt as to whether there be astigmatism or not. In this way one may succeed even in ascertaining the *direction of the principal meridians.*

*Ophthalmoscopic determination of the degree of Astigmatism.*—This we accomplish again most easily by seeking the spherical glasses that correct, successively, the two meridians.

Let us take an example: without knowing whether or not an eye is astigmatic, we wish to ascertain its refractive condition. With the ophthalmoscopic mirror we illuminate the eye. While yet at some little distance from the eye, we distinguish, in the red of its fundus, a few details, as retinal vessels or a portion of the papilla. These objects seeming to move in the same direction as our head, if we move the latter slightly, we know that we see them in the erect image; which is confirmed by the fact that they grow more and more distinct as we approach nearer to the eye under examination. Being ourselves either naturally emmetropic, or made so by the correction of our ametropia, we conclude that the examined eye is either emmetropic or hyperopic. In order to assure ourselves of this, we cause to pass in front of our own eye stronger and stronger convex lenses in the ophthalmoscope. With several of them we see as well as without a glass. There are, however, always certain portions of the fundus that are not so distinct as others.

By paying close attention, we notice that neither the lateral margins of the papilla nor the vertical vessels are ever clearly defined when we see the horizontal vessels and borders of the papilla distinctly. Hence we know that we have here a case of hyperopic astigmatism and that the principal meridians are vertical and horizontal. In order to be more sure of this, we might still resort to the inverted image.

We have, therefore, to determine the refraction in two meridians of the eye. Let us commence with the vertical. In our example we seek the strongest convex glass with which we see most distinctly the horizontal vessels and the upper and lower borders of the papilla. We find that it is the number 2. Hence the examined eye is hyperopic by two dioptries in its vertical meridian.

We proceed in the same way to determine the refraction of the horizontal meridian. We have already noticed, during the preceding examination, that the convex glasses that we tried, for the purpose of correcting the vertical meridian, did not render less distinct the vertical lines of the fundus. From this we conclude that the horizontal meridian is not myopic, and we endeavour to correct it by continuing to pass stronger and stronger convex glasses in front of our eye. The vertical vessels appear more and more distinct, and finally we fix upon the convex 5, as giving the clearest definition to the vessels thus directed, as well as to the lateral margins of the optic disc. The horizontal meridian is, therefore, hyperopic by 5 D and the astigmatism amounts to $5-2=3$ D.

Cases of this kind are not rare; we very often find a strongly hyperopic eye to be at the same time astigmatic, and, in such cases, the astigmatism may reach very high degrees.

It has been, indeed, among hyperopes that I have found the greatest differences of refraction in the different meridians of the eye. It is in such cases, too, that the objective determination of astigmatism achieves its greatest triumphs, inasmuch as the visual acuity is often so slight that ametropes of this kind scarcely appreciate either a difference in the distinctness of the various lines or the effect produced by the different glasses, and no confidence can be placed in their answers. When once the astigmatism has been corrected, or nearly so, by means of the ophthalmoscopic determination, the acuteness of vision is considerably improved and the inclination of the meridians, as well as the strength of the correcting glasses, may be still farther rectified, if necessary, by the subjective examination.

Schœler has added to the ophthalmoscope a mechanism whereby a series of cylindric glasses may be successively superposed on the spherical; each glass being so arranged that it may be rotated on its own axis. Having found the spherical that corrects one of the principal meridians, he adds to it the cylinder that corrects the other.[1]

Astigmatism may also be recognised objectively by means of the *image formed on the fundus of the examined eye*, and that as well with the erect as with the inverted image.

Thus, by placing, in front of the luminous source of the ophthalmoscope, a ring, crossed by threads stretched in its different diameters, this stellate figure is depicted upon the fundus of the examined eye as well as the flame in front of which it is situated. If the eye reflects light normally, all radii of the star appear equally distinct at the *fundus oculi*. If the eye be astigmatic, some of them will be more distinct than others, and the meridian that is best adapted to the rays reflected by the ophthalmoscopic mirror is parallel with the most diffuse line. The cylinder which, placed in front of the examined eye, equalises the clearness of all these lines, evidently expresses the degree of the astigmatism. Sometimes it will be necessary to add a spherical glass to it, in order to obtain the maximum clearness of the image.

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[1] Schœler, *Jahresbericht seiner Augenklinik*, 1875, p. 55; and De Wecker et Landolt, *loc. cit.*, i., p. 849.

This combination of the spherical with the cylindrical glass would give the correction of the total ametropia, but only in case the rays reflected by the mirror are parallel. To procure this state of things, the stellate figure must be placed at the focus of the concave mirror.

The latter has, however, one disadvantage when used in this way; if it be inclined ever so little, its action is greater in one direction than in another, just as in the case of a lens through which one looks obliquely. Under such conditions it produces, itself, phenomena similar to those of astigmatism and may thus lead the observer astray.

In other respects the method does not, moreover, seem to us to be very practical for the determination of the degree of astigmatism. But it may very well serve for the demonstration or diagnosis of this form of ametropia.

Coccius employs, for this purpose, simply a pencil, which he interposes in different directions between the flame and the mirror. Regular astigmatism is present when the shadow of the pencil at the fundus of the examined eye is not equally well-defined in all directions and the greatest differences in clearness indicate the directions of the principal meridians.

For determining refraction successively in the two principal meridians, a combination of Stokes' lens and the ophthalmoscopic mirror might be employed, as has been suggested by L. Purves. Stokes' lens[1] is composed of two cylinders of equal power,—one concave and the other convex. They turn on each other in opposite directions, and, according as they more or less neutralise each other, produce a cylinder whose strength varies from zero to the sum of the two primitive cylinders.

Finally, we have seen (p. 274) that observation of the ocular lustre from a distance (koroscopy) may also furnish symptoms from which astigmatism may be diagnosed.

In order to determine the *degree* of astigmatism by koroscopy, one proceeds, successively, for one and then for the other principal meridian, following the same method that is employed in determining the refraction of a non-astigmatic eye,—that is to say, one seeks the glass which makes the movements of the pupillary lustre disappear in one of the meridians and then the glass that suspends them in the other meridian. Or, again, one meridian having been rendered emmetropic, —in other words, corrected—by a spherical glass, the other is corrected by the addition of a cylinder.

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[1] De Wecker et Landolt, *Traité complet d'ophthalmologie*, vol. i., p. 688.

It may be asked if it be not possible to obtain, by direct measurement of the refractive surfaces of the eye, a more exact objective determination than the ophthalmoscope furnishes? We use the plural form "surfaces" advisedly, for, in speaking of astigmatism, we always understand that of the entire eye, and we know that it may be due to one or several of the dioptric surfaces. As no signs exist that indicate beforehand which of them all is the seat of this refractive irregularity, it is, therefore, necessary to measure them all in order to obtain the total astigmatism.

The cornea is so accessible to all kinds of observation that measuring its curvature is not attended with any difficulty, and may be done by means of any of the ophthalmometric principles that we have elsewhere explained.[1] Thus, it has become very simple, thanks to an instrument devised by Javal and Schiötz,[2] based on Coccius' ophthalmometer, and which gives pretty quickly the difference in refraction between the meridians of this first refractive surface of the eye. De Wecker and Masselon[3] and Seely propose to measure corneal astigmatism with the aid of an instrument similar to Placido's, which will be described farther on.

These instruments may render service when the dioptric system of the eye is reduced to the cornea alone, in *aphakia.* But in an immense majority of cases the determination of the curvature of the cornea is far from sufficient to acquaint us with the amount of astigmatism. We evidently require for this purpose to know, besides, the action of the *crystalline.* But it is infinitely more difficult to make out the lens's share in the astigmatism than that of the cornea. Various obstacles oppose themselves even to the determination of its form. The light, whose reflections inform us as to the curvature of its surfaces, must first penetrate to a certain depth into the observed eye; and these reflexes are, consequently, attenuated when they reach us.

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[1] *Ophthalmométrie,* De Wecker and Landolt, *loc. cit.,* vol. i., pp. 738-776, and iii., p. 309.

[2] Javal and Schiötz, *Congrès international d'ophthalmologie,* Milan, 1880. *Ophthalmomètre pratique* (*Ann. d'oc.,* July 1881), Javal, *Ann. d'oc.,* May 1882, July 1882, January 1883.

[3] De Wecker and Masselon, *Astigmomètre* (*Ann. d'oc.,* July 1882).

Thus the reflex from the anterior face of the crystalline (Figs. 63 and 65) is much less intense than the one from the cornea, because the light which produces it has passed twice through the cornea and aqueous humor. That from the posterior surface of the crystalline is so weakened by the media in front of it that, in order to employ it advantageously, solar or electric light must be used.

Moreover, these crystalline reflexes having once been obtained and measured, we cannot compare them simply with the object furnishing them, and thus draw direct conclusions as to the curvature of the surface, as is the case with the cornea. We must take into account the dioptric action exerted upon these images by the media which separate them from our eye. The first is magnified by the meniscus formed by the cornea and the aqueous humor; the second is subject to the influence of this meniscus and to that of the crystalline itself, through which we look at it. But, before becoming acquainted with their optical action, we must know their form, their dimensions and their indices of refraction. With the assistance of these data, we shall have to compute the actual size of the reflected image, which only then can be compared with its object, and thus serve in obtaining a knowledge of the surface in question.[1]

It will be seen, from this, with what complications one meets in the determination of the curvature of the crystalline and we may readily understand why so many observers have measured the cornea, and so few the crystalline.

Finally, in order to have exhausted all the possible etiology of astigmatism, it would still be necessary to determine the position of the crystalline, to see if it were well centred, or if, on account of its inclination to the axis of the eye, it did not exert a stronger optical action in one direction than in another. This determination would be no less complicated than that of the curvature of the crystalline.

We see that, as soon as we wish to go beyond the cornea, the determination of astigmatism, by direct measurement of the different parts of the dioptric apparatus, meets with difficulties that are incompatible with the necessities of ordinary practice.

It might be thought that, knowing, on the one hand, the astigmatism of the cornea, and, on the other hand, the total astigmatism, as found by one of the subjective methods or by the opthalmoscope, that of the crystalline could be obtained by subtracting the former from the latter. But it is easy to understand why such is not the case. The astigmatism of the crystalline directly increases or diminishes that produced by the cornea, only on condition that the principal meridians of both have the same direction. In any other case crystalline astigmatism is not so easily obtained.

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[1] De Wecker et Landolt, *loc. cit.*, vol. i., pp. 756-761.

It would be necessary, either to correct the corneal astigmatism, and proceed to a second *subjective* examination of the refraction, which would then reveal the astigmatism of the crystalline; or, after having determined the curvature of the cornea, as well as the refraction of the eye, successively in several meridians (Donders has done this for twelve different directions) to deduce from this, with the assistance of rather complicated calculations, the action of the crystalline.[1]

The refractive action of the cornea might also be suppressed by means of an orthoscope.[2] This is a small, tub-shaped glass cup, like an eye-cup, or open at the top and on one side. This open side is applied to the eye and the cup filled with water, which has the same refractive index as the cornea and the aqueous humor. The surface, of this little basin, opposite the eye, now becomes the first refractive surface, and, since it is flat, it does not change the course of luminous rays, which are refracted only by the crystalline.

Excepting the first, none of the methods mentioned could be used in practice for the determination of astigmatism of the crystalline.

But in the vast majority of cases, fortunately, it suffices to know the *total* astigmatism of the eye, without our needing to concern ourselves with the question as to what part of it is due to the cornea, and how much to the crystalline.

It is true that there are circumstances in which astigmatism of the crystalline deserves special attention. We know that this lens may participate passively in astigmatism, by its anatomical form (*static astigmatism of the crystalline*), or, in the second place, actively, by the unequal contraction of the ciliary muscle (*dynamic astigmatism of the crystalline*). This muscular spasm may at times give rise to the phenomena of asthenopia, and these disappear as soon as we replace, by a cylindric glass, the action that the crystalline can produce only at the expense of a pathological effort of the ciliary muscle. Hence it is the active part, taken by this muscle in the production or correction of astigmatism, that is important to know. We could not succeed in doing this otherwise than by exclusion of the accommodation, whether by the ophthalmoscopic examination or, what is better, by the use of mydriatics.

After having determined the general astigmatism of the eye, by means of one of the above-mentioned subjective methods, the accommodation is to be paralysed by instilling atropine or some other mydriatic, and the astigmatism again determined. In this way may be manifested in an eye an astigmatism that did not previously seem to exist, or a pre-existent astigmatism may be increased or diminished, or the principal meridians may change direction. All these modifications in the refraction of the eye are evidently ascribable to the change in form, *i.e.*, to the dynamic astigmatism of the crystalline. Here again, therefore, we accomplish what we desire only by subjective methods.

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[1] See the method of calculation due to Hueck and Buys-Ballot in Donders' *Anomalies*, p. 496.

[2] De Wecker et Landolt, *loc. cit.*, vol. i., p. 881.

## II.—IRREGULAR ASTIGMATISM.

In the normal eye, refraction is the same in every meridian. Through whatever meridian the rays pass, they are focused at the same point.

In regular astigmatism, refraction is still regular in certain planes passed through the axis. But the foci of these planes do not correspond. They are situated on a focal line which coincides with the axis of the eye. However unfavourable may be the influence of this optical defect upon the distinctness of retinal images, we have seen that it is corrigible, for the very reason that refraction is normally produced, at least in the two principal meridians, and that we possess optical means, in cylindric glasses, of producing refraction solely in any one desired direction.

There exists a more serious optic imperfection, *irregular astigmatism*. In this case refraction no longer presents any uniformity. It even varies oftenest in different parts of one and the same meridian, and may attain such an irregularity as to elude all analysis. If it is difficult, if not impossible, to foresee the form, or rather the deformity of retinal images produced by such a system, we can easily conceive, at least, that they can be neither distinct nor regular. The image of a point may present any imaginable alteration. It may appear as a shapeless and ill-defined spot; it may be stellate or, indeed, even multiple.

Any surface and any medium of the dioptric apparatus of the eye may become the cause of irregular astigmatism by irregularity of curvature, inequality of its index of refraction, lack of transparence in certain parts, imperfection of centering or of reciprocal arrangement. This last defect was mentioned (p. 119) at the end of the discussion of the dioptric apparatus of the eye.

The reader will remember that, in all calculations and optical deductions, we have always confined ourselves to the limited territory of a part of the eye very near its axis and possessing an irreproachable conformation.

In reality, only a small number of eyes answer to this ideal premise and, if one considered the entire surface of the cornea, probably no eye could be found which would be exempt from any irregularity in its curvatures, its transparence or its indices of refraction. Hence every eye would be tainted with a certain degree of irregular astigmatism. This is, as a matter of fact, the case. That it is not oftener spoken of and that we are not more affected by it, is due to the simple fact that all mankind is subject to the same defect, and that we only exceptionally meet with those privileged eyes whose infinitely superior acuteness of vision makes us feel all the imperfection of our own. And it has always been so. We have proof of this in the word *star* (Vedic Sanskrit), *(s)tārā* (classical Sanskrit), *çtare* (Zend), whence *ἀστήρ*, star, &c., which have always had the double signification of celestial bodies and radiating figures.

Now the former are spherical bodies. Their images, as formed by any optical instrument that is ordinarily exact, are circular. If man sees them surrounded by rays, so that he takes them for types of bodies bearing barbs, if he finds a striking resemblance between a star and a star-fish, a celestial and a marine star, this seems to prove that the optical apparatus of his eye leaves something to be desired.

The rays that almost every one sees about stars are in fact due to the irregular refraction that light undergoes in the eye. This defect in construction being extremely variable, it is not to be wondered at if stars appear differently to different persons. It is indeed interesting to ask different persons what form these bodies appear to them to have. A faithful reproduction of these appearances would give the best demonstration of the deformity, and even multiplicity, of images to which irregular astigmatism may give rise.

Still another example of this *polyopia* is given by a very simple experiment. Let one eye be closed, and, with the other, look at the balls of the thumb and index finger, which are being approximated slowly until they are in contact. The experiment is most successful when the thumb and finger are held between the eye and a light, which forms the background against which the following phenomenon is seen. Before the thumb and finger touch each other, it appears as if a film of black liquid were formed between them, by the intermediation of which they seem to run together. With a little attention, we easily see that this drop-like image is due to the apparent multiplicity of the contours of the thumb and finger, which approach, and are superposed upon, each other.

It is, therefore, a polyopia that produces this phenomenon and prevents perception of the precise moment at which two bodies enter into contact. This polyopia is due to the irregular astigmatism of the eye. It becomes especially marked when the eye is not adapted to the distance of the two approaching bodies, as, for instance, when it is fixing an object beyond them.

The proof that in this case as well as in that of the radii from the stars, we are dealing with a cause inherent in the eye, is that we can prevent the polyopia and deprive the stars of their radii, by placing in front of the eye a diaphragm in which is a small pin-hole. This minute opening marks off a portion of the dioptric apparatus, so restricted that its refractive defects are inappreciable and the luminous rays passing through it are focused without too much dispersion or aberration.

There are but few cases known of men who are entirely exempt from this defect and who see the stars without radii. The rest of humanity cannot boast of such perfection; therefore we act the wise part by speaking little of it, considering a low degree of irregular astigmatism as included in the normal. We become alarmed about it only when it reaches a sufficiently high degree to make visual acuteness fall below the average that we call normal. Despite this restriction, irregular astigmatism is far from being rare.

The cornea is very often the seat of this trouble. What we call conical cornea, whether congenital or acquired, does not consist solely in the projection of the central portion of this membrane, whose form then corresponds to that of a surface of revolution, but it is nearly always accompanied by a much more irregular deformity. We have the direct proof of this in the reflexes furnished by such a surface. If we place the patient opposite a window, whose image we observe upon the cornea, we see that this image, almost always very unlike the object, changes shape with each variation in the direction of the eye.

The same thing occurs for all irregularities in the form of the cornea, and we know how manifold they are. Each ulcer of this membrane, however superficial it may be, may leave a cicatrix that alters, not alone its transparence, but especially the regularity of its form. Light may enter the eye in floods, but, if its rays are not united in a focus, vision is not distinct. This is the reason why a scarcely perceptible opacity of the cornea sometimes interferes considerably with sight, much more so than the almost total occlusion of the pupil, provided the part remaining uncovered corresponds to a portion of the dioptric apparatus whose surfaces have a regular curvature.

Thus is to be explained also a fact that often enough disagreeably surprises the young oculist in his first operative ventures. I refer to the relatively slight increase in visual acuteness that is frequently obtained by even a well-placed and well-executed iridectomy, in the case of corneal leucoma. If we considered only the transparence of the sector of the cornea behind which the artificial pupil is made, we ought to expect an excellent acuteness of vision. But, unfortunately, this part of the cornea, surrounded by cicatrices, has undergone the traction and alteration of tissue that these entail and has thereby entirely lost the regularity of its form. The retinal images produced by rays that have passed through it are absolutely deformed and but poorly adapted to give a good acuteness of vision.

We may directly convince ourselves of this with the ophthalmoscope. Objects at the *fundus oculi*, as well as the image of the flame that we project upon it, are sometimes not to be recognised at all, and assume very odd shapes when the direction of the line of observation is changed.

Irregular astigmatism very often has its seat in the crystalline. Mauthner[1] has noticed, immediately after the ablation of a corneal staphyloma, multiple folds in the capsule of the lens, and thinks that the inequalities of this refractive surface, where they exist, must have an injurious influence upon the formation of retinal images.

But it is especially the peculiar structure of the crystalline[2] that becomes the cause of irregular astigmatism. This little organ is composed of several sectors. Now, however little they fall short, if at all, of being exactly adjusted to each other, so as to form a regular surface, each one will refract light differently from the others and will furnish its own peculiar image.

If the difference be great, the retinal images corresponding to the sectors of the crystalline will be far enough apart to be perceived separately. The individual sees several objects instead of a single one; there is polyopia. If the images partially overlap, there is indistinct vision, or polyopia, only for luminous points.

The result is the same when the different sectors do not have the same index of refraction. In this case, too, they refract light differently and produce irregular astigmatism, more marked in proportion as the inequality of the indices of refraction is greater.

The crystalline is the principal cause of the polyopia that we have pointed out as existing, physiologically, in nearly all eyes. It is likewise the seat of that which is frequently developed with age. We know, indeed, that the structure of the crystalline undergoes notable modifications, as well with respect to its elasticity (perhaps its form), as with regard to its index of refraction. The component sectors, almost

[1] Mauthner, *Die optischen Fehler des Auges*, p. 800.

[2] Compare p. 117.

impossible to differentiate in early life, become more and more distinct at an advanced age. By oblique illumination, their junctions, forming a sort of star in the pupillary field, may be easily seen.

This differentiation may become so pronounced as to be seen directly, without the observer needing to have recourse to concentration of light. It is not necessary, for this, that the crystalline shall have lost its transparence. If, nevertheless, the different parts are distinguishable from each other, they must certainly refract light differently. This is what gives rise to the most varied phenomena of irregular astigmatism. For some persons the stars seem to take on radii that they did not previously possess; others discover satellites about stars that they have always before seen isolated. Very often the moon, especially when crescentic, seems multiple. The same thing occurs for every very luminous object, notably in the case of street lamps, whose flames radiate in an unusual way and multiply. These phenomena become especially marked when a cataract commences to form and they increase when the pupillary field is extended by the use of atropine. They cease after the successful extraction, but make way generally for regular astigmatism, which results from the change in the form of the cornea that is occasioned by the wound left by the incision. This has been spoken of above.

## Determination of Irregular Astigmatism.

Irregular astigmatism reveals itself first by *subjective* symptoms, which we have already mentioned: weakness of the visual acuteness, monocular diplopia and polyopia and apparent deformity of objects looked at.

These phenomena manifest themselves *objectively*, sometimes by the indistinct aspect of the *fundus oculi*, observed with the ophthalmoscope, but especially by the deformity of the details of this fundus, which appear sometimes elongated, sometimes compressed, partially effaced, or stretched in the most whimsical way. This aspect changes, moreover, according to the direction in which the eye is examined, *i.e.*, according to the part of the dioptric apparatus that furnishes the ophthalmoscopic image.

The anomaly, in the form of the *cornea*, which produces irregular astigmatism, can at times be directly recognised with the naked eye, when viewed in profile, as in karatoconus. But it is especially the catoptric phenomena that furnish us information concerning the form of this refractive surface; they evidently correspond to the dioptric phenomena.

In the case of conical cornea, for instance, we see with the ophthalmoscope a circular shadow surrounding the central part of the red of the *fundus oculi.* This shadow corresponds to that portion of the cornea which, descending from the pointed summit toward the periphery of this membrane, is but slightly convex, affects especially a conical form and reflects light aside instead of allowing it to enter the eye. This median zone seems dark, too, on simple inspection of the eye, and the shining centre appears as if floating on a circular layer of a dark liquid.

The same is true of all irregularities of the cornea. They are most clearly manifested by reflective images produced by this membrane; one needs, for instance, only fix one's attention upon the image of a window or any other light reflected by a cornea which has been the seat of ulcers or traumatisms, in order to immediately recognise the irregular astigmatism of this surface from the odd form assumed by the image of the window or the multiplicity of reflections obtained from the light. Here, again, the phenomenon becomes especially convincing, when the position of the eye, relatively to the object, is changed so as to make the reflex pass over the whole extent of the cornea. The curvature of all its parts is thus examined.

Instead of using, as the object, a window or other source of light, it is quite natural to choose one having a regular form, and which is moveable, so that we may, by moving it toward or from the eye, modify the size of its image, and, by changing its position, change the place where its image is formed. Thus it is that, long before the invention of special instruments, we used a square and a disc of white cardboard in studying the form of the cornea.

The patient is placed with his back to the light, and the card, well illuminated, is brought toward the eye to be examined. The observer looks at the eye through a central opening made in the card. If regular corneal astigmatism be present, the image appears elongated in the direction of the least convex meridian and narrowed in the meridian of strongest curvature, as we have already explained. The circle becomes an oval, the square an oblong, when placed so that its sides are parallel with the principal meridians, and a trapezium when they are not. But the shape of the reflex differs much more sensibly

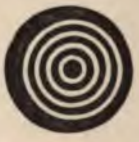

FIG. 114.

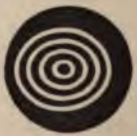

FIG. 115.

from that of its object, when irregular astigmatism exists. It may become so changed as to absolutely prevent its being recognised.

We have always used this method, for purposes of demonstration, in our clinical lectures. It has, however, become still more demonstrative for regular, and specially for irregular, astigmatism, owing to an application, of the principle, made by Placido.[1]

This author uses, as a "*keratoscope*," a disc of card-board, wood or zinc, 23 centimetres in diameter. On one side is drawn a series of concentric circles, alternately black and white. In the centre of the disc is a circular opening, 1 centimetre in diameter. The other side is all painted black and provided at the centre with a small perpendicular tube, 3 centimetres long.

The instrument is held and managed by a handle. The patient's eye and that of the observer should be at the same height. In order to observe the region of the visual line, the person examined is made to fix the centre of the instrument, the observer taking care meantime to direct his own eye in the axis of the tube.

The examined eye being in the shadow, and the disc well lighted up, the observer sees at a glance the images of as many circles as the disc contains black and white zones. The aspect is quite striking, and demonstrates, so to say, the form of the cornea in different regions. Thus, Fig. 114 gives the aspect of a regularly constituted cornea, Fig. 115 that of a regular corneal astigmatism.

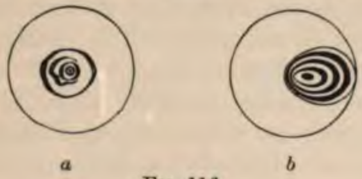

FIG. 116.

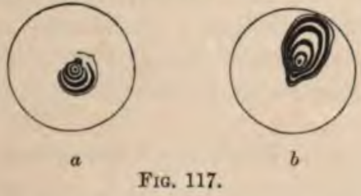

FIG. 117.

In Figs. 116 and 117 *a* and *a* correspond to the summits of two conical corneæ, *b* and *b* to eccentric portions of the same corneæ.[2]

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[1] Placido, *Nouvel instrument pour la recherche rapide des irrégularités de courbure de la cornée: l'astigmatoscope exploratoire* (*Period. di oftalm. pratica*, second year, Nos. 5 and 6, 1880).

[2] Hirschberg, *Centralblatt f. Augenheilk.*, p. 59, 1882.

Javal and Schiötz have adapted a similar disc to their keratometer. The breadth of the black and white circles increases according to the law of tangents, and the degrees are marked upon them. This disc may serve, according to these authors, not only for the demonstration of the corneal curvature in all points desired and well determined by the angle of deviation of the eye, but also for the measurement of that curvature, if a doubly refracting prism be added. The refraction of the cornea is then given by the circle whose double images become tangent to each other. It is here found inscribed, in dioptries, on one of the radii. One of the white zones, which corresponds to the region that is usually measured, is divided, by means of two black circles, into five parts. Each of these divisions corresponds to one dioptry.[1] By observing the cornea, at first in the centre and then in the vertical and horizontal medians, at equal degrees from the centre, Javal[2] obtained very demonstrative drawings for the normal cornea, as well as for its divers anomalies of form; notably in keratoconus, where the images furnished by the excentric portions affect the ovoid form represented by Figs. 116 and 117.

Irregular astigmatism of the *crystalline* is infinitely more dificult to study than that of the cornea.

Every time that we find symptoms of irregular astigmatism that cannot be attributed to the first refractive surface of the eye, they are placed to the crystalline's account.

Irregularities in the refraction of this lens are rendered manifest by oblique illumination, as well as by the ophthalmoscope. They may or may not be accompanied by opacities and are not rare. But the crystalline being but slightly accessible to practical investigations, they escape the precise demonstration and measurement which can be made of those of the cornea.

It is self-evident that there are multiple transitions between the two species of astigmatism that we have been studying. The form of the cornea or of the crystalline is not always as regular as the first kind requires, nor sufficiently abnormal, even in irregular astigmatism, so that there is no longer a single meridian that possesses a tolerably regular curvature.

Often, too, regular astigmatism of the cornea is accompanied by irregular astigmatism of the crystalline, and *vice versâ*. Although these causes frequently interfere with the result of the correction in the first case, the discovery of a meridian that is corrigible by a cylindrical glass, in irregular astigmatism, is often, too, the precious recompense of an attentive and conscientious examination.

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[1] Javal, *Contribution á l'ophthalmométrie* (*Ann. d'oc.*, lxxx., p. 213, 1882).

[2] Javal, *Troisième contribution à l'ophthalmométrie* (*Ann d'oc.*, lxxxix., p. 1, 1883).

# CHAPTER V.

## CLINICAL PORTION.

## I.—ANOMALIES OF REFRACTION.

IT is perfectly logical, correct and practical to classify the different refractive conditions of the eye as *hyperopia, emmetropia* and *myopia*. In this classification emmetropia represents the average *normal* refraction; ametropia, the refraction which deviates from this standard —hyperopia remaining below and myopia surpassing it.

But, from a clinical point of view, it would be wrong to consider emmetropia as the only physiological condition of the eye, having a pathological type on either side—hyperopia on one and myopia on the other.

We are not to suppose that, whenever an eye does not have its principal focus on the retina, it ought to cause anxiety and demand optical correction or even medical care. We have already seen that, for the requirements of practice, the limits of emmetropia have been notably extended. While in holding strictly to the physical definition, only such eyes as were adapted to parallel rays would be entitled to be called emmetropic, we admit in this group all those which, without accommodation and without a correcting-glass, see distinctly at a distance of 5 metres. Inasmuch as we make our optometric examinations at a distance of 5 metres, it follows that our emmetropia is, in reality, a myopia of 0·2 of a dioptry. Since, moreover, the convex and concave number 0·25 represent our weakest test-glasses, it follows that we give to emmetropia a latitude of more than 0·25 D.[1] If we do not use these very weak glasses, and are content with half dioptries, our emmetropia oscillates even between the limits of nearly one dioptry.

If we have thus extended the definition of *emmetropia*, we shall have to broaden still more the conception of *the normal eye.* Indeed, it would be surprising and contrary to Nature's habits that she should never, in the formation of our eyes, have erred by a millimetre in the length of the eye, or by a fraction of a millimetre in the radii of curvature of refractive surfaces. And this is all that is needed to make *the refractive condition of the eye* vary by several dioptries (see the Table on p. 140). Optical regularity, in order to be perfect, would require a mathematical accuracy which Nature scarcely ever realises.

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[1] If the examination of refraction is made at a distance of 5 metres, myopia amounting to as much as 0·37 D is still regarded as emmetropia, whereas myopes of 0·08 D are already considered as hyperopes. Indeed, only when, in the first case, myopia exceeds 0·37 D, will the person see better with the concave 0·25 D, for, with this glass, the circles of diffusion formed on the retina become smaller than those produced by the naked eye. On the other hand, as soon as the refraction falls below a myopia of 0·08 D, vision will be improved by convex 0·25, that is to say, the circles of diffusion become smaller than without a glass.

Hence a certain amount of deviation from this regularity does not necessarily imply a defect in the anatomical structure or the physiological working of the organ. And, in fact, the majority of slightly hyperopic or slightly myopic eyes are perfectly healthy. Their visual acuteness, after correction, differs in nowise from that of natural emmetropes, a fact which permits the conclusion that their nervous apparatus is irreproachably organised. The functions of the intrinsic and extrinsic muscles are executed no less normally, and the harmony between the two eyes—binocular vision—leaves nothing to be desired. Moreover, up to a somewhat advanced age, the accommodation suffices for the correction of a certain degree of hyperopia. So that, at least so far as distant vision is concerned, persons affected by this anomaly would generally not be aware of it, if the time for commencing to wear reading-glasses did not arrive sooner for them than for others.

This slight inconvenience does not exist in the case of myopes. It is true that, on the other hand, they do not see so well at a distance as emmetropes. A weak myopia is, of itself, sufficient to markedly decrease the range of vision. But it is likewise true that one rarely needs the maximum of visual acuteness for distant vision. Very few vocations, as those of coast-guards, pilots, surveyors, chamois hunters, &c., require it. Moreover, there is developed, in eyes myopic from infancy, an astonishing ability to profit by the imperfect retinal images that they receive. These myopes, whose defect is of low degree, do not find themselves at all annoyed by it in everyday life. They easily recognise people several metres distant, they read the names of streets and even succeed in becoming pretty good marksmen, without the aid of glasses.

Slight myopia demands the clinician's intervention only in the minority of cases, when it takes on a progressive or malignant course, with which we shall become acquainted later on. But, for the same reason, emmetropia may require the assistance of therapeutics, when it constitutes a stage in the morbid development of an eye which, from having been hyperopic, tends to become progressively myopic. But then it is not the refraction, properly so-called, which holds first place as a cause. It is not the myopia or emmetropia that we have to combat; it is a real disease of the organ, of which disease the peculiar refractive condition is but a symptom.

In general, then, it is understood, and it is a point we insist upon, in opposition to a tendency to polytherapy which we have often remarked, that a certain degree of hyperopia or myopia does not of itself render an eye pathological.

After this *slight* degree of ametropia, we distinguish the *medium* degree, about two to four dioptries for hyperopia and three to six dioptries for myopia.

Here the conditions are already very different; the ametropia, in itself, nearly always calls for optical correction. Nevertheless, the eye, properly so-called, may still be normal. Its acuteness of vision may reach the average. The accommodation is still the same as that of the emmetrope. However, the differences in the form and dimensions of the globe commence to manifest themselves, the hyperopic eye being visibly smaller and the myopic eye larger than the typical emmetropic eye. These differences find expression, among others, in the field of fixation, which shows greater excursions for the former than for the latter.[1]

But the peculiar characteristic of these medium degrees of ametropia is the fact that, although each eye alone may still work normally, the *co-operation* of the two is found to be more or less hindered. The very fact that the vision of each is still good enough, inspires in the individual the legitimate desire to use them simultaneously. But, in order to fuse the distinct impressions of each eye in single, binocular, stereoscopic vision, the degree of convergence required by the position of the object must be made to agree with the degree of accommodation which corresponds to the ametropia.

Now we have seen (p. 195 *et seq.*) that these two functions are so intimately associated that they can be dissociated only within certain limits. Although simultaneously fixing the same object, the eyes can exert or relax their accommodation several dioptries, or, what amounts to the same thing, they can change their degree of convergence several metre-angles without varying their accommodation. The space within which these relative variations of the two functions move, does not, however, exceed certain limits. It seems to have been arranged, by Nature, with special reference to the irregularities, which she cannot avoid, in the relations between the length and the refractive power of the eye. The relative range of accommodation usually oscillates, indeed, between the limits that we have assigned to the refraction of healthy, though slightly ametropic, eyes.

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[1] Donders, *loc. cit.*, pp. 179 and 248. Landolt, *Transactions of the International Medical Congress* (London, 1881, vol. iii., p. 25), and *Arch. d'ophthal.*, i., p. 589 (1880-81).

When the ametropia is of higher degree, this correction no longer suffices for the realisation of binocular vision. The maximum of accommodation, associated with the convergence requisite for binocular vision, is powerless to adapt the hyperopic eye to this distance, and, inversely, if the optical correction is obtained, it provides an excess of convergence which renders binocular vision impossible.

In the case of myopia, on the contrary, the regular convergence tends to bring about an excess of accommodation, while the weakness of the accommodative tension, or its complete relaxation, as required by the degree of the myopia, excludes a sufficient impulsion for the convergence. Hence the latter remains below what it ought to be or is realised only at the price of an excess of accommodation.

This struggle, between the desire for distinct vision and that for binocular vision, engenders, in the case of ametropes of medium degree, the most diverse manifestations of asthenopia, and they frequently escape from this struggle only with the relative loss of one eye. Hyperopes acquire convergent, and myopes divergent, strabismus. They have obtained rest by sacrificing binocular to distinct vision. In other cases, the co-operation of the two eyes is retained, but to the detriment of perfect optical adaptation.

We shall see, later on, all the inconveniences, torments and dangers to which eyes of a medium degree of ametropia are exposed. It will suffice, for the present, to call attention to the fact that it is especially in *binocular vision* that the troubles due to this class of ametropia are manifested.

After the medium degrees, we shall meet with *high degrees* of ametropia.

Eyes of this category may generally be regarded as *pathological*, the hyperopic ones as imperfectly developed, the myopic as affected by real disease. Both kinds work equally badly, although for different reasons. Visual acuteness is weak and falls below the normal in proportion as the degree of ametropia is high. Accommodation is almost always defective and the motor apparatus of the eyes is damaged, often very considerably.

For this latter reason, alone, binocular vision would be very difficult to obtain. But it sinks almost into insignificance in the presence of the difficulties which each eye alone encounters when striving to procure fair vision. Frequently even this end is not attained ; but if, owing to the combination of all the means at the disposition of the organ and of all those that the oculist's art can add, both eyes secure vision, it is not rare to observe that even the desire for binocular vision is lacking. The individual feels pretty happy—nor without reason—to be able to make profitable use of at least one eye, or of either alternately.

If, in the first category of ametropia, one should guard against a too zealous intervention, with regard to medicaments as well as glasses, in the second an acquaintance with ocular physiology and attentive observation achieve their greatest victories. The last class constitutes a vast field open for optical correction, as well as for therapeutic action, especially in malignant myopia.

The general considerations, that have just been occupying our attention, refer to the majority of cases of ametropia, and we think it useful, for practice, to arrange the matter of which we are treating, and which is so difficult to manage, in conformity with the teachings of experience. We shall proceed in the same way in the chapters that are to follow. The normal type being once characterised and the cases which belong to it being once arranged and studied, the atypic forms, which still constitute a very respectable minority, will be easier to explain and better understood.

It seems advisable to state, at this point, that, however intimate the relationship between the doctrine of ocular refraction and physics, we must be careful not to consider the former from a too mathematical point of view. Although we may establish rules, exceptions to them are numerous, and the figures cannot, often, lay claim to greater authority than those of physiology in general. We shall always succeed just so much better in adapting eyes to what they are destined to accomplish, in proportion as we associate more closely perfect knowledge of theory with the most careful clinical observation.

Let us now consider somewhat more minutely the various refractive conditions of the eye.

## EMMETROPIA.

Concerning emmetropia, as such, that is to say, as a refractive condition, there is really little more to be said than is contained in its definition.

The emmetropic eye, in a state of repose, is adapted to infinity; it produces, on its retina, distinct images of distant objects, provided that the refractive media are transparent. The latter condition has, however, nothing to do with the refraction, that is to say, with the relation between the refractive power and the length of the eye. The cornea, crystalline lens, or the vitreous body may be opaque, and this defect may interfere with the distinctness of retinal images, but the eye is nevertheless emmetropic, provided it is so formed that the focus of its dioptric system falls upon the retina.

If the refractive surfaces are irregularly curved, so that the distinctness of the retinal images is thereby injuriously affected, then emmetropia is complicated by astigmatism; or, to speak more accurately emmetropia exists in only one meridian, and that part is the one which corresponds to the definition, while the others are affected with curvature-ametropia.

On the other hand, it is evident that an emmetropic eye may be attacked by all possible diseases of the vascular and nervous apparatus and, notwithstanding its faultless optical formation, be unable to see anything. Acuteness of vision has, indeed, nothing to do with the refractive condition of the eye. Emmetropia affords no protection against atrophy of the optic nerve. Even a dead eye is emmetropic, when the focal distance of its dioptric apparatus corresponds to its real length.

In this definition, the *absolute* length of the eye need not be taken into account. As we have already pointed out, an eye may be longer or shorter, and still emmetropic, provided only that it answers to the above requirements.

A healthy emmetropic eye ought, therefore, without an auxiliary glass, to see distinctly at a distance. One might think, therefore, that a person who sees distant objects more distinctly without, than with, the aid of glasses, is emmetropic; but such is not always the case.

The definition of emmetropia has reference to the *static refraction* of the eye—to that which it has when in a condition of repose. But we have already seen, in the chapter on methods of examination, and we shall often find, hereafter, that this repose is at times involuntarily disturbed by a spasm of accommodation. In this case a certain amount of dynamic refraction becomes associated with the static, causing the refractive power of the eye, in its presumed state of rest, to appear greater than it really is. Thus, it may happen that a hyperopic eye simulates emmetropia, simply because its refractive defect is compensated by a spasm of the ciliary muscle.

On the other hand, a similar spasm may conceal emmetropia and convert it into apparent myopia. Where this is the case, the patient sees distinctly, at a distance, with concave glasses but not with the naked eye, and emmetropia might thus be mistaken for myopia.

It is of primary importance to properly recognise this condition, since asthenopic troubles are frequently the consequence of such an over-exertion of the muscle of accommodation. With the aid of the ophthalmoscope, the diagnosis is easily made, since, as we have already pointed out, the accommodation is relaxed, during the ophthalmoscopic examination, and only the static refraction of the eye appears; or we may have recourse to mydriatics, which force the ciliary muscle to remain in a state of rest. As we shall hereafter see, in the discussion of myopia, in which affection ciliary spasm plays a *rôle* of capital importance, these mydriatics, with local and general hygiene, constitute the principal means to be employed for its relief.

Aside from this complication, there is nothing more to be said concerning emmetropia, as regarded from a clinical standpoint, at least in so far as distant vision is concerned. This complication, too, usually arises only in young patients, and emmetropic eyes are less likely to suffer from it than those which are ametropic.

As regards *near vision*, the case is very different. However perfectly emmetropia may adapt the eye for infinity, it is totally incapable of doing so for finite distances. For the latter, emmetropic refraction is too weak, and the static must, to this end, have the aid of the dynamic and often even that of auxiliary glasses.

For civilised nations near vision possesses very great importance. Hence we may well devote a special paragraph to it and shall be obliged to return to this topic in connection with the discussion of hyperopia, as well as with that of myopia.

For adaptation to near distances, the eye calls into play its apparatus of accommodation. We have already (p. 277) seen how, for every refractive state and for every degree of accommodative power, the nearest point to which the eye can adapt itself is to be determined. For emmetropia this calculation is especially simple. The shortest distance to which the eye can adapt itself is inversely proportional to the number of dioptries represented by its range of accommodation. This is clearly shown by the diagram (p. 174), or by the table (p. 178) derived from it.

At the age of 10 years the emmetropic eye sees at a distance of $\frac{1^{\text{m.}}}{14} = 7$ cm.
" 20 " " $\frac{1^{\text{m.}}}{10} = 10$ cm.
" 30 " " $\frac{1^{\text{m.}}}{7} = 14$ cm.
" 40 " " $\frac{1^{\text{m.}}}{4{\cdot}5} = 22$ cm.
" 50 " " $\frac{1^{\text{m.}}}{2{\cdot}5} = 40$ cm.
" 60 " " $\frac{1^{\text{m.}}}{1} = 1$ metre.
" 70 " " $\frac{1^{\text{m.}}}{0{\cdot}25} = 4$ metres.

One might be led, by this, to assume that an emmetrope of forty years could easily work at a distance of 22 centimetres, or one fifty years old at 40 centimetres; and that, for any eye, the accommodation would be sufficient for working, so long as the point, at which the work must be held, is not nearer to the eye than its *punctum proximum*. This is, however, very far from being the case.

The *punctum proximum* corresponds, it is true, to the nearest point at which vision is distinct. Such vision is, however, secured only by bringing into play the maximum effort of accommodation. It is self-evident that, if muscular work is to be continued for a certain length of time, it must not require, from the outset, the whole strength at command. For instance, I may be able to hold a weight of forty pounds at arm's length, but nobody can reasonably expect me to hold it in that position all the afternoon, for, since that weight corresponds to the entire power of my arm, I shall be obliged to lower it at once, my strength being exhausted. If I wish to continue the muscular effort for some time, the weight must be lighter in proportion to the length of time the effort is to last. In this way I keep in reserve a certain amount of strength, which makes good the amount used up during the continuance of the work.

The case is precisely the same for the ciliary muscle. Nobody can read long at the distance of the *punctum proximum*. The task must not, from the outset, require the maximum of accommodation. A certain quantity of accommodation must remain in *reserve*. What amount of such reserve accommodation is requisite for ordinary work? From a clinical point of view, this question is of the utmost importance. In the first place, it is easy to see that such a reserve fund cannot have an absolute value, for instance, one dioptry for all cases, The number of dioptries does, indeed, correspond to the effect of the contraction of the ciliary muscle, but not to the expenditure of power which is required by it. One dioptry of increase in refraction calls for a greater effort on the part of the ciliary muscle, in proportion as the latter has already furnished more units of refraction.

Let us take the case of two emmetropes, one of whom is fifty-two and the other twenty years old. The former has a range of accommodation amounting to 2 dioptries, while that of the latter is of 10 dioptries. In order to see distinctly at a distance of 1 metre, each must bring into action 1 dioptry of accommodation. This dioptry, however, represents half of all the accommodation which the former has at disposal, but only one-tenth of the total possessed by the latter. If, moreover, the younger of the two continues to accommodate, and brings the object nearer, he will notice a marked difference between the effort which permits him to bring it from infinity to a distance of 1 metre, and that which enables him to bring it from 11 to 10 centimetres. It is true that, in both cases, distinct vision demanded a like amount of increase in accommodation—1 dioptry. But, in raising it from 0 to 1 D, the person in question expends the first dioptry of all that he possesses, and still has nine in reserve; the tenth dioptry, however, is his last; it is brought into play after nine have already been expended, and when he has none besides it remaining. It is much more difficult to bring this last dioptry into action than it was to do the same with the first one.

These examples suffice to show that the accommodative effort is not to be measured by any fixed standard, but finds its expression in the *relation between the effect produced and the entire amount of accommodative power at disposal.* It follows from this that the reserve fund of accommodation must have a *relative,* and not an absolute, value; it must be a *quota* of the range of accommodation.

I have tried to determine experimentally what this quota is, and I think myself justified, from the experiments, in assuming that it amounts to one-third or one-fourth of the total range of accommodation. I mean this: that a continued effort of the ciliary muscle is practicable only when it calls for but two-thirds or, at the utmost, three-fourths of the total power of accommodation.

Distinct vision of an object $\frac{1^{\text{m.}}}{3}$ distant requires, from the emmetropic eye, 3 dioptries of accommodation. If this effort is to be kept up for a certain length of time, the individual must have a range of at least 4 dioptries. In this case the requisite 3 dioptries represent three-fourths of the accommodation which he has at disposal.

With an amplitude of 9 dioptries, an eye will be able to work at a distance of one-sixth of a metre. For, after having used 6 D, it still has 3—*i.e.*, one-third of all it possesses—in reserve.

But, with a range of even 2 D, something can still be done, though only at a comparatively greater distance. Two-thirds of 2 D amount to 1·33 D, to which corresponds a focal distance of 75 centimetres. Hence the eye could still work at this distance, for it is holding, unused, 0·66 D or one-third of its amplitude of accommodation. The object might, perhaps, be brought even to a distance of 66 centimetres. This would require 1·5 D of accommodation,—that is to say, three-quarters of the total in our example. Such conditions may be found in the case of a musician, whose notes are at some distance from him, or in that of a painter, who adds, to the length of his arm, that of his brush.

When the accommodation does not suffice for the adaptation of the eye, we have recourse to convex lenses. Their refractive power is added to that of the eye and is, consequently, able to supply any defect in the latter. Such auxiliary lenses should not, however, altogether exclude accommodation but only bring it up to the normal standard deduced from the above-mentioned experiments. The individual should contribute, for the vision of near objects, from two-thirds to three-fourths of the power of his ciliary muscle.

Let us take, for instance, the case of an emmetrope whose range of accommodation amounts to 3 dioptries. If he is to work with ease, he must hold in reserve from 0·75 to 1·0 D. Hence he may bring 2·25 or 2 D into action. If he wishes to work at a distance of one-third of a metre, he must possess 3 D; consequently he lacks 1·0 D. We furnish him this amount in the form of a convex glass. The latter, with the 2 D which he has at disposal, makes up the requisite 3 D, the desired amount, and hence he still keeps 1 D of accommodation in reserve.

What glass shall we prescribe for an emmetrope whose range of accommodation amounts to 2 D and who wishes to work at a distance of 28 centimetres $=\frac{1^{\text{m.}}}{3{\cdot}5}$? This distance calls for 3·5 D of positive refraction. Our emmetrope could bring two of those to bear, but still he must reserve at least 0·5 D—that is to say, one-fourth of his accommodative power. He can use only 1·5 D. He therefore lacks $3{\cdot}5 - 1{\cdot}5 = 2$ D, which we give him in the form of a convex glass.

The amount of accommodation, which the individual must keep in reserve, diminishes, necessarily, in proportion as the range of accommodation decreases. Consequently, the part of the convex lens which we have to prescribe, in addition to the absolutely required number, gradually diminishes and becomes *nil*, when the accommodation is reduced to zero. This is perfectly correct and means nothing else than that, where no accommodation exists, the glass to be used is that exactly corresponding to the distance of the object and the static refraction of the eye, but not a stronger one. An emmetrope who is a septuagenarian, and whose power of accommodation is therefore extinct, or a young emmetrope whose accommodation is paralysed, requires, in order to work at $\frac{1^{\text{m.}}}{3}$ distance, number 3 convex, but no higher. If we prescribed for him number 4, this glass would not enable to work more easily at the distance of $\frac{1^{\text{m.}}}{3}$, but simply oblige him to bring his work up to the distance of a quarter of a metre.

The same rule holds good, *mutatis mutandis*, for hyperopes and myopes.

On the other hand, the required quantity of accommodation in reserve must increase with the increase of the range of accommodation. This is equally true, and is every day illustrated in practice.

An emmetrope of forty, who has 4·5 D accommodation at disposal, can still read at a third of a metre. Indeed, he employs only 3 D, or two-thirds of the power of his ciliary muscle, and keeps one-third (1·5 D) in reserve. But this 1·5 D would be quite insufficient for a child of fifteen with 7·5 D hyperopia. The latter's amplitude of accommodation being, according to Donders' diagram, 12 D, he woul still, after the correction of his ametropia, retain $12-7{\cdot}5=4{\cdot}5$ D, or 1·5 D more than required to see at a third of a metre. But this amount of accommodation is, in this case, too small to permit of continued work. It represents but $\frac{1{\cdot}5}{12}$ of the accommodative power. The patient would therefore be obliged to work with $\frac{7{\cdot}5}{12}$ of it. In order to be under the same conditions as the emmetrope of forty, the hyperope of fifteen would have to keep $\frac{12}{3}=4$ dioptries in reserve. These we provide by means of a convex glass of 2·5 dioptries. The labour performed, by the ciliary muscle of each person, would then be the same.

Of course this rule, which we here recommend for the choice of spectacle-glasses, can make no claim to the exactitude of a physical law. This follows directly from the latitude which we have left, between one-third and one-fourth, for the quota in reserve. Hence it is only in cases of very limited ranges of accommodation that one will carry the calculation to so fine a point as to one-fourth of a dioptry. We may generally approximate sufficiently to call the fraction a half-dioptry, and simply guard against over-stepping the limits of one-third or one-fourth. In connection with the discussion of hyperopia we shall treat more fully of convex glasses, inasmuch as they there have a very important part to play.

Let us now turn to the other factor which is indispensable for normal binocular vision, that is to say, the *direction* of the eyes,—the convergence, as we have called it.

In the general portion of this treatise we have already called attention to the fact that perfect vision demands not only the proper optical adaptation of each separate eye, but also the simultaneous direction of both toward the object of fixation. In this way, by the fusion of the two distinct retinal images in one common impression, *binocular vision* is produced.

We have seen that the normal direction of the eyes is under the control of the motor apparatus, and have designated the latter's function, in so far as it serves in binocular vision, simply by the term "convergence."

We may assume, without notable error, that the eyes have a parallel direction when they are at rest, and that their convergence is then, consequently, *nil*. We know that, starting from this condition, the convergence may be changed in two opposite directions. By the common contraction of the abductor muscles, it becomes less than zero, *i.e.*, *negative*. By the common action of the adductors, it becomes *positive*.

*Negative convergence* (*i.e.*, divergence) is never brought into action in normal vision. No object requires, for its binocular perception, a divergence of the lines of fixation. The latter is rendered appreciable only by the aid of abducting prisms. Nevertheless, the negative portion of the range of convergence has frequently great significance. It may serve in pathological cases, as we shall hereafter see, to make good a lack on the part of the positive portion.

Normal binocular vision has to do only with the positive portion of the range of convergence. We have heretofore seen that the maximum of convergence ($p$) amounts, on an average, to from 9 to 10 metre-angles. It might be inferred from this that the convergence would more than suffice for ordinary work—that it might, indeed, fall below its average without prejudice to binocular vision and without giving rise to asthenopic troubles. As a matter of fact, one's work is seldom brought nearer than one-third or one-fourth of a metre, and, consequently, requires only 3 or 4 metre-angles of convergent power. With 9 metre-angles, which represents a maximum of convergence of but a small amount, the individual would have a surplus of 5 or 6 metre-angles of convergence. But, after what we have explained, when discussing the quota of accommodation, it will readily be understood that such *a priori* reasoning is not admissible. It is plain that the individual must keep a certain part of his convergence in reserve, if he is to continue his work without fatigue. Before we know the amount of the quota of convergence demanded by continuous work, we shall not be in a position to judge as to whether or not nature has been too liberal in the endowment of the eyes with adducting power. We therefore have first to determine, as in the case of accommodation, the quota of convergence that it is necessary to hold in reserve.

The experiments which I have made, with a view to furnishing the answer to this question are of recent date and not yet as numerous as I could wish.[1] They tend to show that the quota of convergence, which can be utilised for continuous work, ought not to amount to more than one-third or even one-fourth of the positive convergence power; hence that two-thirds or three-fourths of the latter should be kept in reserve. Whence it results that a person with a $p$ of 9 metre-angles has just the absolutely necessary amount of convergent power for work at the ordinary distance of $\frac{1^{\text{m.}}}{3}$ of a metre, but no more. For one-quarter of a metre (25 centimetres) the strength of his adductors would no longer suffice. They would soon become fatigued. In fact, continued binocular vision at this distance requires at least 12 metre-angles of positive convergence ($p$), 4 metre-angles for immediate use and twice as many (8) to make up for the force expended.

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[1] Landolt, "L'amplitude de convergence," *Arch. d'Opht.*, v., p. 108, 1885.

This large amount of the reserve quota might at first sight seem astonishing, but only so to ophthalmologists, who involuntarily compare the quota of convergence with the corresponding quota of accommodation. It naturally occurs to us that the relation of the two parts to each other, in the two cases, is just reversed. One may employ two-thirds of one's accommodation in working and need hold back but one-third; on the contrary, two-thirds of the convergence must be held in reserve, in order that one may work with the remaining third.

This seeming contradiction disappears at once, however, when we call to mind the great differences existing between the muscles which preside over the two functions, as well as between the obstacles which they have to overcome. The muscles of convergence are striated, while those of accommodation belong to the smooth variety. First of all, however, we should take into account the fact that the work performed is very different for the two sets of muscles. The adductors act in antagonism to the constantly increasing tension of the external *recti*, of the optic nerve and of the elastic tissues, which endeavour to keep the eye-ball in a state of equilibrium. The contraction of the ciliary muscle, on the other hand, is opposed by the tension of the choroid. The elasticity of the lens, however, which tends to retain that organ in the more convex form which it has when left to itself, aids the effort of the muscle of accommodation. In fact, it is not the *direct* result of the latter's work which we observe—not the stretching of the choroid, but the change in the form of the lens which results from it. Hence the optical effect produced by the contraction of the muscle of accommodation is not directly comparable with the geometrical effect due to the muscles of convergence. We shall therefore not be greatly astonished when we find that a considerably greater portion of the existing force is requisite for the production of the former than for that of the latter.

If it now becomes a question of ascertaining whether, in a given case, the convergence-apparatus is able to furnish the required effort—whether an asthenopia, which arises in the course of work, is ascribable to insufficient activity of the adductors—we shall proceed as follows:—We first ask: At what distance does the patient work? This may depend upon the nature of the work. It may be $\frac{1^{\text{m.}}}{2}$ for piano playing or painting, $\frac{1^{\text{m.}}}{3}$ for reading, $\frac{1^{\text{m.}}}{4}$ for sewing, sometimes $\frac{1^{\text{m.}}}{5}$ for engraving and embroidering. The distance may also be determined by the refractive condition, *e.g.*, by myopia, and in this case is often very short.

Let us assume, for the sake of illustration, that the patient wishes to work at a distance of $\frac{1^{\text{m.}}}{3}$. Binocular vision, at this distance, calls for 3 metre-angles of convergence. In order, however, that this effort may be kept up, the individual needs to possess three or four times this amount of convergence-power. Can his muscles furnish the required quantity? This is the next question.

The answer to it is easily obtained: one need only determine, with the aid of my dynamometer (p. 283), the maximum of convergence ($p$). If this, in the case under consideration, amounts to 9 or even 10 metre-angles, the individual will be able to easily perform the required work and the cause of any existing asthenopia is to be sought elsewhere than in weakness of the motor apparatus.

Suppose, however, that, instead of 9 or more, only 7 metre-angles had been found as the maximum of convergence. The patient lacks therefore $9-7=2$ metre-angles of convergence power. How are we to make good these metre-angles that are wanting? This is an inquiry into the therapeutics of muscular asthenopia.

In myopia, as we shall see, this question plays a very important part, and we shall therefore, in that connection, discuss the matter more thoroughly. We may, however, remark just here, that we possess various ways and means of treating muscular asthenopia, or insufficiency of convergence.

The first of these is the removal of the fixation-object to a greater distance. The farther away the object is, the less convergence is required for its binocular perception. Hyperopes and emmetropes have usually brought into play and exhausted this means of relief—that is to say, carried their work as far away as possible before they consult a physician. On the other hand, we may often, in accordance with this principle, render great service to myopes, whose range of distinct vision, on account of the proximity of their *punctum remotum*, is very limited. A myope of 4 D does not see distinctly at any distance greater than $\frac{1^{\text{m.}}}{4}$. If his convergence power is insufficient for work at this distance, he cannot move his book or writing paper off from $\frac{1^{\text{m.}}}{4}$ to $\frac{1^{\text{m.}}}{3}$ (from 25 centimetres to 33 centimetres), where he need converge less, since he cannot see distinctly beyond 25 centimetres. In this case, a concave glass will help him. The concave 1 D would, in our hypothetical case, diminish the myopia from 4 D to 3 D, and proportionally lessen the necessary convergence.

But, in a case where, as in our example, the work must be at a given distance from the eyes $\left(\frac{1^{\text{m.}}}{3}\right)$, insufficiency of convergence power is not to be remedied in this way.

Under such circumstances, we must have recourse to other methods of treatment. The first means which suggests itself is furnished by optics—that is, the use of *prisms*. We have already (p. 193) discussed the action of prismatic glasses. When held before the eye, with its apex turned toward the temple, a prism deflects rays, which emanate from a fixation-object, in such a way that they appear to come from a more distant one. The effort of convergence required for binocular vision is thus diminished in proportion to the strength of the prism. According to an explanation previously given (p. 193), it is easy to calculate what prism is requisite for the production of a given effect. The diagram, or the formula (pp. 188-190 and 194) will aid us in this. We have given to the latter the simpler form of $Z=\frac{X}{7}$, in which expression $Z$ represents in metre-angles the deflection which a prism of X degrees of aperture (number marked on the prisms in our trial cases), placed before *one* eye, produces in *each*. A prism of 21°, for example, with its apex toward the temporal side, would diminish the convergence of each eye by $Z=\frac{21}{7}=3$ metre-angles. If the question is, as in the case under consideration, to find a prism of a certain desired action, we shall give the formula the following form: $X=7\,Z$, *i.e.*, we need only multiply the required number of metre-angles by 7, in order to obtain the angle—that is to say, the number of the corresponding prism.

The patient, in our example, lacks 2 metre-angles. Hence he would need a prism of $2 \times 7=14$ degrees, in case the whole effect were to be produced by a single prism. In practice, however, we do not proceed in this way. Although the action of a monocular prism is really divided between both the eyes, we prefer, for very obvious reasons, to furnish each eye with a prism having half the requisite strength. In our case, therefore, it would be necessary to place a prism of 7 degrees, with its apex toward the temporal side, in front of each eye. But the use of prismatic spectacle-glasses is limited. Besides the property of deflecting luminous rays, they possess others, which are injurious to vision as soon as the prism reaches a certain strength: for instance, chromatic aberration, an apparent alteration of surface form and seeming recession of the objects looked at. Moreover, prisms, whose strength exceeds a few degrees, are so heavy that their use, as spectacle-glasses, soon becomes burdensome. At all events, it is scarcely practicable to wear prisms of more than 5°. Prisms of this strength would, according to our calculations, produce a deflection of 1·5 metre-angles (more accurately stated, 1·43 for a base-line of 58 millimetres, such as is found in children, or one of 1·36 metre-angles for the average adult base-line of 64 millimetres). But prisms of 7°, such as our hypothetical patient requires, can never be employed in practice. It is especially important to appreciate the fact that prismatic glasses can be usefully employed only when the defect in convergence scarcely exceeds 1 metre-angle.

It may now and then happen that prismatic action may be advantageously combined with removal of the fixation-object to a greater distance. In a case of myopia, for instance, prisms of 3·5° may diminish the necessary convergence by 1 metre-angle, while concave glasses of 1 dioptry carry the *punctum remotum* 1 metre-angle farther away. A myope of 5 D, with concave glasses of 1 D, will be able to work at one-quarter, instead of one-fifth, of a metre ; he will then need only 4 instead of 5 metre-angles of convergence, and, with the prisms above mentioned, will require only 3 metre-angles. Of course we we should not, in such a case, superpose two pairs of glasses of the two different kinds, but would give the surfaces of the prisms the proper concavity, so that they would act at once as both deflecting and dispersive glasses. In the same way the collecting and prismatic actions of glasses may be combined, if they are ground with convex surfaces, as is demanded when we have a defect in convergence and one of refraction to correct simultaneously. But we shall hereafter be concerned with such cases in the chapters on ametropia. Let us resume the consideration of our example.

We have convinced ourselves that the insufficiency of convergence is too great to admit of compensation by optical appliances. Hence we must have recourse to *surgical methods*. Instead of compounding with the affection and labouring further with the inadequate muscular power, in the way of artificial deflection of rays, we shall augment the convergence power, and extend its province, by operative means.

The activity of the adductors may be increased, in the first place, by lessening the strength of their antagonists, the abductors. This end is attained by tenotomy of one or both of the *recti externi*. On the other hand, the convergence may be strengthened by giving the muscles, which preside over this function, a more favourable point of action. This is to be accomplished by *advancement* of one or both of the *recti interni*. These then become inserted farther forward on the eyeball, nearer the cornea and, moreover, in the process of cicatrisation, they undergo a certain amount of shortening. In this way their action on the rotation of the eyeball—*i.e.*, on convergence—is certainly strengthened, and the more so in proportion as the muscle has been brought further forward.

Finally, we may combine both of these operations—advancement of the adductors and tenotomy, or setting back, of the abductors. In this way, any desired degree of convergence may be obtained, provided that there does not exist a pathological weakness of the muscles, or of their innervation. On the other hand, it may at times become a question as to whether or not the range of convergence will allow of the diminution, at the extremity of its *punctum remotum*, which will be produced by the operation. For it will readily be perceived that, when there is an operative increase of the maximum of convergence, by bringing the *punctum proximum* nearer, the risk is sometimes incurred of bringing the *punctum remotum* too near, through weakening of the abductors.

But we shall discuss these questions more thoroughly in connection with the subject of divergent strabismus. Let us now return to the consideration of our main theme, the refraction of the eye, from which, in the interest of the study of binocular vision, we have made so marked a digression.

## HYPEROPIA.

Let us at first recall the definition, of hyperopia, that has been given on page 134, and which may be briefly stated as follows :—*Hyperopia is characterised by the fact that the principal focus of the eye is behind the retina.*

As we have already pointed out at the place above cited, the definition prejudges nothing as to the cause of ametropia. And, in fact, taking the typical emmetropic eye (pp. 109-121) as a term of comparison, all the conditions that may cause the focus of the system to fall behind the retina, adapt the eye for convergent rays—in short, render it hyperopic—are found realised. Sometimes it is too short an eye, whose dioptric system is equal in power to that of a typical eye; at other times, it is an eye of ordinary length, with too weak a dioptric apparatus. And this insufficiency of refractive power, which prevents parallel rays forming a focus on the retina, and carries them too far behind the retina, may be due to various causes—flattening of the refractive surfaces, absence of a part of the system (aphakia), diminution of the index of refraction of the lens, or increase in that of the vitreous body. Moreover, all these causes may arise in turn from different influences and, finally, may combine with each other in most diverse ways.

According to its etiology, too, hyperopia takes on different characters and requires different kinds of treatment.

Being given this multitude of forms of hyperopia, it is again necessary to proceed in an orderly way, so as not to become confused and not to confound the important with the accessory, the exception with the rule.

Therefore we shall attempt to establish a *typical hyperopia*, which we shall discuss in the first place, and shall devote a special chapter to the various atypic hyperopias.

## I.—TYPICAL HYPEROPIA.

The typical hyperopic eye may generally be considered as *imperfectly developed.*[1] This arrest of development may vary from the normal human eye, to similarity with the eye of the higher mammalia, in the class, and to microphthalmos in the species. It is, indeed, important to note that the eyes of the lower orders of animals are nearly all hyperopic. Those that we have examined presented a hyperopia averaging 3 dioptries. Moreover, in the eyes of children under eight years of age hyperopia is the rule. They become emmetropic or myopic only when, later on, the body has acquired its full development.[2] Finally, this refractive condition is usual in ill-formed eyes,—those affected by any defect in evolution. We may, in this connection, mention eyes affected by coloboma of the iris or by more or less complete congenital atrophy of the optic nerve, with or without nystagmus. Let us also especially bear in mind the microphthalmos already cited, and all eyes that resemble it, in which the arrest of development very often makes itself felt outside the eyes, in the accessory organs, in the orbit and even in the face and entire cranium.

We may cite, as an example, a characteristic case of trophoneurosis of the face, with typical hyperopia, that we have described.[3]

The case was that of a man of forty-five years, affected from infancy with an arrest of development of the left half of the face, resulting, very probably, from a lesion of the left cervical sympathetic. The eye on the dystrophic side presented a hyperopia of 3 dioptries, with acuteness of vision equalling 0·1, and a limitation of the field of fixation. The right eye, corresponding to the normal side of the face, was hyperopic by only 0·5 D and enjoyed visual acuteness of 1.

Schmidt-Rimpler[4] observed a high degree of hypermetropia in two eyes, presenting a macular coloboma of the choroid alone, without ectasiæ or defect of the visual field. Indeed the incomplete closure of the choroido-retinal fissure is usually due to some impediment to the perfect evolution of the visual organ.

Desjardins[5] found that almost all congenital deaf-mutes were hypermetropic.

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[1] Donders, *loc. cit.*, p. 245.

[2] Ely, *Archives of Ophthalmology*, ix., 4. 1880. Hortsmann, *Danziger Naturforscherverein*, 1880. Schleich, *Mittheilungen aus der ophth. Klinik in Tübingen*, 1882. Cohn, *Klin. Monatsbl. f. Augenheilkunde*, p. 460, 1871; *Arch. f. Ophth.*, xvii., 2, p. 305 1871.

[3] Landolt, *Archives d'Ophthalmologie*, iii., p. 109, 1883.

[4] Schmidt-Rimpler, *Arch. f. Ophth.*, xxvi., 2, p. 221, 1880.

[5] Desjardins, *Ann. Ocul.*, t. lxxxiv., p. 103, 1880.

Schaefer[1] was also surprised at the small number of myopes and emmetropes which he met with among the students of an institution for the deaf and dumb. It is allowable to admit that there exists a certain relation between the arrest of development of the organ of hearing, and the much less marked defect in the visual organ.

A. *Hyperopia of Low Degree.*—In the low degrees of hyperopia this arrest in development is scarcely perceptible. Even the normal eye shows so many deviations in its length and in the degree of curvature of its surfaces, that, without direct determination of its refraction, it is impossible to distinguish a slightly hyperopic from an emmetropic eye.

Moreover, we have already said that, so far as their anatomical structure or their working is concerned, these eyes are generally as perfect as emmetropic ones. We need only cite, in support of this assertion, certain tribes of savages, examined with reference to their ocular refraction, who, with their hyperopia, combined an acuteness of vision twice as good as the average in our own lands. Nor need we go even so far in order to verify this fact. We prove its truth every day among ourselves. Eyes that are hyperopic in a slight degree, and such probably constitute the majority, nearly all have a very good acuteness of vision.

Their range of accommodation is the same as that of emmetropic eyes, and their excursions are at least equally extended.

B. *Hyperopia of Medium Degree.*—In the *medium degrees*, on the contrary, the hyperopic type is already easily discernible. It even betrays itself sometimes in the conformation of the cranium and face. The latter is characterised by its lack of relief. It appears flattened, especially in the region of the root of the nose, the forehead, the orbital borders and the zygomatic process. In short, the hyperopic type of face resembles that of the Mongolian. It might seem as if smallness of the eye corresponded to the lack of depth of the orbit.[1]

The relation between the conformation of the cranium and that of the eye is still more apparent in certain cases of asymmetry of the cranium and of the face. We very often meet people in whom one-half of the head is visibly smaller than the other. One side of the forehead is only partially developed, receding, the cheek-bone more or less flattened and the diameter of the cheek less than that of the other side. The same difference is also found between the two halves of the palate. One side of the chin is, as it were, atrophied relatively to the other. The median line of the face is not straight, but slightly curved and turning its convexity toward the better developed portion, as if the latter tended by its more active growth to surround the other, which has been arrested in its evolution. Very frequently the measurement or even simple inspection of the cranium denotes a similar difference.

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[1] Schaefer, *Centralbl. f. Augenheilk.*, p. 65, 1884.

[2] Donders, *loc. cit.*, p. 252.

In the majority of cases we find, corresponding to the less developed half of the head, the eye whose refraction is weaker (the smaller one), oftenest hyperopic, or presenting a higher degree of hyperopia, if this refractive anomaly exists in both eyes.[1]

It is true that this rule admits of frequent exceptions, upon which we have never failed to insist. The general hyperopic type, above spoken of, may also be wanting in cases of medium hyperopia, but the peculiar, characteristic form of the eye itself is almost never lacking.

Such eyes are generally smaller than normal ones. Their arrest of development is especially pronounced in the antero-posterior diameter, so that, on a meridional section, their form approaches that of the diagram on page 133, and their analogy with the eyes of animals is very striking. To be convinced of this it is necessary only to compare such sections with the faithful reproduction of a rabbit's eye, which Fig. 118 represents.

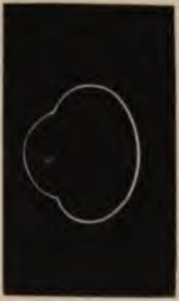

FIG. 118.

Nor can the form of the typical hyperopic eye escape the attentive observer, even without his having recourse to autopsy. Its smallness and great mobility strike one at first glance, and when the lids are separated and drawn toward the temple, and the eye strongly rotated toward the nose, the ellipsoidal form of the eye is discernible from the strong curvature of the equatorial region. Its smallness is revealed by the possibility of following its contour with the eye and, even better, with the finger, behind the equator nearly to the optic nerve. This hyperopic form is striking, especially when compared with that of the myopic eye, which, as we shall see, is distinguished by a sort of flattening of its equatorial portion. In hyperopia the contour mounts rapidly from the cornea toward the equator, to descend again with equal rapidity toward the posterior pole.

If the shortening of the hyperopic eye is so marked as to be noticeable at first glance, without, however, its producing more than a medium degree of this ametropia, it must be presumed that the dioptric system cannot be very different from that of the emmetrope; that is to say, typical hyperopia is due to a defect in the length of the eye rather than to flattening of its refractive surfaces.

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[1] Donders, *loc. cit.*, p. 558. Landolt, "Relations between the Conformation of the Cranium and that of the Eye," British Med. Ass. Meeting at Cambridge, Aug. 1880; *British Med. Journal*, April 1881.

Indeed, ophthalmometric measurements, far from revealing a weaker curvature of the cornea, have now and then shown it to be even greater in the hyperope than in the emmetrope.[1]

With this class of hyperopes, regular astigmatism already commences to show itself; a very important fact to notice, in view of the other visual difficulties with which hyperopes struggle, in proportion as the degree of their ametropia increases.

According to Iwanoff (page 156 of this work), the ciliary muscle seems to take on a peculiar form in the hyperopic eye. It is not stronger than that of the emmetrope, although the condition of the static refraction claims a more powerful intervention of the dynamic refraction.

As to the nervous apparatus, this commences to show imperfections, visual acuteness scarcely reaches the average and is oftener inferior, than superior, to it.

The field of vision, according to Uschakoff and Reich, is of somewhat greater extent than that of emmetropes or myopes. We have explained this phenomenon by the position of the eccentric portions of the retina relatively to the dioptric apparatus of the eye.[2] The form of the globe, strongly bulged in the case of hyperopes, permits luminous rays, that form a large angle with the axis of the eye, to still fall upon percipient portions of the retina, while, in elongated eyes, they fall in front of them. Hence this slight difference in the extent of the visual field in favour of hyperopes is not due to superiority of the nervous membrane, nor does it even render them appreciable service.

Although, in the case of hyperopes whose ametropia is of low degree, the accommodation may suffice, until an advanced age, to adapt the eye for vision at a great, as well as for a short, distance, the defect in the static refraction makes itself felt much sooner in the medium class of hyperopes. The insufficiency of the accommodation, when the latter is compelled to assume the double task of supplying the lack in static refraction and of adapting the eyes to near work, necessarily makes itself more and more felt as the degree of the hyperopia is higher. We find, too, in the case of hyperopes, a very marked tendency to increase the distance of the object of fixation, with a view to keeping it within the domain where accommodation is still exerted without too much effort.

The proximal extreme of this domain, however, recedes pretty rapidly as age advances. It arrives at the limit where the maximum of accommodation scarcely suffices for distinct vision at working distance. Symptoms of asthenopia appear. Gradually the *punctum proximum* becomes so far removed from the eye that all reading, writing, &c. is impossible on account of the imperfection of the retinal images. Distant vision alone is still distinct. But even this becomes imperfect, sooner or later, according to the degree of the hyperopia. Thus, the diagram on page 174 shows us that a hyperope of four dioptries can no longer see clearly at the age of forty-three years, since at that age his *punctum proximum* is already beyond infinity.

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[1] Donders, *loc. cit.*, p. 246.

[2] Landolt, "Il perimetro e la sua applicazione;" *Annali di ottalmologia*, i., 1871.

The motor apparatus, alone, of hyperopes seems to work better than that of the normal eye, since the field of fixation is generally of greater extent in the medium degrees of hyperopia than in emmetropia.[1] This great extent of the excursions of the eye is not, however, attributable to a superior development of the muscles, but only to the greater mobility of the globe, which is small and short, and whose centre of motion is situated farther back than in the emmetrope or the myope.[2]

The advantages of the hyperopic conformation, for the rotations of the eye, become especially apparent when it is compared with that of eyes which are strongly myopic, elongated, amplified and whose movements are, as a natural consequence, impeded by the walls of the orbit and the lack of elasticity of the over-stretched muscles.

Notwithstanding their great mobility, hyperopic eyes do not attain a range of convergence markedly higher than that of emmetropes. (Schuermann, Reich, Landolt, Hoffmann.)

Donders' investigations seems to indicate that, in such eyes, the macula is further from the temporal border of the papilla than it is in either emmetropic or myopic eyes. He has shown, indeed, that the line of fixation, which nearly coincides with the line of vision going from the *fovea centralis* to the object fixed, passes *to the inside* of the axis through the centre of the cornea, and that the angle included between these two lines, and which may be called $\gamma$, is generally positive and greater than in other eyes.[3]

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[1] Donders and Schuermann, Donders, *loc. cit.*, p. 409; Schneller, *Arch. f. Ophth.*, xxi., 3, p. 133, 1875, and xxii., 4, p. 147, 1877; Landolt., *Arch. d'opht.*, i., p. 586. 1880-81.

[2] Donders (*loc. cit.*, pp. 181 and 248) found in twelve eyes, whose hyperopia varied from 2·5 to 10·5 D, an average ratio, between the distance of the centre of motion from the posterior pole and that separating it from the anterior pole of the eye, of 8·885 to 13·22, *i.e.*, nearly 2 : 3, while it is 9·99 to 13·54, or about 5 : 6·8 in emmetropia.

[3] Donders calls this angle $\alpha$. But in reality he measures the angle formed by the line of fixation and the axis passing through the centre of the cornea. Hence it is proper to call it $\gamma$, although it does not rigorously correspond to the definition of this angle that has been given on page 118. Compare also what the author says concerning these two angles, p. 746, vol. i., of De Wecker and Landolt's *Traité complet.*

This fact may be due either to a displacement of the macula toward the temple, supposing the eyeball to remain the same, or to a displacement, *i.e.*, a faulty setting of the cornea, whose axis would, in this case, form an angle with that of the eyeball, and whose centre would be directed toward the temple.

It is apparently the former hypothesis that Donders has admitted, for he attributes to this divergence, between the visual line and the corneal axis, the advantage of facilitating the convergence of the visual lines. This is the case only when the macula has changed its position relatively to the rest of the eyeball, especially the muscular apparatus, while the eccentricity of the insertion of the cornea could in no way have any influence in this sense.

Our own experiments[1] and those of Dobrowolski,[2] which have proved that in hyperopia the distance between the papilla and the macula is generally greater than in emmetropes or myopes, lend a high degree of probability to this opinion, unless we suppose an abnormal insertion of the optic nerve.

From the fact established by Donders, it results that the corneal axes of hyperopic eyes always converge relatively to the visual lines, and that the former are in absolute divergence when the latter are parallel with each other. A hyperope, therefore, who presents large positive angles $\gamma$, may have the appearance of squinting outward, without really being affected with strabismus. The observer, indeed, judges of the direction of the eyes from that of the corneæ, which he sees, and not from that of the visual lines, which he does not see.

This apparent divergent strabismus is comparatively frequent in hyperopes. It rarely, however, reaches very high degrees,[3] and is easily distinguished from a true strabismus. It is necessary only to let the patient fix a near object, while one of his eyes is covered. If, at the instant of uncovering this one, the other be covered in turn, the former does not need to execute a convergence movement in order to fix the object; its line of fixation is properly directed, and there is no strabismus.

Real divergent strabismus is, in fact, rare among hyperopes, especially among those of the class now under consideration. *Convergent strabismus* is, on the contrary, very frequent among them.

As has just been said, Donders thinks that the size of the angle $\gamma$ disposes these eyes to convergence by the fact that, other things being equal, the lines of fixation converge, when the eyes are at rest, in their position of equilibrium, which nearly corresponds to parallelism of the corneal axes. However this may be, the angle $\gamma$ is certainly not the principal cause of the convergent strabismus of hyperopes.

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[1] Landolt, *La distanza tra la macula lutea e la papilla del nervo ottico* (*Annali di ottalmologia*, ii., p. 1, Jan. 1872).

[2] Dobrowolski, *Ann. d'oculistique*, lxvi., p. 217, Dec. 1871.

[3] Donders, *loc. cit.*, p. 248. Landolt, Article "Strabisme" in *Dict. encyclopéd. des sciences médicales*, by Dechambre, pp. 223-226, 1882.

Donders also explains its production in an infinitely more satisfactory way by the relation which he has pointed out as existing between accommodation and convergence. We know that the two functions are associated in such a way that, for a given degree of convergence, there is always a nearly equal degree of accommodation, and *vice versâ*.

Now, if the emmetrope always calls into play as many dioptries of accommodation as the distance of the object requires metre-angles of convergence, the hyperope is obliged to draw on his accommodation, over and above this amount, for a number of dioptries equal to the degree of his ametropia.

So long as this degree does not surpass the positive quota of his relative range of accommodation—*i.e.*, the quota by which he can increase his accommodation without varying his convergence, binocular and distinct vision is possible. This is the case with hyperopes whose ametropia is of low degree, and who are still young enough to have at their disposal a considerable degree of accommodation. But when the hyperopia exceeds certain limits, it is no longer possible to harmonise the two functions that preside over binocular and distinct vision. The individual, while directing both eyes simultaneously toward the object of fixation, is no longer able to join with this convergence, necessary for single vision, the amount of accommodation requisite for distinct vision.

The latter could be obtained only at the expense of a more considerable degree of convergence. But, in this case, there is no longer binocular vision; there is an excess of convergence—that is to say, convergent strabismus.

In fact all hyperopes whose ametropia is of a certain degree, find themselves in this dilemma; they may see binocularly but indistinctly, or distinctly but with detriment to association of the two eyes.

For example, a hyperope of five dioptries, fifteen years old—*i.e.*, having at his disposal twelve dioptries of range of accommodation—needs, in order to fix an object $\frac{1^{m.}}{4}$ distant, and requiring four metre-angles of convergence, $5 + 4 = 9$ D of accommodation. If, for this degree of convergence, the positive part of his relative range of accommodation amounts to only 2 D, he cannot see clearly, being insufficiently adapted. In order to realise the three dioptries that he lacks, he is obliged to make, with each eye, a convergence effort of about 3 metre-angles more. Hence his lines of fixation will no longer be directed simultaneously toward the same object, one-fourth of a metre distant. He will either turn his head, or move the object so that this shall be on the line of fixation of one of his eyes, leaving the other to make all the necessary convergence. If, in our example, the right eye were directed toward the object, the left would be found executing a pathological convergence of $2 \times 3 = 6$ metre-angles, which, if the object were $\frac{1^{\text{m.}}}{4}$ distant, would give this eye a total convergence of $4 + 6 = 10$ metre-angles.

In spite of this inequality of convergence, the accommodation thus gained would, nevertheless, be divided equally between the right eye (the one fixing the object, in our example) and the left (which converges excessively). Each, indeed, receives a like amount of innervation from the nervous centres; only, in the left eye, it is all brought to bear upon the adductor muscles, while in the right eye it is uniformly distributed to the adductor and abductor muscles, thus maintaining the eye immoveable in its state of fixation.

We do not pretend that all this takes place with mathematical exactness, as the figures might lead one to suppose. The individual succeeds, indeed, in rendering his convergence, to a certain degree, independent of his accommodation, as is shown by the *relative* amplitude of these functions. The figures serve only, like the lines of a schematic diagram, to make an example more striking.

But the principle is absolutely true. Donders has already brought proofs to its support, and we can prove it, at any time, by the following experiment. If we possess perfect binocular vision, let us fix a near object; cover one eye, the left for instance, and then place a concave glass before the right. This eye will not change its direction, and will continue to see clearly. But the effort of accommodation that it is forced to make in order to neutralise the negative glass, imposes itself, at the same time, on the other eye, and provokes in the latter a convergent strabismus of a degree corresponding to the power of the concave glass. The existence of this strabismus may be easily established objectively, and manifests itself subjectively by a homonymous diplopia at the moment when the diaphragm is removed from in front of the left eye.

The anisometrope, whose one eye is hyperopic and the other emmetropic, and who has binocular vision, does not obtain distinct retinal images in the hyperopic eye, because the amount of accommodation brought into play is the same for each eye. Whenever he wishes to see clearly with the hyperopic, his emmetropic eye is deviated inward by an exaggerated contraction of its internal rectus muscle.

Moreover, if the convergent strabismus of hyperopes is not too inveterate, it can be suppressed by the use of convex glasses, which relieves the eyes from the necessity for this exaggerated accommodation, which can be exerted only in conjunction with a correspondingly excessive convergence effort.

The production of the convergent strabismus of hyperopes is perfectly explained by Donders' theory.[1] What is more difficult to understand is, why certain hyperopes are cross-eyed, while many others, whose ametropia is of medium degree, are not. Donders thinks the reason for this to be the differences presented by the angle $\gamma$. But mensuration does not seem to confirm this opinion, since the size of this angle is not in sufficiently constant relation with the degree of convergent strabismus.

It seems more natural to suppose, as does also that great physiologist, that, if hyperopia tends to produce convergent strabismus, the latter is developed especially in cases in which one of the eyes is far inferior to the other, and, consequently, of less value in binocular vision. This eye would then be sacrificed, in this sense, that, not being able to render great service by its co-operation in binocular vision, it facilitates clear vision of the other eye by assuming the excess of convergence requisite for clear vision. Thus it is that a hyperopia of higher degree in one eye than in the other, a strong astigmatism, opaque spots on the cornea, or alterations in the *fundus oculi*, in short any cause that diminishes the visual acuteness of one of the eyes, may, in a hyperope, give rise to convergent strabismus of that eye.

This way of thinking seems to us not only very plausible, but often confirmed by clinical observation. We are not, however, oblivious of the exceptions, which are numerous. We frequently see hyperopes who, though one eye is weaker than its fellow, are not, however, cross-eyed. On the other hand, convergent strabismus is met with in persons in whose cases both eyes are of equal strength, or nearly so. But until somebody has proved, by ample statistics, that such cases constitute the majority, we shall hold that the inferiority of a hyperopic eye may become, for it, the cause of convergent strabismus, and shall seek elsewhere an explanation of the exceptional cases.

Some have supposed they had discovered this explanation in a tonic spasm, with or without a consequent alteration, of the internal rectus, arising from an inflammation of the cornea propagated to the sheaths of this muscle. It has been thought to explain thus the frequency of opaque spots on strabismic eyes generally.

Professor Schweigger, having observed that very often both eyes, and not alone the deviated one, have defective visual acuteness, thinks that this circumstance forces the person to procure very large retinal images. He succeeds in so doing by bringing objects very close to his eyes, thereby inducing an exaggeration of convergence and accommodation.

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[1] Donders, *loc. cit.*, pp. 291 and 306.

Others attribute this form of strabismus to conditions which facilitate convergence, such as a stronger development, or more favourable insertion of the internal rectus, or a more powerful innervation of the muscles of convergence; but no positive arguments have been furnished in support of this hypothesis, which, however, seems very plausible.

Mauthner[1] thinks that, if all hyperopes do not squint, it is because only a small number of them come to possess this artifice calculated to procure, for them, distinct images. In order that they shall discover how to do this, certain favourable conditions are necessary, as, for instance, paresis of the accommodation (Javal), or the example given by a condisciple, expert in the art of squinting, who excites in them the desire of imitation. In this way the strabismus would be the punishment for imitating the infirmity of a companion.

The hypothesis of our learned *confrère* of Vienna may have much about it that is true, but neither this theory, nor any of those that we have enumerated, fully satisfies us, and we prefer to acknowledge that many cases of convergent strabismus are still beyond our comprehension. This avowal does not cost us much, and certainly less than the building up of theories without foundation, and far-fetched attempts at explanation. We shall encounter still other obscure points in the course of these studies. We must know how to look them calmly in the face, in order to undertake with courage the task of elucidating them. General medicine, which has for ages been the object of investigation, affords many more unfilled gaps than does ophthalmology, which was, so to say, born yesterday. Hence let us not despair, but seek diligently, analyse the many facts that practice presents every day, and light will eventually dawn upon us.

The convergent strabismus of hyperopes is generally developed in early infancy. It is not rarely met with at the age of from two to four years.

Periodical and alternating at the outset, *i.e.*, manifesting itself only under the impulsion of attentive fixation, and sometimes in one eye, sometimes in the other, it becomes constant, as the child grows older, and localises itself upon one eye. This is generally the poorer of the two, if a difference between them exists.

In this connection, it has been asserted that the inferiority of the deviated eye, in point of visual acuteness, was not the cause, but the consequence of strabismus. The eye which was excluded from binocular vision, it has been supposed, either does not acquire, or else loses, its normal acuteness of vision, from want of exercise. We do not participate in this way of thinking. An eye may for years be excluded from vision without experiencing any alteration in visual acuteness. Hence the amblyopia of a squinting eye seems to us to be primitive and not secondary.

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[1] Mauthner, *Die optischen Fehler des Auges*, p. 557.

It is interesting, on the other hand, to observe with what facility the individual learns to ignore the retinal impression of the deviated eye, and the small value he seems to set upon binocular vision. Indeed, one never hears complaints of diplopia from such cases. The latter may, however, almost always be produced, though this is contrary to the generally entertained opinion. It is a question merely of facilitating this. Choosing some small object which vividly impresses the eye, such as a candle-flame, the eye that is in the habit of fixing is covered with a coloured glass, and the attention of the deviated eye is attracted to its own retinal image by the successive covering and uncovering of its fellow. It is, however, curious to notice that, when once having succeeded in seeing double, these persons have the greatest difficulty in projecting the retinal image of the squinting eye. They can scarcely tell on which side the flame, seen with that eye, is located; most frequently it is impossible for them to tell how far apart the images are from each other, and so to give the element of a subjective determination of the degree of strabismus.

This measurement is, however, easily made by the objective method, *i.e.*, by perimetric measurement.[1] The impossibility of obtaining it by the first method is not due to inconstancy of the deviation, or to a vacillation of the eye causing an analogous variation in the reciprocal position of the two images, but rather to the loss, by the squinting eye, of the faculty of locating retinal impressions.

It is this important fact which distinguishes this form of strabismus, which has been called *concomitant*, from *paralytic* strabismus, in which the diplopia is so marked, so distinct, so troublesome and so persistent.

The difference is, moreover, susceptible of a very natural explanation. Paralytic strabismus with diplopia is nearly always produced rapidly, and at an age when the eyes have for a long time been accustomed to binocular fixation. The convergent strabismus of hyperopes, on the contrary, is almost always developed slowly and gradually in childhood, when binocular vision has not become firmly established, and especially in an eye whose retinal impressions are less distinct, and perhaps less vividly perceived than those of its fellow.

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[1] See Landolt in *Traité complet d'Ophthalmologie*, De Wecker et Landolt, vol. i., p. 910.

Convergent strabismus is, however, occasionally produced rapidly in hyperopic children, especially when consequent upon general diseases, or even very slight affections of the eye, such as phlyctenular conjunctivitis and keratitis. In the first case, the result has been ascribed to a weakening of the power of accommodation; in the second, to a direct irritation of the internal rectus muscle, or, what seems more probable, to the influence of the occlusion or exclusion of one eye. The latter would, so to say, constitute an apprenticeship to strabismus. It suppresses binocular vision and gives to the excluded eye its freedom of direction. If the individual finds an excess of convergence advantageous, it is not surprising that he brings it about with that eye, which is of no use to vision properly so called.

Even in cases of concomitant strabismus of rapid development, spontaneous diplopia is lacking. But we must not conclude from this that young hyperopes attach so little value to binocular vision that they renounce it for ever on the slightest, and even transient, occasion. On the contrary, the great number of hyperopes who do not squint and who, nevertheless, have passed through the same incidents, have resisted the temptation and have been saved from strabismus by the stability of their binocular vision. It must be admitted that there is, with the minority of those who squint, a predisposition to strabismus, with whose exact nature we are not yet entirely acquainted.

C. *Strong Hyperopia.*—The *highest degrees* of hyperopia bear, most of all, the imprint of the arrest of development which characterises this form of ametropia and best present certain consequences of it.

Here a certain kinship between the conformation of the cranium and that of the eyes is oftenest recognisable. The hyperopic type of head is more pronounced. The eye itself is reduced in all its dimensions. The cornea is visibly smaller, and this smallness seems to us to explain the diminution, in its radius of curvature, which has been found to exist in such cases. It seems, indeed, that this membrane is more curved only because the diameters have been shortened by the same cause to which the smallness of the entire eyeball is due. One would say, moreover, that this phenomenon of constriction of the circumference of the cornea was effected in an unequal way, according to the various diameters or even in a totally irregular manner. Indeed, the cornea often presents considerable regular astigmatism, with which is frequently associated an incorrigible irregular astigmatism.

The smallness of the eye might seem a condition favourable to extended excursions. But, in reality, the field of fixation of hyperopes of this category is restricted and shows a degree of mobility inferior to the normal. The cause of this is, without any doubt, the weakness of the extrinsic muscular apparatus, which has also suffered from the general dystropy of the visual organ.[1]

The ciliary muscle, even, does not escape this influence. Contrary to what we might expect, it is less vigorous than that of a normal eye, or of one affected by hyperopia of moderate degree, and the range of accommodation in such cases is generally restricted, so that this function is almost always insufficient for the correction of the imperfect retinal images produced by an insufficient dioptric apparatus.

This would, indeed, of itself be a satisfactory explanation of the incapacity of strongly hyperopic eyes for sustained vision. The principal cause of it must, however, be sought in the inferiority of the nervous apparatus, which also participates in the incomplete formation of the eye. In spite of perfect correction of the refractive error, not only is the visual acuteness often considerably reduced, but the eyes are unable to work long at a time without experiencing a sensation of unbearable fatigue which soon obliges them to cease work. It would seem that the provision of nervous substance, requisite for prolonged vision, is less than in others and is exhausted with greater rapidity.

The ophthalmoscope, moreover, often furnishes strong indications of the imperfections of the nervous apparatus. The papilla presents itself in the form of a disc, sometimes of abnormal pallor, sometimes, on the contrary, of a deeper colour than is normal, and which scarcely differs from that of its surroundings. Its limits lack distinctness and are often irregular.

Moreover, we may here add a peculiar disposition of the retinal vessels, which are extraordinarily sinuous, so that they may, in certain cases, suggest the appearances of optic neuritis. This appearance may be explained, however, according to Horner, in the following manner: —The retinal vessels are developed in a cavity previously formed, that of the ocular globe. The latter being remarkably small in high degrees of hyperopia, the vessels are hindered from extending themselves to their full length and, therefore, become twisted. We may mention, finally, the peculiar lustre of the retina, which at times resembles asbestos.

That which specially characterises high degrees of hyperopia, and authorises us to make of them a separate group, is the peculiar way in which their vision is accomplished.

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[1] Landolt, "Etudes sur les mouvements des yeux, &c.;" *Archives d'opht.*, i., p. 587, 1881.

The *vision of hyperopes* is variable, according to the degree of their refractive defect. When the latter is not very pronounced, the only inconvenience resulting from it is, as we have seen, the precocious appearance of symptoms of presbyopia, from insufficiency of the accommodation, which, in this case, is compelled to perform the double task of supplying the lack in static refraction, and of adapting the eyes to near work.

We remember that moderately hyperopic eyes do not differ much from the normal, and that it is only with advancing age that a necessity arises for holding the book farther away from the eye, or for having recourse to glasses, while distant vision is still for a long time perfect. We have noticed that, in the medium degrees, the characteristic tendency to hold the object farther away is still more marked, unless the hyperope renounces binocular vision, giving way to a no less pathognomonic convergent strabismus.

The phenomena presented by an individual affected with *strong* hyperopia are quite different. Far from moving the book away from him, he brings it, on the contrary, very close to his eyes—much nearer than his *punctum proximum*—in such a way as to simulate true myopia. This error is soon dissipated by an examination of the eyeballs, which present a noticeable smallness, or, at all events, a form very different from that which myopic eyes, reading at so near a *punctum remotum*, would have. The hyperope, indeed, so to say, applies his eyes to the paper; being unable to obtain distinct retinal images, he seeks to obtain as large ones as possible. His circles of diffusion are not an obstacle for him, for, in the first place, their diameter, generally limited by the narrowness of the pupils, increases only in proportion to the square root of the decrease in distance, while the size of the image increases proportionally to this decrease itself. In the second place, there exists, in his case, a singular aptitude for analysing imperfect retinal images, and making out the form of the object in spite of the diffusion-circles of which its image is composed.

That does not prevent his having recourse to all possible means of reducing them. Thus, he will seek, by preference, a very strong light, which contracts his pupils; he will limit the size of the latter still more by partially closing the lids, or, when he uses only one eye, by carrying the object to the opposite side of the median line. In this way, he sees over his nose, the bridge of which intercepts a part of the luminous rays and thus restricts the diffusion-circles.

A second capital difference, which exists between the second and the third class of hyperopes, is that convergent strabismus is exceptional in cases belonging to the latter.

This fact has, on account of its singularity, occasioned much surprise; to such a point, indeed, that Stellwag and Mauthner have wondered if statistics had not deceived us in this regard, and if the rarity of strabismus in hyperopia of high degree was not due to the comparative infrequence of the latter itself, or if it had not been overlooked in many cases of hyperopic convergent strabismus.

Other authors, without being stopped by such scruples, have seen in this phenomenon a formal contradiction of Donders' theory. They think that, if the convergent strabismus of hyperopes really be due to the considerable effort of accommodation that they are obliged to exert in order to correct the insufficiency of their refraction, and which they purchase at the price of exaggerated convergence, this convergence ought to increase proportionately to the refractive defect,—*i.e.*, to the degree of hyperopia. So that, among hyperopes whose defect is greatest, we ought also to meet with the greatest frequency and the strongest degrees of convergent strabismus.

Donders has already answered this argument by saying that the hyperope, whose refractive defect is very considerable, renounces even the effort to neutralise it by means of his convergence, because of a feeling of its inability to wholly correct it.

Then a new objection has been raised: hyperopia, even when strong, rarely exceeds a degree which cannot be largely overcome, by the accommodation at the disposal of the individual, at the age when convergent strabismus is usually developed. Thus, supposing the range of accommodation, which amounts to 14 dioptries at the age of ten years, to be at least great enough prior to that age—(it has not been measured in persons younger than that),—it amply suffices to correct a hyperopia of 8 dioptries, and still to bring the *punctum proximum* to a focal distance of $14-8=6$ dioptries—say 16·6 centimetres.

But two things have here been forgotten. In the first place, a hyperope of 8 dioptries is obliged to furnish accommodative work which is always 8 dioptries greater than that of the emmetrope. If to each dioptry of accommodation there corresponds 1 metre-angle of convergence, these 8 metre-angles would represent, for a young person whose base-line is 57 millimetres long, 13° of convergence (see p. 184). And if, as is generally true, the deviation is wholly confined to the eye excluded from vision, the objects fixed being held directly in front of the other, the squinting eye would have to furnish, even in distant vision, a convergence of $2 \times 13 = 26°$, and a much greater amount when looking at near objects. The requisite convergence would soon have exceeded, and that greatly, the limits of what an eye can attain.

But even the less degrees of convergence are not so easily realised as seems to be believed. We have already called attention to the fact that, in order to keep up work for any length of time, there must be in reserve a certain quantity of the force that produces it (p. 173, *et seq.*). This quota is rather large for convergence. Hence it is easy to understand that, even though the individual should still succeed in realising, for an instant, the degree of convergence requisite for his perfect accommodation, he could not sustain it; for this degree represents his entire range of convergence, or a very large portion of it, when it ought to be only a very small part.

Convergent strabismus is not produced in those cases simply on account of the absolute or relative weakness of the adducting muscles.

Something similar to this may be produced with the accommodation. However superior the strength of the ciliary muscle may be to that of the extrinsic muscles of the eye, the accommodation is, nevertheless, not at the *absolute* disposal of the individual. It is certain that the greater the quota of accommodation already expended, the more trouble its production costs. The last dioptry of amplitude of accommodation requires, indubitably, a larger quota of innervation and of muscular effort than does the first. Hence, neither the accommodation brought into play, nor the convergence, can be estimated by the absolute number of dioptries, but by quantities *relative* to the total accommodation at the individual's disposal—in short, by quotas.

The accommodation, too, must have a quota in reserve, in order that ocular work may be accomplished for any length of time. If the hyperopia is of very high degree, it is quite possible that the required accommodation may be more than the ciliary muscle can furnish, especially if the latter is insufficiently developed. Hence the individual dispenses with an exaggerated effort of convergence, not only because his convergence power is too slight, but also because his accommodation would not be sufficient in any event. This may be another one of the reasons why these hyperopes acquire the habit of renouncing optical adaptation and bring objects so near their eyes.

If convergent strabismus is exceptional in hyperopes whose refractive error is of high degree, *divergent strabismus* is not rarely met with in their cases.

This phenomenon will not surprise us, if we bear in mind the constitution and workings of such eyes. They are very imperfectly developed, as was said above, and work badly in all respects. One would think that, having already sufficient difficulty in seeing with each eye separately, these hyperopes would not undertake the much more complicated task of binocular vision.

It is quite possible that the tendency to the continued working of both eyes, which is the privilege of the higher orders, is itself as little developed as the means of satisfying it, in visual organs arrested in their evolution.

At all events, these eyes being amblyopic, sometimes to a very high degree, they must easily take the direction that amblyopic eyes usually do, *i.e.*, divergence. This seems, indeed, to be the true position for eyes that are not in use. The explanation that we have given of this fact,[1] that we may perhaps see in this a phenomenon of atavism, a reminiscence of the inferior orders, in which the axes of the eyes diverge like those of the orbits—this explanation would nowhere find a greater justification than in the cases of divergent strabismus of strongly hyperopic eyes, which, in so many regards, are like the eyes of inferior orders.

Whether or not the desire for binocular vision exists, such vision is alreadly excluded, and the divergence, at least relative, is already established, in the case of hyperopes who bring objects nearer than the *punctum proximum* of their convergence. Nor is it a matter for surprise, either, if this relative divergence becomes absolute, *i.e.*, subsists for all distances, even for infinity. The divergent strabismus of extreme hyperopes is, in fact, easier to explain than is the convergent strabismus of myopes.

*Symptoms and Diagnosis of Typical Hyperopia.*—The symptoms and diagnosis of hyperopia result in a very natural way from the facts that we have just pointed out. In order to give a description of it, at once easy to understand and corresponding to clinical facts, we shall maintain our division of hyperopes into three categories.

The hyperope whose defect is of *low degree*, and who is of medium age, presents the greatest resemblance to an emmetrope. Like the latter, he sees well at a distance; if his accommodation still be strong, he sees equally well near at hand and nothing in his anamnesis makes him distinguishable from an emmetrope. The hyperopia will make its presence suspected only when the person who consults the doctor, although still relatively young, tells him that he sees well at a distance, but experiences a certain difficulty in fixing near objects in reading or writing.

This resemblance to normal refraction may extend, in youth, even to *medium degrees* of hyperopia. The latter, though, scarcely fail to strike the observer, whether on account of the patients' complaints, since they experience decided difficulty in seeing near at hand, or by an *ensemble* of external signs, which we have described, and which might be called the hyperopic *habitus;* a brachycephalous cranium, a flat face, with small and mobile eyes, already make us suspect the presence of hyperopia, and if to this aspect a tendency to exaggerated convergence, or an already established convergent strabismus is added, one may be almost sure of having to deal with a certain degree of hyperopia.

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[1] Landolt, Article "Strabisme," in Dechambre's *Dictionnaire Encyclopédique des Sciences Médicales*, p. 268.

This hyperopic *habitus* is still more striking in the *higher degrees.* Though combined with a tendency to bring the objects fixed excessively near, it will at once be distinguished from extreme myopia, which gives rise to the same peculiarity. We have, moreover, in this category of typical hyperopia, insufficiency of visual acuteness, sometimes divergent strabismus, and improvement of vision under the influence of strong convex glasses.

The hyperopia is, indeed, indubitably established only when *optometry* has brought out the defect of static refraction which characterises it. It is, as has been shown above (p. 253, *et seq.*), the *ophthalmoscope,* that represents the surest means, not only of establishing the nature, but also of determining the degree, of the ametropia. Spectacle-glasses placed before the eye, or any other of the subjective methods of optometry above described, can by no means give an equally trustworthy result. The reason for this has already been given. It is the interference of the dynamic refraction, the *accommodation.*

This ability to increase refraction plays a very important part in hyperopia, because it serves to correct or neutralise a portion, if not the whole, of this defect in refraction and to adapt the eye for finite distances (p. 172).

The power of accommodation, employed with a view to correcting hyperopia, is so great in young persons that it not only adapts their eyes perfectly for long and short distances, but it is exerted unnecessarily, and even when detrimental to the eyes, when they are supplied with convex glasses. It happens, in fact, that young hyperopes reject such glasses, just as an emmetrope would. Nothing but the ophthalmoscope and mydriatics permit the refractive condition to be ascertained in such cases.

Theoretically, it might be supposed that seeking the *punctum proximum* ought also to lead to the same thing. The range of accommodation being the same in all healthy eyes, whatever their refractive condition, the *punctum proximum* ought to be further away in hyperopia than in emmetropia. In other words, the refraction which they present, when fixing their *punctum proximum,* must be less in eyes that expend a certain quota of it to neutralise their defect in static refraction than in the eyes of emmetropes at the same age who employ all of it within zero, that is to say, in the domain of positive refraction (p. 183).

This means of diagnosis would not, however, prove very practical. Hyperopia that is capable of being thus confounded with emmetropia is never of a very high degree. The difference between the maximum refraction of which the eye is susceptible ($p$), in normal and such hyperopic eyes, is, therefore, only very slight. The corresponding difference of P, *i.e.*, of the distance between the *punctum proximum* and the eye, is likewise very small, because the phenomenon in question is present only in very young people, whose accommodation is very powerful, whose *punctum proximum* is, therefore, very near to the eye, and in whom, consequently, a difference of several dioptries in refracting power implies only a very slight difference in focal length. For instance: two children, ten years old, have each an amplitude of accommodation amounting to fourteen dioptries. In the first, an emmetrope, $p$ amounts to 14 D, and the *punctum proximum* is at $P = \frac{1^{\text{m.}}}{p} = 71$ millimetres. The second, hyperopic by 3 D, has a $p$ amounting to $14 - 3 = 11$ D. Consequently $P = \frac{1^{\text{m.}}}{11} = 91$ millimetres. Despite the difference of 3 D between the static refraction in the two cases, that between the position of the *punctum proximum* is, therefore, not greater than 2 centimetres. Now, a difference of 2 centimetres, in the distance of vision, is difficult to estimate absolutely in subjective optometry. It would be necessary, in order to increase the linear difference, to furnish the eyes with concave glasses, to remove further the *punctum proximum*, as we have pointed out on p. 285.

But, besides being more complicated than the objective determination, this method would always be blemished by a source of errors, because the constancy of the range of accommodation is not absolute. It is subject to variations, even in a physiological state, but particularly under the influence of general or local debilitating causes (weakness or paresis of the ciliary muscle). Hence one could not draw an absolutely certain conclusion as to the degree of hyperopia, or as to the absence of this form of ametropia, from the situation of the *punctum proximum* of a young child.

## LATENT—MANIFEST—TOTAL HYPEROPIA.

It often happens that a spasm of accommodation conceals the *whole* of the hyperopia, when the latter is not of very high degree, but especially when the accommodative power is very great, as is generally the case with children.

In other cases, the young hyperope, although seeing well at a distance, *i.e.*, *being able* to neutralise his ametropia by means of his accommodation, accepts, notwithstanding, the help of convex glasses. His visual acuteness, already at the maximum without a glass, remains so until the convex glass reaches a *certain power*. Say, for instance, one dioptry. A knowledge of this fact is of great value in diagnosis, because it establishes, with certainty, the presence of hyperopia; the emmetrope would not have seen so well at a distance with the convex glass. But, if this hyperope be examined with the ophthalmoscope, it may be found that, in reality, his hyperopia amounts to 5 D. Hence the convex glass has rendered manifest only *a part* of the total hyperopia, while another part, equal to $5 - 1 = 4$ D, in our example, has remained latent and remains latent in spite of the strongest glasses. These glasses only lower again the acuteness of vision, because the young hyperope is not able to relax his accommodation beyond a certain limit.

The *total hyperopia* ($H^t$), which remained latent in the first case, is therefore composed in the second of a part that the individual can at will, or at least with the aid of a convex glass, render *manifest* ($H^m$), and of another part which, in spite of him, remains *latent* ($H^l$). $H^t = H^m + H^l$.

When *total* hyperopia is very strong, relatively to the power of accommodation, the latter does not suffice to entirely correct the former, however great an effort the individual may make. In this case weak convex glasses *increase* visual acuteness immediately, up to a certain limit. This limit being reached, the acuteness of vision remains stationary, in spite of the upward progression of the number of the convex glass, and, finally, decreases when a certain strength of the correcting-glass is exceeded.

Thus it may happen that a person does not see well at a distance, and obtains the maximum of his visual acuteness only with the aid of the convex 1. If we give him the 1·5, 2, 2·5 or 3, vision is no longer bettered, but it is not lessened, either—it remains the same. But with the convex 3·5, the patient instantly sees worse. What is here the amount of manifest hyperopia? Evidently 3 D.

It is well, however, to distinguish between this manifest hyperopia and that of the preceding example. In that case all the manifest hyperopia could at will be corrected by the accommodation; it was wholly *facultative*. In the second case a portion of it (1 D) is *absolute*, and only $3 - 1 = 2$ D are facultative.

Hence manifest hyperopia ($H^m$) may be composed of two factors—an *absolute manifest hyperopia* ($H^{ma}$), which no effort of the accommodation can conceal, and which is represented by the *weakest* glass with which the individual acquires his maximum of visual acuteness; and a *facultative manifest hyperopia* ($H^{mf}$), which he can correct at will, and which is represented by the difference between the strongest and weakest glass which procures for him the best vision at a distance—$H^{mf} = H^{m} - H^{ma}$.

With all that, a portion of the total hyperopia may still remain latent; so that, according to our example, a total hyperopia of 5 dioptries may be made up as follows:—

$$\begin{array}{rll} & H^{l} = 2\ D & \\ & H^{m} = 3\ D, \text{ of which} & H^{mf} = 1 \\ \text{Total,} & \overline{5\ D} & H^{ma} = 2 \\ & \text{Together,} & \overline{3\ D} \end{array}$$

In proportion as age advances and the range of accommodation correspondingly diminishes, a greater part of the total hyperopia becomes manifest and a greater part of the facultative manifest passes over to the absolute.

Thus it happens that all the hyperopia is revealed by the subjective examination and that our hyperope of 5 dioptries, for instance, sees perfectly well at a distance with the convex 5. In this case the total hyperopia is evidently equal to the manifest, $H^{t} = H^{m}$. But, if this patient still sees as well with a weaker glass as with the number 5, this proves that he can still correct a part of his ametropia by means of his accommodation. We will try to find what is the weakest glass that suffices for him: if it be the $+3{\cdot}5$, we say that his manifest hyperopia amounts to $3{\cdot}5$ D, and the facultative to $5 - 3{\cdot}5 = 1{\cdot}5$ D. Finally, when accommodation is *nil*, there is no longer any means of correcting, spontaneously, even the smallest fraction of the hyperopia. All of it will have become not only manifest, but *absolutely* manifest.

This is what happens to every hyperope after the age of sixty-five years, because, with every one, the power of accommodation is lost at about this epoch. We then write $H^{t} = H^{ma}$. In this case, visual acuteness, after having gradually increased, with the number of the convex glass, up to the one that raised it to its maximum, diminishes as soon as a stronger glass is tried. It does not remain invariable, because facultative manifest hyperopia has disappeared at the same time with the accommodation, $H^{mf} = 0$. This is a condition just opposite that which we considered in the first place, where, in a child, all of the hyperopia was latent, so that no part of it could be rendered manifest, $H^{m} = 0$.

If we recapitulate this discussion, we shall say, therefore, the *total* hyperopia is indicated, aside from the ophthalmoscopic examination, by the convex glass which gives the maximum acuteness of vision to an eye whose accommodation is entirely paralysed. The latter being intact, on the contrary, the *strongest convex glass* that gives the subject the best acuteness, indicates the degree of *facultative manifest hyperopia*, and the *weakest convex glass* gives the *absolute manifest hyperopia*. The *latent hyperopia* is constituted by the *difference between* the *total* and the *manifest hyperopia*.

These distinctions being established, it is self-evident that the relations between these divers quantities are susceptible of very extended variations. The relation between latent and total hyperopia, generally the same for a determinate range of accommodation and a given degree of hyperopia, oscillates with these two factors. The stronger the total hyperopia is, the greater is also the effort of accommodation exerted by the hyperope to overcome it, and, consequently, the more considerable is the fraction of such effort that is incapable of relaxation. Since accommodation diminishes as age advances, latent hyperopia will diminish correspondingly until the accommodating power has been entirely lost.

Thus practice shows us, every day, hyperopic children with whom even the weakest convex glass produces a decrease in the acuteness of vision. The physiological spasm of accommodation, in such cases, is equal to the whole of their hyperopia. Past the age of sixty-five years, on the contrary, all latent hyperopia has disappeared, which is equivalent to saying that all the hyperopia has become manifest.

The same factors influence the proportion between facultative and absolute manifest hyperopia.

Dr. v. Schroeder of St. Petersburg,[1] at that time our *chef de clinique*, has attempted to deduce a law governing the relation between total and latent hyperopia (and, consequently, between the latter and manifest). He assumes that the latter is the expression of a constant effort of the ciliary muscle, designed to conceal, in the young child, all the hyperopia. The latter being of variable degree for different individuals, the effort in question varies correspondingly, according to the hyperopes; but the individual effort remains constant during the entire lifetime.

Only, as the elasticity of the crystalline diminishes as age advances, the effect of this constant effort will grow less and less. Hence latent hyperopia is a constant function of accommodation, which may be expressed by the following formula :—

$$H^l = \frac{H^t \times a}{14},$$

in which $a$ designates the accommodation corresponding to the age of the subject, and the number 14 the amplitude of accommodation, as it is supposed to be for youth,—*i.e.*, at the moment when the individual has deducted from it the quota necessary for the neutralisation of his total hyperopia.

The most elementary discussion of this formula clearly demonstrates the diminution of latent hyperopia with accommodation, or, in other words, with the advance of age, and its increase with the degree of total hyperopia. Thus, let us take the case of a person nineteen years old affected with a total hyperopia of 4·5 D—What is his latent hyperopia? The measurement of his amplitude of accommodation gives us, conformably with Donders' diagram, $a = 10$ D. Hence we shall have—

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[1] Th. de Schroeder, *Arch. d'Oph., Juillet et Août*, 1882.

$$H^l = \frac{4{\cdot}5 \times 10}{14} = 3{\cdot}21 \text{ D},$$
$$H^m = 4{\cdot}5 - 3{\cdot}21 = 1{\cdot}29 \text{ D}.$$

It is easy to see that $H^l$ is weak in proportion as the degree of $H^t$ is low.

A series of measurements made by von Schroeder showed a remarkable coincidence between the figures for manifest hyperopia, computed in this way, and those found empirically by the use of convex glasses.

The formula remains absolutely valid for cases in which the diminution of the range of accommodation is not due to the resistance of the crystalline, but to paretic weakening of the ciliary muscle. The result is, indeed, the same; an identical nervous impulsion produces, in both cases, an effect upon the accommodation which is less than in the normal state.

We have seen that the quota of accommodation representing $H^l$ varies with the degree of total hyperopia. It is a fraction whose denominator equals 14 (the amplitude of accommodation in extreme youth) and whose numerator is the degree of the total hyperopia. Von Schroeder calculated and made a synoptic table of the various amounts of the latent hyperopia for different ages (from ten to seventy-three years), and for degrees of hyperopia ranging from 1 to 4 dioptries.

*Visual troubles produced by hyperopia.*—The visual troubles to which hyperopes are exposed are due especially to the effort of accommodation which their refractive defect imposes upon them.

Although, in youth and with a not too strong hyperopia, this surplus of work is so easily borne by a vigorous ciliary muscle that it gives rise to no complaint or inconvenience, there are degrees of hyperopia and there comes an age, with and at which this work is not easily furnished. The ametropia may still be corrected for a distance, but near vision commences to be fatiguing. The patient seeks to diminish the effort of accommodation by holding the book away, or withdrawing his head from the desk. He is obliged to discontinue work, from time to time, to let the ciliary muscles rest. Thanks to this stratagem, many hyperopes are able to continue their work fairly well. Others, on the contrary, are early tormented by symptoms of *asthenopia*, provoked by the excessive working of their accommodative muscle. Sight becomes dim, objects disappear and reappear, dance before their eyes and even seem, at times, to become double. A sensation of heaviness, which may pass into real pain, invades the eyes and forehead. This cephalalgia may take on the form of a genuine migraine and render impossible, not only all work, but even the fixation of the most distant object. Such persons can escape from their discomfort only by closing the eyes and thus precluding the possibility of any accommodative stimulation. Asthenopia arises all the more easily in proportion as the hyperopia is of greater degree and the power of accommodation less.

Weakness of the accommodation may originate in either of two very distinct ways. It may be due to the crystalline, which, having lost more or less of its elasticity, requires a more energetic muscular contraction for the production of the desired increase in refraction. This is the cause which, in all eyes, brings about a diminution of the range of accommodation as age advances, beginning at that of ten years. This explains, too, why all hyperopes are doomed, sooner or later, to asthenopia, unless they have recourse to the preventive means which we shall point out later on.

But insufficiency of accommodation may also be brought about by a weakening of the ciliary muscle. Thus we often see asthenopia in anæmic children and young persons, those that are poorly nourished or convalescent from diphtheria, or affected with paresis of the ciliary muscle of some other origin. The accommodative power, being thus reduced, no longer suffices to adapt the eye or keep up a prolonged adaptation. We have, therefore, no reason to be astonished if we find, pretty frequently, asthenopia with a low degree of hyperopia and in young persons who ought to possess a very extended range of accommodation. They do not always have it.

Finally, the visual troubles mentioned may also be produced, in spite of a weak hyperopia and powerful accommodation, when the peculiar nature of the individual's work, or the imperfection of his sight, does not permit him to choose at will the distance at which objects, with which he is to busy himself, shall be placed. These objects may be so small—as, for instance, the meshes of the needlewoman's embroidery canvas, or the lines that an engraver must copy, &c.—and demand such precision, that work is possible only on condition that the retinal images be rather large. To obtain such images the person is obliged to get very near to his work.

The same is true when the visual acuteness is insufficient whether this be in consequence of astigmatism, opacities of the refractive media, or changes at the *fundus oculi*. Here, too, the patient tries to render the retinal images as large as possible, in order to make up for their imperfection, or for the insufficiency of the impression which they produce upon a weakened nervous apparatus. The hyperopes of whom we have spoken, whose defect is of high degree, furnish us the most striking examples of this kind of vision.

Now the (axial) hyperope, who adapts his eyes with the aid of his accommodation alone, finds himself under the most unfavourable circumstances for obtaining large retinal images. A single glance at the accompanying figures will at once make this clear. The three eyes, M, E and H, have each the same optical power, and all fix the same object, A B, situated at the same distance, C *c*, from each, and each obtains a clear retinal image of it. But the size of this image is very different for the three eyes. This size is determined by drawing straight lines (lines of direction, or secondary axes—pp. 23 and 60) through the nodal point (the centre of curvature of the refractive surfaces of the reduced eye), from the extreme points, A and B, of the object, to the retina, where they indicate corresponding points of the image. The nodal point is the same in all three eyes, so long as their refractive systems are of the same strength,—at rest, for instance.

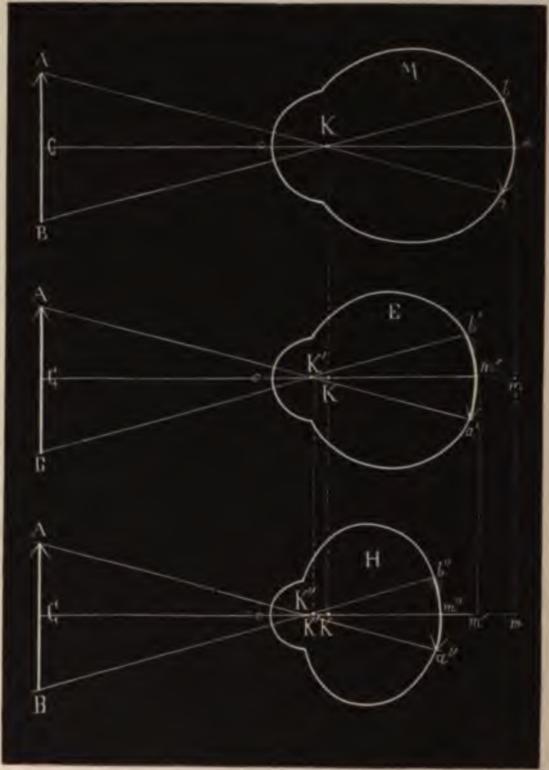

FIG. 119.

If the eye M be so myopic that its *punctum remotum* is located just at C, it will see the object A B distinctly, without an effort of accommodation. Its dioptric system does not vary, K remains where it is, when the eye is at a state of repose, and the retinal image will have the size *a b*.

The emmetropic eye E, when at rest, does not see at a short distance. Hence it makes an effort of accommodation. The strength of its dioptric system increases and the nodal point advances a little, from K to K′.

The effect of the increase of refraction upon the position of the nodal point could not be made more evident, or more easy to understand, than in the reduced eye, as in the annexed diagrams, in which the dioptric system of the eye is represented by a single refracting surface, whose nodal point is the centre of curvature itself. The increase of refraction, due to the accommodation, is necessarily represented, in the reduced eye, by an increase in the convexity of its refractive surface. Hence its radius of curvature diminishes and its centre approaches it, coming up to K′ for the emmetropic eye E.

In spite of this advancement of the nodal point, the retinal image *a′ b′*, of the emmetropic eye, is notably smaller than that of the myopic one. The reason for this is that the distance K′ *m′*, between the nodal point and the retina, is less in the emmetropic eye than the corresponding distance, K *m*, in the myopic one. The increase K K′, to which it has been subject in consequence of the advancement of the nodal point, is much less than the difference, *m m′*, between the lengths of the emmetropic and myopic eyes.

In fact, it may be said, generally speaking, that the adaptation of an eye to short distance, which is produced by the elongation of its axis, supposes a much greater distance between the nodal point and the retina, and much larger retinal images, than does adaptation by means of the accommodation. This explains why myopes see so well near at hand. It solves, too, remarkably, the puzzle presented by those extreme myopes who, notwithstanding the most perfect correction, obtain a visual acuteness, for distance, infinitely poorer than that of an emmetrope, and still analyse much smaller objects, near at hand, than the latter can.

The difference in near vision is still more pronounced for hyperopia. The eye H (Fig. 119) shorter than E, and therefore not so well adapted for vision at the distance *c* C, increases its refraction still more—makes a more considerable effort of accommodation. The radius of curvature of its refractive surface is shorter than in E, the nodal point is still displaced forward, but yet this advancement K′ K″ is less than the amount *m′ m″*. Hence K″ *m″* is less than K′ *m′* and, consequently, the image *a″ b″* smaller than *a′ b′*, proportionally to the difference between K″ *m″* and K′ *m′*. It results from this that the stronger the axial hyperopia is, the smaller are the retinal images obtained by means of the accommodation. Here is, for the hyperope, a new cause of difficulty in working.

The higher the degree of hyperopia, and the more the range of accommodation diminishes, the more distant will be the point at which asthenopia commences to manifest itself. Far from being limited to working-distance, it may vary so as to include infinity. Thus a hyperope of three dioptries, aged forty-seven years, must necessarily experience very sensible fatigue when he wishes to see distinctly at a distance, because he succeeds in doing so only by employing all of the accommodative power at his disposal.[1] With all the more reason is vision at a finite distance absolutely impossible for him. At a still more advanced age he will no longer see clearly even in the street. This is what happens whenever the accommodation is less than the degree of hyperopia. Such persons are then really to be pitied, since there is nowhere between heaven and earth an object that they can see distinctly with the naked eye.

We have explained how many hyperopes purchase the necessary accommodation at the price of binocular vision; how they thus acquire *convergent strabismus.* This strabismus, which is generally developed at a very early age, often does not cause the patients to complain. They are scarcely ever annoyed by diplopia and, having never known the advantages of binocular vision, they hardly suffer from its absence. It is more likely to be cosmetic considerations, in such cases, that bring them to the doctor, or their indifference to considerations of this sort which prevents their seeking his advice. However this may be, we know what capital importance attaches to the collaboration of the two eyes. Whether or not the individual thinks himself disfigured by convergent strabismus, we know enough of what he lacks, when he has lost binocular vision, so that we consider convergent strabismus one of the most deplorable consequences of hyperopia.

It also happens that, when this discordance in the movements of the eyes is developed at a more advanced age, the patients are extremely troubled by transient diplopia and the painful fatigue brought about by exaggerated contraction of the ciliary muscles. Often they complain only of vague symptoms of asthenopia, pains in the head, vertigo, &c.; but they are not aware that they sometimes see double or that they squint. The examination with a candle and coloured glass brings the phenomenon into evidence.

Finally, a cause of visual troubles, that is frequently connected with hyperopia, is *amblyopia.* It may be due to regular astigmatism, and is then corrigible by means of glasses; or to irregular astigmatism, against which we possess scarcely any remedy. Not less important is the amblyopia due to imperfection of the nervous apparatus of the eye, such as is often met with in the highest degrees of this form of ametropia.

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[1] Compare the diagram on page 174.

These are the principal visual troubles that are susceptible of being produced by hyperopia. It must not be forgotten, however, that the latter does not preserve the eye from any disease; in other words, that the hyperope may be attacked by any kind of ocular trouble, just as the emmetrope or myope. But we must guard against attributing, under such circumstances, everything to his refractive condition, as has often been done in an excessive spirit of system.

## THE TREATMENT OF HYPEROPIA.

Hyperopia may be spontaneously rectified. Statistics, however fallacious they may be in general, clearly prove this. The proportion of hyperopes is much greater among children than among adults. Hence many emmetropes are cured hyperopes; and many myopes were formerly hyperopes; nor have they much cause for rejoicing in this change of refraction, as we shall see.

This diminution and disappearance of hyperopia, which takes place gradually in certain individuals during the period when the entire organism is growing and acquiring its definitive form, may easily be followed with the aid of the ophthalmoscope. The eye participates in this general development, and, from being imperfect, as it has been, becomes normal.

Nature alone can bring about this happy transformation. We are incapable of producing it. Although hyperopia is spontaneously rectified, we possess no means of changing the form of the eye, elongating its axis or increasing the power of its dioptric system.

But, in default of a radical cure, we possess the most effective palliative remedies. These are, before all others, *convex lenses.* Such glasses, by adding their refractive power to that of the eye, can supply any deficit in the latter and correct no matter what degree of hyperopia. More than this, if the accommodation is weak, they come to the aid of the dynamic refraction, as well as supply the lack of static refraction. In short, we have in convex lenses the means, not only of correcting all possible degrees of hyperopia, but of adapting the eye for vision at any desired distance, by relieving its accommodation to any amount that we may wish.

Convex glasses are certainly most powerful and most precious remedies for the hyperope; but they are a benefit to him only on condition that he uses them judiciously; otherwise they may do harm, as is the case with any active remedy. In fact convex lenses, and spectacle-glasses generally, must be regarded as remedies. As such they are destined to *second the action of nature*, but in nowise to replace or force it. Hence their use must be limited to strict indications, and we must not try to impose them upon an eye that can dispense with their aid, nor must we give, to him who needs convex-lenses, stronger ones than are necessary.

Let us say here, once for all and generally, for all kinds of lenses: the use of spectacle-glasses, even though they be well chosen, has not only its advantages, but also its inconveniences. It is not solely that they are unbecoming, as they are often reproached (and not alone by ladies) with being, nor that they may be broken while being worn, and a piece of them enter the eye. This is certainly a serious disadvantage, if it occurs. But it should not be forgotten that glasses easily become greasy, dusty and covered with all sorts of dirt, and may thus trouble vision more than their refractive effect increases it. Besides, their borders are sometimes very annoying, and many persons find it extremely difficult to accustom themselves to this cloudy circle surrounding everything looked at. Moreover, their action is quite different when one looks through them in the direction of their axes and when they are looked through obliquely. In the latter case, the objects looked at may assume very odd shapes, according to the kind and strength of the glasses; they may appear displaced, doubled for both eyes, or even for a single eye, when the edge of the lens is looked through. This fact obliges the wearer to direct his eyes so that the visual lines shall be as nearly as possible perpendicular to the plane of his spectacle-glasses, and thus restricts the field of the eyes' excursions. He has to replace the latter by rotations of his head, which is much more complicated.

Another great inconvenience of spectacle-glasses consists in the reflection-images, which they produce, of objects situated behind them. These images are the more annoying in proportion as the objects toward which one's back is turned are, relatively, better illuminated than the general field toward which one is looking. Let one try, for instance, to cross a room, starting from the windows, the floor being occupied by the usual articles of furniture and lighted only by the glimmer of twilight; one will be stopped every moment by the reflections of the windows, whose brightness destroys the faint impression that one receives of the objects before one. The same thing is observable, too, when one reads with a bright light behind, and a little to one side of, the head.

We shall abstain, therefore, from giving spectacles except in cases of absolute necessity, or unless they render the patient so great service that the attendant inconveniences disappear in comparison with it.

In case of the trouble which we are at present considering—hyperopia—we shall have recourse to spectacles only when they are demanded on account of asthenopia or insufficiency of sight. But we shall not prescribe them, for instance, for young persons having low degrees of hyperopia, who see perfectly well both near at hand and at a distance, and have no trace of either asthenopia or strabismus.

When spectacles are ordered, it should be done with the greatest discernment. One must not give way to the idea that, if the glass is not strong enough, the patient will return to have another prescribed, and that, if it is too strong, he should be glad of it, since, in this case, his accommodation will be so much the more relieved. Whoever should proceed on this plan would grossly deceive himself. Between convergence and accommodation, which are customarily associated together, there exists an intimate relation which cannot be interfered with without the production of serious inconveniences as regards vision. Even he who sees with only one eye has accustomed himself to a certain position of the body, a fixed distance for his work, a given size of retinal images and a nearly constant effort of accommodation, all of which conditions he can renounce only with difficulty, even when his own interest demands it. Why should he be forced, by too strong glasses, to do this when he has nothing to gain by it?

The choice of glasses is a delicate operation; he alone is successful in it who, to a perfect theoretical acquaintance with the subject, adds the intelligent observation of each patient. It does not suffice to know the action of lenses, from every point of view, and the workings of the visual organ. We must know, more than this, how to individualise cases,—that is to say, we must take into account the state of refraction and accommodation, that of the muscles of the patient's eyes, the particular purpose of his wearing glasses, his peculiar habits, and many other circumstances to which attentive observation in practice teaches us to attach importance.

It is impossible, therefore, to make rules, in this matter, which may be blindly followed, which would answer all questions by anticipation and relieve the oculist from the necessity of reflecting and combining for himself. We believe, however, that we can give, for the use of convex glasses, a general principle, which has long served us as a guide, and concerning which we have occasion for satisfaction only, especially because we never close our eyes to the exceptions to any rule.

We consider the static and dynamic refraction as *one whole*, and we say—

*The convex glass ought to correct the whole of the refractive defect, and, moreover, disengage a certain amount (quota) of accommodation, which will help the person to keep up his ocular work the desired length of time.*

Thus, a hyperope of five dioptries, who has an amplitude of accommodation amounting to three dioptries, has, for distant vision, a refractive defect of $3 - 5 = -2$ D. We shall give him a somewhat stronger glass than the $+ 2$ D, because, otherwise, he would be obliged, from the outset, to use the whole of his accommodation. He would not be able to maintain this correction. His ciliary muscle would be exhausted at once. Consequently, we shall give him, with the correcting lens, a certain amount of accommodation in reserve.

If this reserve quota of accommodation makes up (according to what we have explained, p. 339) one-third of the amplitude of accommodation, we would prescribe, for the patient of our example ($a = 3$ D), at least 1 D more than if his entire power of accommodation were at his disposal. He would therefore, for seeing at a distance, require $1 + 2 = 3$ D. In the same manner should we calculate the strength of working-glasses. Vision at the distance of 28 centimetres $\left(\frac{1\text{ m.}}{3{\cdot}5}\right)$ requires 3·5 D of positive refraction. In order to get this, the patient must increase his static refraction ($- 5$ D) by $5 + 3{\cdot}5 = 8{\cdot}5$ D. The total amount of his accommodation would furnish 3 D. There consequently still remain $8{\cdot}5 - 3 = 5{\cdot}5$ D, which must be corrected with a convex lens. But the patient would never be able to work with these convex glasses of 5·5 D, because, in order to see distinctly, he would have to employ his entire power of accommodation. We enable him to do continuous work by giving him an additional 1 D, which allows him to keep 1 D of accommodation in reserve. The working glass would thus amount to $5{\cdot}5 + 1 = 6{\cdot}5$ D, or perhaps a trifle more, for in high degrees of hyperopia the reserve-quota is sometimes a little greater.

It follows from our law, if we may so call it, that a hyperope may need the aid of a convex glass, even though his amplitude of accommodation suffices for the correction of his ametropia. This is true when the former is just equal to the degree of the latter. Thus, a hyperope of three dioptries, who has at his disposal exactly 3 D of amplitude of accommodation, will generally find it to his advantage to use a weak convex lens for distant vision, unless he has acquired the habit of seeing with images of diffusion, which is oftener the case than is generally supposed.

From the same law there follows, moreover, an important rule concerning the correction of both the *absolutely* and of the *facultatively* manifest hyperopia. The absolutely manifest hyperopia may always be not only corrected, but even somewhat over-corrected, so long, at least, as any accommodation at all exists. Where, however, there is no accommodation, that is, where the entire hyperopia is absolutely manifest, there cannot, as we have already shown, be any question of a reserve-quota. For distant vision, the entire hyperopia must be corrected and, for looking at near objects, the hyperope must be further provided with the glass corresponding to the required distance. For instance, a hyperope of four dioptries, who, whether from advanced age or in consequence of paralysis, has lost his faculty of accommodation, wants, for distant vision, convex number 4; and for working purposes, at the distance of say $\frac{1^{\text{m.}}}{3}$, neither more nor less than convex $4 + 3 = 7$ D.

But *facultative*-manifest hyperopia will be totally corrected only in special, exceptional cases, of which we shall speak in connection with asthenopia and when on the subject of convergent strabismus especially.

We have, indeed, said that, for the application of our rule, static and dynamic refraction must be regarded as forming, together, but one whole. The accommodation which conceals, at will, a portion of the manifest hyperopia, that which is facultative, is added to the general refraction, as well as that which masks the latent hyperopia. Suppose, for instance, a total hyperopia of 6 D, of which 1·5 D is absolutely manifest, 2 D facultative-manifest, and, consequently, 2·5 D latent. To correct it for distance, we shall not prescribe the convex 1·5, because, with this glass, distant vision would still require all the accommodation that the individual has at disposal. Nor shall we give him the 3·5 (the amount of manifest hyperopia), because, in this way, too great a portion of his accommodation would be disengaged (3·5 in 4·5 dioptries). He needs an intermediate number, 2 to 2·5 D, according to what we have explained above.

The quota of one-fourth seems to be sufficient in the great majority of cases of emmetropia, or low degrees of hyperopia. But daily observation shows that, in the case of hyperopes of high degree, who are obliged to call upon their accommodation, even when looking at distant objects, for a considerable permanent contribution, the quota of accommodation, for near work, must be increased and made one-third. Thus, let us take the case of a hyperope of seven dioptries, whose amplitude of accommodation amounts to 9 D; with what glass will he most conveniently work at 33 centimetres (3 D)? The quota of accommodation is represented, in this case, by $\frac{9}{3} = 3$ D. Adding these to the 3 D of positive refraction necessary for vision at 33 centimetres, we obtain 6 D. Besides these 6 D, he needs seven for the correction of his hyperopia, the sum of both being thirteen. Of these thirteen, his accommodation will furnish nine; hence there remain four to be furnished by the glass.

The same example can be done in another way. In order to see at a distance of one-third of a metre, a hyperope of 7 D requires $3 + 7 = 10$ D. The one under consideration has 9 D of amplitude of accommodation. Hence, he needs $10 - 9 = 1$ D to adapt him to the desired distance. But in order to be able to work continuously at this distance, he will need a quota of accommodation in reserve; this being the third part of his 9 D, or 3 D, we add this last number to that of his distance-glass, which gives $3 + 1 = 4$ D.

If the same hyperope's amplitude of accommodation no longer amounts to more than 3 D, only one dioptry, beyond what his accommodation can furnish, will be necessary for the same distance; but, of himself, he can correct only two of the requisite 10 D. The remaining eight must be artificially procured for him by a convex lens. Thus he will work with two-thirds of his accommodation (2 D in 3), just as before (6 in 9 D).

I may, in fact, repeat here what I have already remarked; namely, in the first place, that in those cases where, on account of reduced amplitude of accommodation, the quota is very insignificant, the attentive practitioner soon learns what glass is to be given, without having to make all the calculations above indicated; and, secondly, that in all cases it is only actual experiment which can confirm the deliberations of the physician and finally determine the glass to be prescribed.

Our rule, like all rules, has its exceptions. It sometimes happens, for instance, that a patient has, by the prolonged use of strong convex glasses, accustomed himself to working with a very small degree of accommodation, thus keeping a too great quota in reserve. In such a case, it is obvious that our correctly calculated glass would be found too weak. On the other hand, there are some persons who, either from vanity or in consequence of a widely spread prejudice, have either avoided too long the wearing of convex glasses, or neglected getting, at the proper time, glasses of higher numbers. If we, at once, supply such patients with the proper spectacles, they will be equally dissatisfied, because they continue straining, more or less, their ciliary muscles, in spite of the glasses which were to relieve them from doing so. We must, therefore, facilitate the transition from the less to the more powerful glasses, by prescribing, for a certain time, glasses of intermediate strength.

It is, indeed, always of the greatest importance to take into account the habits and fancies of the patients, and especially the glasses they have hitherto worn.

SECONDARY ACTION OF CONVEX SPECTACLE-GLASSES.—The *number* of the lens is not the only thing to be taken into consideration in adapting an eye.

*The distance between the glass and the eye* may increase or diminish the influence exerted by the former upon the refraction of the latter. We have already met with an example of this (p. 137), and shown that, when employed as a means of adaptation for distant vision, convex glasses act as so much the stronger lenses, in proportion as they are further removed from the eye. But that is only one special case. Convex glasses do not serve alone for distant vision and are not prescribed solely for hyperopes. The effect of their recession from the eye varies with the conditions in which they are employed, as well as with their number. Hence it is important to be acquainted with the general rules governing their action.

They are the following:

In order to adapt an eye to a distance *greater than double the focal distance of the convex lens*, the latter will need to be weaker in proportion as it is farther removed from the eye; or, what amounts to the same, a weaker glass, farther from the eye, will have the same effect as a stronger glass, nearer to the eye.

This law is applicable to the case to which we have just referred, that of the hyperope's correcting-glass. The distance, to which this glass adapts him, is infinite, and, consequently, greater than 2 F, if we call F the focal distance of the glass.

The same is true of a finite distance of adaptation, as long as this is greater than 2 F, *e.g.*, in hyperopia of high degree, especially in aphakia. In order to adapt a hyperope of 10 D to 25 centimetres $\left(\frac{1\text{ m.}}{4}\right)$, we should need, if the glass were placed in actual contact with the cornea, $10 + 4 = 14$ D. The focal distance of this is $F = 71$ millimetres, and the double of it, $2F = 142$ millimetres, hence less than 25 centimetres. If we place the glasses farther away, a lower number will be necessary, in order to give, to rays coming from a distance of 25 centimetres in front of the cornea, a convergence toward a point situated 10 centimetres behind it.

This is the reason why all hyperopes can increase the effect of the glass which adapts their eyes for distant vision, by moving it away from the eye. Extreme hyperopes even enjoy this advantage with reference to the glass which serves them for near vision.

The contrary is true when the distance, for which the eye is adapted by means of the convex glass, is *less than double the focal* *distance of the latter.* In this case, the glass will need to be stronger in proportion as it is farther from the eye; its recession weakens it, so to say, if we consider it in its relation to the dioptric system of the eye.

This is what happens in the cases of hyperopes, whose ametropia is of low degree, of emmetropes and of myopes who are obliged to have recourse to convex glasses, in order to see well close at hand. Let us take an example: A hyperope of 2 D, deprived of accommotion, will need, in order to see at a distance of 33 centimetres, number $2 + 3 = 5$ D. The corresponding focal distance is 20 centimetres; twice this is 40 centimetres, hence more than 33 centimetres. If the glass be withdrawn from the eye, it will act as if it had become weaker, or, in other words, it will need to be replaced by a stronger glass, in order that a like effect be produced. This fact will be more easily observable if the hyperope still disposes of a certain degree of accommodation. He is then comparable with an emmetrope, or even with a myope, when his accommodation is greater than the degree of his ametropia.

The emmetrope deprived of accommodation demands a positive lens, whose focal distance is equal to that for which he wishes to be adapted. This is, necessarily, less than double the focal distance. The emmetrope who still possesses a certain amount of accommodation, or the myope (their refraction amounting to the same thing in this case) will need still weaker glasses,—that is to say, glasses whose focus is more remote—in order to adapt himself for the same distance. Hence he will feel still more the unfavourable effect of withdrawing the glass. For instance, an emmetrope, disposing of 2 D of accommodation, or a myope of 2 D deprived of accommodation, will need the number 1, with which to see at 33 centimetres. Double the focal distance of this glass is 2 metres, hence much greater than the 33 centimetres, for which this glass ought to adjust the vision.

Hence it is quite inaccurate to say that presbyopes put their spectacles on the end of their nose in order to increase their effect. On the contrary, they weaken the effect of the glasses by moving them from the eyes, and would see nearer at hand if the glasses were brought closer to the eyes. If the presbyope sometimes moves his spectacles towards the end of his nose, it is because, in so doing, he places them on a lower level than the eyes, and is thus, at the same time, able to incline the glasses a little. These are two advantages for near vision, in which the eyes are generally directed downward.[1]

Proofs of what we have just been pointing out are easily furnished by a very simple experiment. Let a person who is not naturally hyperopic, make himself so by means of a concave lens, and he will see that, upon correcting his hyperopia with a weak convex glass, the latter's effect will be increased in proportion as it is withdrawn from the eye. On the other hand, if he takes a strong convex lens and reads with it, at the distance of his *punctum proximum*, it will be noticed that the farther the lens is removed from the eye, the farther he will be obliged to remove the book, and *vice versa.*[1]

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[1] Landolt, *Ann. d'ocul.*, January 1878, p. 95.

The convex lens has not for its sole object that of giving the eye the optical adaptation for a certain desired distance, *i.e.*, of procuring, for it, distinct images of objects located at that distance. These images are, moreover, *enlarged* relatively to those obtained by the naked eye, adapted solely by its accommodation.

The magnifying influence of the convex lens makes itself all the more felt in proportion as the lens is stronger and farther removed from the eye. A hyperope whose ametropia is of high degree may, in this way, considerably increase the size of his retinal images. Furnished with convex glasses of different powers, placed at the distances at which they correct his hyperopia, his eye is very like an opera-glass whose magnifying power may be ten or more.[2]

Positive glasses exert still another influence on vision, which depends upon their magnifying action, upon the advancement of the nodal point and also upon the partial relaxation which they cause the accommodation to undergo. This is, first, with regard to the *estimation of the distance of an object.* This estimation, in monocular vision, results from three principal factors: the size of the image produced on our retina, the real size which we suppose the object to have and the effort of accommodation necessary to adapt the eye to the distance at which it is situated. The convex glass, by increasing the extent of the retinal image, thereby increases the apparent size of the object, and makes it seem nearer. It is true that the relaxation of the accommodation, produced by the convex glass, ought to make us suppose the distance of the object greater; but if this factor predominates, when we use a magnifying glass, for instance, the accommodation being wholly relaxed, it would disappear before the magnifying effect of a weaker glass, which only relieves the ciliary muscle of a portion of its task.

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[1] Maimonide Levi, *Sull' influenza dell' allontanamento delle lenti positive dell' occhio* (communication made to the Congress of Italian Oculists at Florence, Sept. 1877), and Landolt in de Wecker et Landolt *Traité complet d'ophth.*, iii., p. 359.

[2] Landolt, in de Wecker et Landolt's *Traité complet d'ophthalmologie*, vol. i., p. 502. "The effect of an opera-glass may be explained in this way: the concave ocular renders the observer very hyperopic, and the convex objective is simply his correcting-glass, placed at some distance from the eye." In this way he obtains very large retinal images of the objects looked at.

The amplification of the retinal image has another effect, of a purely geometrical order, by virtue of which *the relief of an object appears to be changed.* It is easy to show this, by first constructing the retinal image of any solid body on the retina of the naked eye, and then uniformly increasing all the proportions of this image (as a convex glass really does). Let us now project outward, through the nodal point, the simple and the amplified images. We shall see that the perspective of the object is changed in the second case; the distant portions of the solid are magnified more than the nearer ones. This fact leads us to consider the former as farther away than they really are, and the latter as nearer; hence the relief of the solid has a tendency to be lessened in its anterior parts, and to be exaggerated in its posterior portions. This is the reason why a plane surface—the ground, for instance—appears, to a person wearing convex glasses, as if it were raised. The same influences make themselves felt, still more exaggerated, in binocular vision. But it must be said that these phenomena are rapidly corrected by the patient's judgment, and that he attains, in a short time, an absolutely normal perspective vision, and an estimation of distance which is more nearly in conformity with the reality.[1]

Convex glasses have, moreover, the effect of *diminishing* somewhat the *amplitude of accommodation.* This effect is due to their distance from the eye. But the diminution is so slight that it may be disregarded in practice. In the case of a myopia of 2 D ($r = +2$), and a maximum refraction of $p = 10$ D, hence of an *amplitude of accommodation* $a = p - r = 10 - 2 = 8$ D,[2] this amplitude becomes equal to 7·72. Hence it is diminished 0·28 D by the convex No. 1, placed 2 centimetres from the eye.[3]

That which, on the contrary, changes notably under the influence of the use of a convex glass is the distance between the *punctum remotum* and the *punctum proximum*, our $R\,P$ (p. 170), the distance which Donders calls the *region of accommodation.*[4] This modification is so evident that there is scarcely any need to speak of it.

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[1] Compare, on this topic, Donders, *loc. cit.*, pp. 153-169, and Nagel, *loc. cit.*, pp. 349-358.

[2] See p. 166 *et seq.* of this work.

[3] Nagel, *loc. cit.*, p. 348.

[4] Donders, *loc. cit.*, p. 150. Theoretically, the *region* of accommodation has, for its extreme limits, the *punctum proximum* and the *punctum remotum*, whether the sign of the respective distances of these two points from the eye be positive or negative. However, as the negative part of the region is of no practical value, since it possesses no utility for the individual, we take into account only the positive portion of this region—that which is in front of the eyes. Hence convex glasses will increase the extent of this space, when, while bringing the *punctum remotum* to, or simply keeping it at infinity, they bring the *punctum proximum* nearer (as is the case with correcting-glasses for manifest hyperopia). They

An emmetrope, whose amplitude of accommodation amounts to 4 dioptries, has a region of accommodation included between 25 centimetres and infinity. With the convex 2, his $r$ becomes equal to $+2$; *i.e.*, his *punctum remotum* ($R$) is brought to $\frac{1^{\text{m.}}}{2} = 50$ centimetres.

His $p$ becomes equal to $4+2=6$ D. The *punctum proximum* ($P$) is, therefore, located at a distance of 16·6 centimetres, and $R\,P$, his region of accommodation, now amounts to only $50-16{\cdot}6=33{\cdot}4$ centimetres.

On the contrary, if the convex glass adapts the hyperopic eye for infinity, as is the case in manifest hyperopia, it increases the region of accommodation.

Convex glasses modify, moreover, *the relation between accommodation and convergence.* An emmetrope who sees, binocularly, at a distance of $\frac{1^{\text{m.}}}{4}$, associates 4 D of accommodation with 4 metre-angles of convergence. If he sees at the same distance through convex glasses of 3 D, his convergence-effort still remains the same (4 metre-angles); but his accommodation now amounts to only $4-3=1$ D.

An emmetrope (or corrected ametrope) can easily watch the effect of a discord between these two functions by fixing, binocularly, an object situated at the focus of two strong convex lenses. In order to fix this near object binocularly, he is obliged to make a considerable effort of convergence, whilst, in order to see it distinctly, the accommodation must be totally relaxed. The result is that, at first, the object is seen with crossed diplopia, because the sudden exclusion of the accommodation disposes the eyes to assume a parallel direction, or, at least, renders convergence difficult.

They diverge, therefore, relatively to the position of the object. Even when one has succeeded in uniting the double images, a certain disagreeable feeling often persists in the forehead and eyes, and proves that the latter perform only with difficulty this unusual and anti-physiological function. Something similar occurs when too strong convex spectacles are worn.

But the influence of well-chosen convex glasses, upon the relations between accommodation and convergence, far from being troublesome, becomes one of the most valuable therapeutic means of treatment of anomalies of refraction and accommodation. In fact, let us bear

will diminish it, on the contrary, when the *punctum proximum* is not brought so much nearer as is the *punctum remotum*, which is the case with the correcting-glasses for presbyopia.

Concave glasses, being constantly destined to carry the *punctum remotum* away from the eye, without ever causing it to pass beyond infinity, and without bringing about a like recession on the part of the *punctum proximum*, will always increase the region of accommodation.

in mind a remark made in a preceding paragraph,—that is, that the figure expressing the amplitude of accommodation does not represent the measure of the muscular or nervous effort made, since the latter acts through the intermediation of the crystalline, whose greater or less elasticity will determine the optical effect produced by this effort. Now, since the elasticity of the crystalline is diminishing throughout life, the accommodative effort ought to increase proportionately, in order to produce the same number of dioptries. And, inasmuch as the amplitude of convergence, essentially dependent upon the strength of the muscles, does not diminish with the accommodation, but remains nearly stationary, the individual is forced to vary, even in a physiological state, the effort of accommodation which he associates with a given effort of convergence. The 3 dioptries of accommodation, which an emmetrope must have when fixing an object 33 centimetres distant, will require from him a much greater muscular effort at the age of forty than at the age of twenty years, while the effort

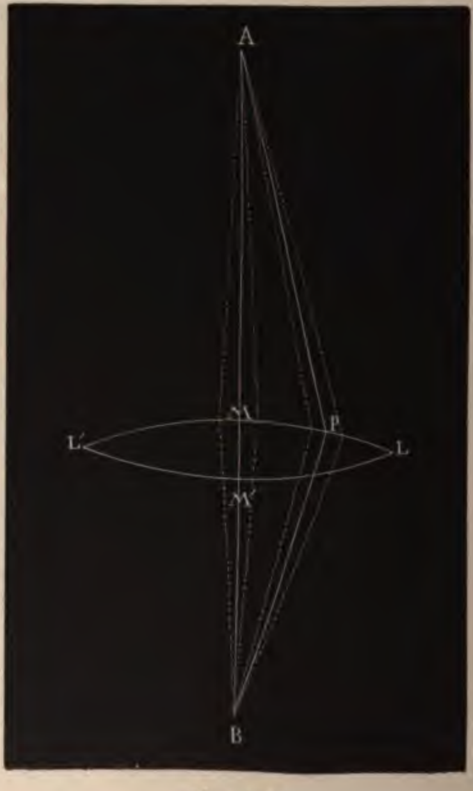

FIG. 120.

of convergence will remain the same—3 metre-angles. The difference is still more perceptible in the cases of hyperopes, especially of the medium or high degrees, in which the defect in static refraction, of itself, determines a breach of the equilibrium between the two functions.

In all these cases, convex glasses render inestimable service. By relieving the accommodative muscle of a quota of its work, they equalise the efforts of accommodation and convergence, by establishing between them, at least, the relation most agreeable to the individual.

But convex lenses can also exercise a DIRECT *influence upon convergence*—that is to say, upon the direction of the eyes. Indeed, a convex lens in section may be regarded as two prisms, having convex surfaces and their bases joined, the apices corresponding to the edges of the glass, M M′ L, and M M′ L′ (Fig. 120).

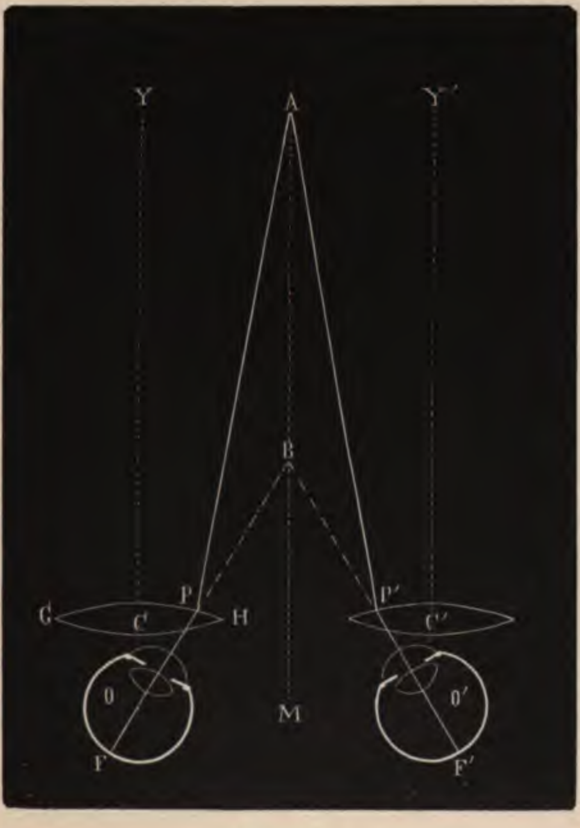

Fig. 121.

Let A be a luminous point, and B its conjugate focus. While a very thin pencil of luminous rays passing through the lens L L′, along its principal axis A M B, does not undergo any deviation, only the reciprocal direction of the rays among themselves being modified, rays falling upon an eccentric point P are still turned aside from their general direction; instead of following a straight line, they are obliged to form the angle A P B, in order to reach the conjugate focus. Hence the portion P of the lens, besides the action due to its convexity, has, moreover, acted upon the rays as a prism would have done: it has deviated the luminous rays toward its base M M′.

Now, each particle of the surface of the lens may be regarded as plane. The planes about the points M and M′ are parallel, while the corresponding particles of the two surfaces are more and more inclined to each other as we approach the edge of the lens. The prisms thus formed by the different particles of the surfaces are, therefore, stronger and stronger; their action increases from the centre toward the periphery of the lens. It could not be otherwise, if all the rays are to be brought toward the point B.

Therefore, an eye, looking through a convex glass, will see objects in their real position only when looking along the axis of the lens, while they will appear displaced toward the periphery of the glass, exactly as if they were seen through a prism, when the line of fixation passes through an eccentric portion of the glass. This prismatic effect will be perceptible in proportion as the part through which one looks is near the edge of the lens, and as the lens is strong. It is easy to convince one's self of this with the aid of any convex lens.

The *prismatic effect* of convex glasses is extremely important to consider in practice. Let us suppose, for instance, two eyes (O and O′, Fig. 121) furnished with convex glasses, and whose axes Y C and Y′ C′ are directed parallelly. If the spectacles are used in looking at distant objects, these objects are seen in their true positions, because, in this case, the lines of fixation coincide with the axes of the glasses. But, if the glasses thus placed are destined for use in near vision, their prismatic action will immediately make itself felt, because, on converging, the eyes look through portions of the lenses to the nasal sides of their axes. The glasses then act as prisms with their apices toward the nose, and deviate the rays toward the temples.

Thus, rays coming from the object A will undergo, at P and P′, a deviation toward F and F′, and the object will be seen at B. The eyes are, therefore, in this case, obliged to make a more considerable effort of convergence; instead of an angle P A M (approximately), they converge through an angle P B M.

The effect of convex spectacles upon convergence makes itself felt even for the lower numbers, especially if the muscles are somewhat weak, as is often the case. They then determine an asthenopia which is inexplicable to those unacquainted with the action of the glasses that they prescribe. From four dioptries on, their influence on convergence ought never to be disregarded. The higher numbers, badly mounted, can produce very troublesome diplopia. It happens thus, that persons, from whose eyes cataracts have been successfully removed by operations, see objects only monocularly, closing one or the other eye. As soon as they open both eyes, they see double, and are extremely annoyed by this phenomenon.

What is to be done in such cases? Theoretically, it would be necessary to give the glasses such an inclination, that the lines of vision should coincide with their axes directed toward the object, or, what amounts to the same, the plane of the glass would have to be perpendicular to the line connecting the object with the *fovea centralis*

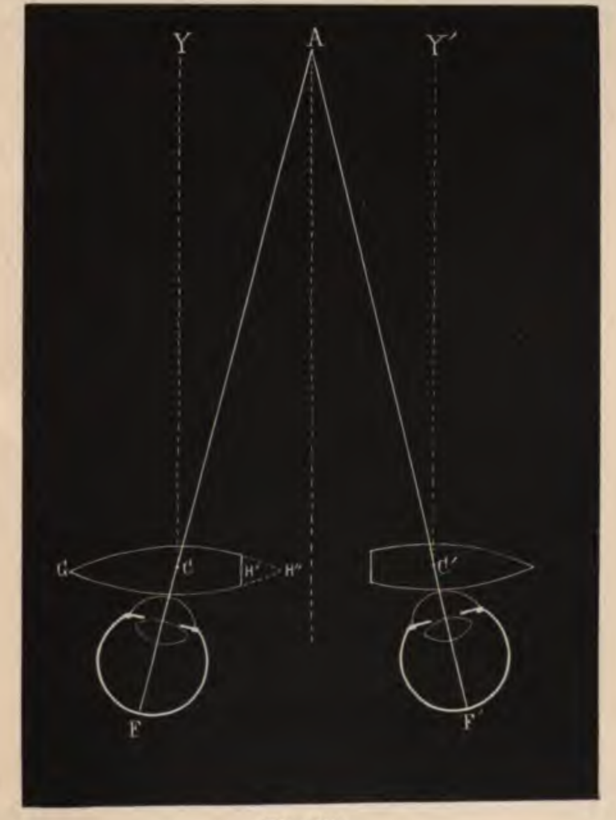

Fig. 122.

of the eye. It is evident that it would be impossible to wear such spectacles, at least for very near vision.

It is better to *decentre* the glasses, *i.e.*, to grind them so that, for the distance for which they are designed, the lines of vision shall pass through their centres (Fig. 122). This is perfectly practicable, so long as the glasses are not too strong, *i.e.*, too convex.

Since the thickness of a centred glass increases uniformly from its periphery to its centre, following each of its radii, it is evident that a glass of this kind will present, for a constant curvature and diameter, the minimum thickness that a lens can have. But this will no longer be true of a decentred glass of the same diameter and the same curvature. As soon as its centre is displaced in any direction, the radius of the lens, in which it is cut, must necessarily increase, and, with it, the thickness of the glass. It will be seen, indeed, that the lens G H in Figure 121 is narrower and thinner than the lens G H″ in Figure 122, from which has been cut the decentred lens G H′, the latter having been given the same diameter as G H.

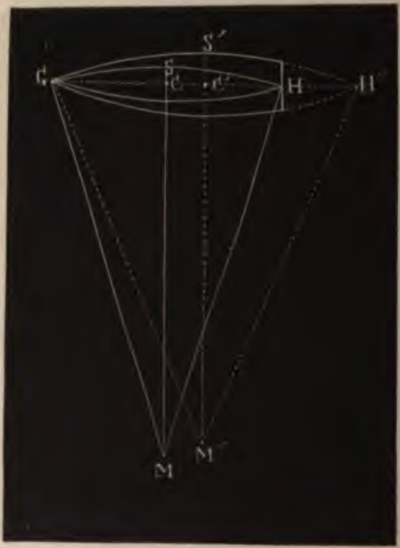

FIG. 123.

Figure 123 gives the geometrical demonstration of this. The surfaces of the lenses G H and G H″ are described with equal radii. The centre (C′) of the second is displaced toward the right, relatively to the centre (C) of the first, and we see that, at those two points, the thickness of the glasses is very different for the same length G H.

This difference depends upon the greater diameter of the lens G H″, of which the decentred glass constitutes a fragment.

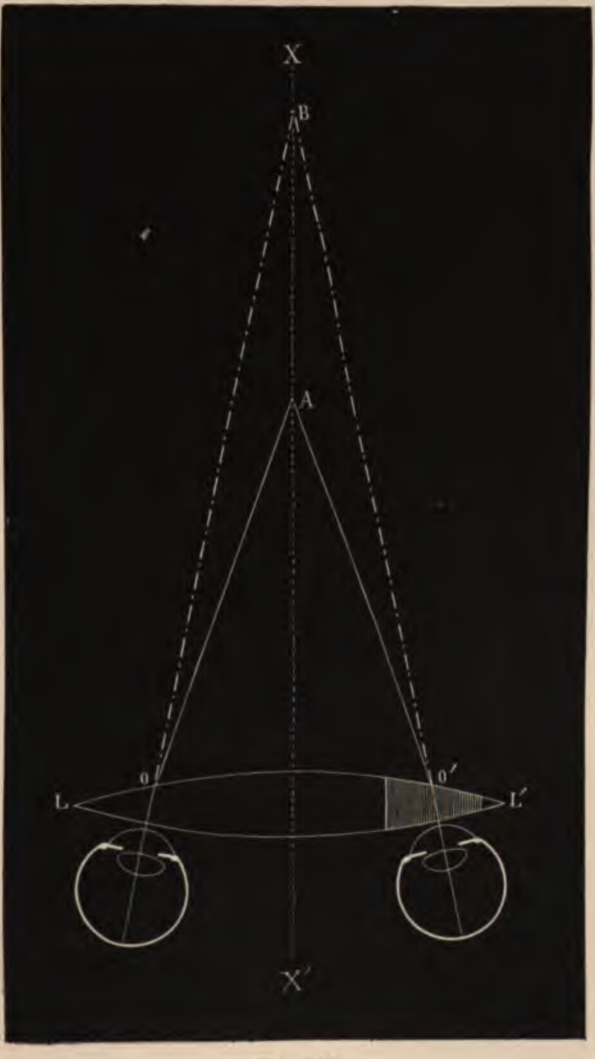

FIG. 124.

The ideal correction would be realised by a single lens, large enough so that (its axis X X′ (Fig. 124) coinciding with the median line) both eyes might simultaneously look through eccentric portions of the lens. In this case, the effect, produced by the latter upon the refraction of the eye, is equal to that which it has upon the convergence. If it diminishes, by three dioptries, the accommodation requisite for distinct vision, it at the same time diminishes, by 3 metre-angles, the convergence required for binocular vision.

Let, for instance, A (Fig. 124) be an object situated 20 centimetres from each eye; distinct and binocular vision of this will require 5 D and 5 metre-angles. If this same object be looked at through the lens L L′ of 3 dioptries, the eyes will need 3 D less of accommodation, *i.e.*, $5-3=2$ D, if they are emmetropic. Hence the lens has had, for the accommodation, the same effect as if we had carried the object from $\frac{1^{\text{m.}}}{5}$ to $\frac{1^{\text{m.}}}{2}$ (from 20 to 50 centimetres), from A to B (Fig. 124).

At the same time, thanks to the prismatic effect of the lens, rays emanating from A will undergo, at O and O′, such a deviation that they will seem to come from B. Hence the angle of convergence will be diminished, proportionately with the accommodation, by three units—that is to say, by 4 metre-angles.

It will be noticed, moreover, that this is true no matter what the distance between the eyes. The farther they are from each other, the farther the points of the lens, through which they look, are from the centre. The prismatic effect of the lens increases from its centre towards its periphery, proportionately with the base-line, *i.e.*, with the requisite effort of convergence.

In practice it would not do to order a single lens of this kind, to be worn in front of both eyes, and it would hardly be convenient. Two spectacle-glasses, however, may be cut out of it, in such a way that each eye may be furnished with that portion of the lens which is on the eye's axis, when the lens is whole. In Figure 124 this portion, for the right eye, is shaded.

Spectacles made in this way are called *orthoscopic*. In order to test their accuracy, it is necessary only to place them at such a distance from the wall that the glasses will produce upon it the image of a candle-flame. The images of the two glasses ought to be exactly superposed, if they form the desired segments of the same lens, and if they are separated by the distance, which would be between them in the whole lens.[1]

The following is the simplest way of representing to one's self this double action of a convex lens:

Let O and O′ (Fig. 125) be the two eyes, and M M′ the median line; *A* a point situated on the latter, at a distance A from each eye. In order that it be seen distinctly, a refractive power of $\frac{1}{\text{A}}=a$ is necessary.

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[1] Scheffler, *Die physiologische Optik.*, Braunschweig, 1865, vol. ii., p. 95. Compare, also, Bruecke, *Arch. f. Ophth.*, v. 2, p. 180, 1859: dissecting spectacles, formed by two glasses, which are neither more nor less than two halves of a lens whose focal distance is 22 centimetres (4·5 D).

If we wish each eye, when fixing *A*, to need no more refraction than it would when looking at a more distant point, *B*, for instance, at a distance B, which requires only $\frac{1}{\mathrm{B}} = b$ of refractive power, we evidently need a convex lens whose refractive power shall be $f = a - b$, or, expressed in terms of distance, $\frac{1}{\mathrm{F}} = \frac{1}{\mathrm{A}} - \frac{1}{\mathrm{B}}$, if F be the focal distance of the lens.

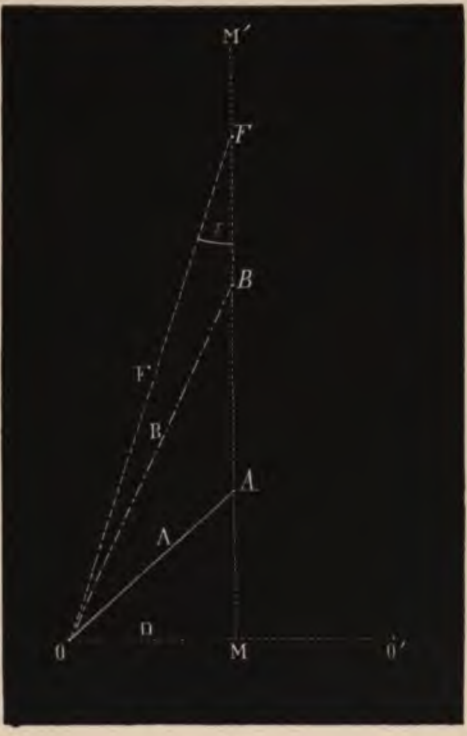

FIG. 125.

Let us mark off, on the median line, the focus *F* of the lens at the distance F from either eye.

In order to fix, binocularly, the point *A*, each eye must have a convergence-angle, which is also the reciprocal of the distance A, or $\frac{1}{\mathrm{A}} = a$ (see p. 188).

In order to fix the point *B*, there must be a convergence of $\frac{1}{\mathrm{B}} = b$. And, to change convergence toward *A* into convergence toward *B*, we evidently need a prism whose strength is $a - b = f$, or $\frac{1}{\mathrm{A}} - \frac{1}{\mathrm{B}} = \frac{1}{\mathrm{F}}$.

This expression is identical with the preceding one, which ought to be the case, according to what we have shown on page 190.

Hence the angle of deviation of this prism will be the angle O *F* M. It represents *f* metre-angles, if F is measured with the metre. To obtain its absolute value, let us call it $\delta$, and designate by D = O M half the base-line.

We may then write :

$$\sin \delta = \frac{D}{F},$$

or, supposing the sine equal to the angle :

$$\delta = \frac{D}{F}.$$

Suppose that we have obtained the desired refractive effect by means of a large convex lens, placed before the eyes as is indicated in Fig. 126. Then the left eye, looking through the point O of the lens, will see the point *A* as clearly as if it were at *B*.

What will be the prismatic effect of this point O of the lens? To ascertain this, let us first draw a tangent at this point, *i.e.*, a line perpendicular to the radius C O of the lens. Repeating the same thing for the posterior surface, which we suppose to have the same curvature as the anterior, we obtain the prism M L P, which corresponds to the two points, of the respective surfaces of the lens, through which the left eye looks.

Now, the angle M L S, or $\lambda$, is equal to the angle M C O, because their sides are mutually perpendicular.

For the angle M C O, we have the expression :

$$\text{tg } MCO, \text{ or tg } \lambda = \frac{D}{R},$$

if we designate by D the distance M O, and by R the radius of curvature of the surfaces of the lens.

The angle M L P of the prism = 2 $\lambda$ or $\Lambda$ is, therefore :

$$\Lambda = \frac{2\,D}{R},$$

supposing the tangent and the angle equal.

Now, the angle of deviation of a prism is, according to a well-known law of physics :

$$\Delta = \Lambda\,(n-1).$$

From our formula 20ª (p. 55), it results, on the other hand, that

$$R = 2\,F\,(n-1);$$

hence

$$\Delta = \frac{2\,D\,(n-1)}{2\,F\,(n-1)} = \frac{D}{F}.$$

This D is nearly the same as half the base-line. The deviation $\Delta$, produced by the lens at the point O, is, therefore, identical with that, $\delta$, which we have found above as being requisite, in order to change convergence for *A* into convergence for *B*. Hence, the lens thus employed fulfils the aim of relieving the *accommodation* and *convergence of the eyes, by identical quantities.*

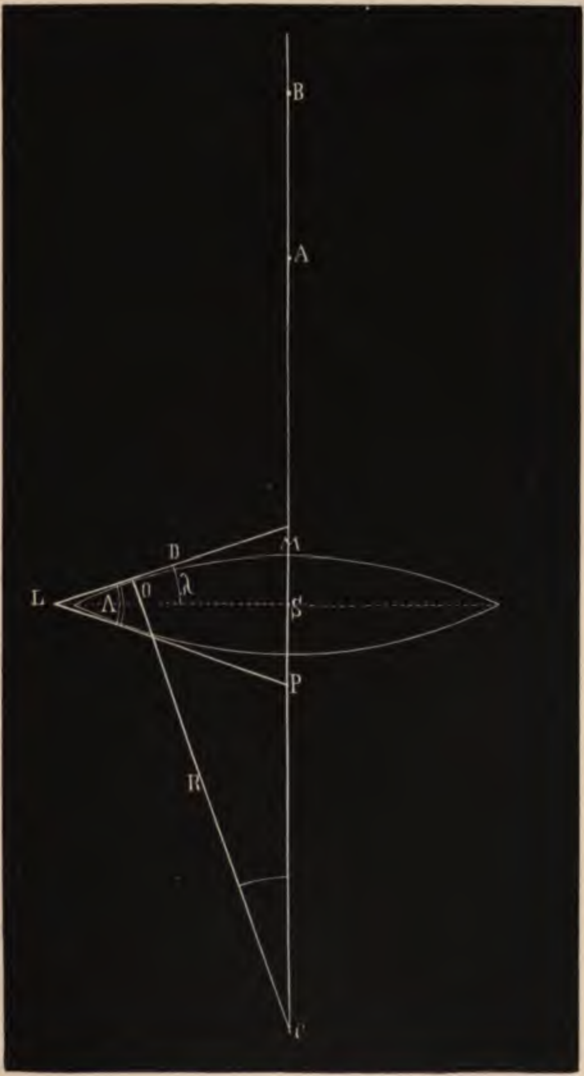

FIG. 126.

It should not be forgotten that the prismatic action of spherical glasses does not make itself felt solely in the horizontal, where it diminishes or increases convergence, and may bring about a homonymous or crossed diplopia, but also in the *vertical*, or in any other direction.

This action manifests itself in a vertical or oblique diplopia, much more troublesome than a similar disturbance of equilibrium in the horizontal, whenever somewhat strong glasses are placed at different heights before the two eyes. Hence, we should be careful that the line, joining the optical centres of the glasses, is parallel with the base-line. It must, moreover, be contained in the plane of fixation passing through the latter and through the lines of fixation, so that objects seen binocularly shall appear to be where they really are.

When one looks through the *edge* of a convex glass, placed at a certain distance from the eye, and in such a way that its edge is just opposite the corresponding pupillary circumference, the point of fixation disappears and, with it, a more or less extended portion of its surroundings. This gap is greater in proportion as the glass is stronger and farther removed from the eye. This is easy to understand. Among the rays coming from the point of fixation, some pass outside the glass, but do not enter the pupil, since the glass entirely covers this; others are deviated by the prismatic edge represented by the border of the convex glass, so that they fall beyond the opposite margin of the pupil.

If the centre of the lens is placed in front of the pupil, the same result will, evidently, be produced for all objects situated on a line joining any point of the edge of the glass with the corresponding point of the pupillary circumference. The *field of vision* of an eye furnished with a convex glass will, therefore, present a larger or smaller annular hiatus corresponding to the circumference of the lens.

When one looks through the latter along its axis, this suppression of a part of the visual field is less troublesome in proportion as the glass is weaker and as the gap remains more peripheral. But it may become a serious inconvenience for persons who, like those who have undergone cataract operations, use strong glasses and sometimes look through their edges.

The *spherical* and *chromatic aberrations* presented by the convex glasses used in ophthalmology are not noticeable enough to constitute serious defects. They scarcely manifest themselves except in strong glasses, when the line of vision passes through them elsewhere than through their centres. In this case these secondary effects of convex glasses are the stronger in proportion as the part of the glass in front of the pupil is farther from the centre.

It has been attempted, without much success, to remedy these defects by making achromatic and aplanatic lenses. Crown-glass lenses are preferable to those of ordinary glass, because they are much less dispersive. Perfect achromatism is obtained by the combination of a crown-glass lens with one of flint-glass. But these combinations give such heavy glasses that their advantages do not counter-balance this inconvenience.[1] Aplanatism is nearly realised by the juxtaposition of two plano-convex glasses, touching each other by their curved surfaces.[2]

We remember, too, that when one looks *obliquely* through a spherical glass, or, what amounts to the same, inclines it in front of the eye, its refractive action increases in the direction perpendicular to the axis of its rotation. Thus, if we rotate a convex lens about a horizontal line, or if we look from above downward through the same glass held vertically, it will act as if we had added to it a convex cylinder with its axis horizontal. Objects may, therefore, undergo an apparent distortion, more or less perceptible, according to the angle formed by the line of vision with the plane of the glass. To avoid this inconvenience, the glass should be given such a position that the line of vision shall pass through it as nearly as possible perpendicularly to its surface. Thus, glasses destined for distant vision, the correcting-glasses of ametropia, are to be placed vertically, because generally, when looking at distant objects, the eyes are directed straight ahead. On the contrary, glasses destined to assist near vision are inclined, because near objects are almost always fixed by directing the gaze downward.

We have required, moreover, that the line joining the optical centres of the glasses shall be in the plane of fixation, and that the latter shall be perpendicular to the plane of the lenses. This is equivalent to saying that the glasses ought to be not only *inclined*, but also *lowered* for near vision, as if they had turned with the eyes, about the centres of rotation of the latter.

There results from this, as to the position of spectacles, this very important fact: supposing that, as is desirable, the optical centre is on the horizontal diameter of the mounted glass, this glass must be placed lower, and its upper edge inclined forward, in spectacles destined for near vision. This position is possible only when the summit of the bow, joining the two glasses, is higher than the middle of the latter. Most often it is necessary to raise it even to a level with the upper borders of the frames, and let it project beyond their plane, according to the height and shape of the nose. The glasses may, moreover, be inclined relatively to the temple-pieces, in order to satisfy the second condition, of which we have spoken above. For this purpose we have had spectacle-frames constructed and furnished with a hinge near the glasses, so that the latter may be inclined as desired.

We regret our inability to devote more space to a discussion of the mounting and adjustment of spectacles. This question is generally too much neglected by the practitioner, as if the slightest detail in the execution of his orders were not of the highest importance for the patient. Thus it is that the fruit of the most conscientious examinations and of the most rational considerations may be lost, simply because the glasses that have been chosen are badly placed before the patient's eyes.

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[1] Donders, *loc. cit.*, p. 139.

[2] Hunter, *The Amer. Ophth. Soc.*, xi., Ann. Meeting, p. 356, 1875.

In order to avoid this serious difficulty, one should bear in mind the principles, which we have just given, concerning the position of spectacle-glasses. One should test them, when made, not alone from an optical point of view, but also from the point of view of their adaptation to the patient's face and to the conditions under which they are to be used. If ophthalmology has made progress, industrial optics have done the same, and still more may be accomplished in this latter line, under the guidance of educated practitioners.

It often happens that a person needs different glasses for distant and for near vision. In trying to avoid the constant changing of spectacles, we may proceed in different ways. The patient may permanently retain his distance-glasses, and add to them the extra amount, necessary for near vision, in the shape of eye-glasses or hand-lorgnettes. Thus a person whose manifest hyperopia amounts to 1 D, and who needs 4 D for near vision, may wear convex glasses number 1 constantly, and put on extra eye-glasses number 3 for reading.

But, if it is a question of changing, frequently, the distance of vision, as, for instance, in the case of a painter, who glances back and forth from his model to the canvas before him, it would not do to subject him to the inconvenience of constantly putting on and taking off a pair of eye-glasses. Emmetropes, and those who are but slightly ametropic, who see fairly well at a distance with the naked eye, may then use glasses whose upper edge is cut horizontally, so that they cover only the lower part of the field of sight. Their having an oval or semi-elliptical form does not prevent the optical centre of each being at the middle of the glass.

For cases in which two different glasses are necessary, whether both are convex or concave, or one convex and the other concave, *double-focus glasses* have been devised. The same glass is formed in such a way, that its upper and lower halves have different refractive powers.

The same effect is obtained, though with less elegance, by means of such glasses as we have mentioned, made of two pieces divided horizontally, and joined by their cut surfaces. They are called *Franklin glasses,*[1] from the illustrious *savant* who seems to have been the first to use them. They render very great services, and I have seen an eminent painter who was so charmed with the Franklin glasses that I had suggested for his use, and which he preferred to wear in round frames, that he faithfully reproduced them in his portrait, not omitting even the fine luminous line marking the junction of the halves of each.

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[1] Donders, *loc. cit.*, p. 138.

It is self-evident that, if the hyperopia of the eye is complicated by astigmatism, the latter is to be corrected by means of an appropriate cylinder (see p. 303 *et seq.*), which is to be added to the correcting-glass. This glass then presents, on one of its faces, the spherical curvature destined to neutralise the hyperopia, and, on the other, the cylindric curvature which remedies the astigmatism.

It is not to be forgotten that, occasionally, though rarely, near vision requires a cylinder somewhat different from that which adapts the eye for a distance. Hence we should always test the correction for both cases, for distant and near vision.

### *Treatment of the Convergent Strabismus of Hyperopes.*

We have seen that the hyperope, in order to obtain the accommodation that he needs, very easily becomes the victim of an exaggerated convergence, and we have pointed out the natural tie which exists between the refractive defect and convergent strabismus. It is this pathogenic mode upon which we shall base our therapeutic attempts to combat the deviation.

We frequently meet with young hyperopes whose strabismus is still only in its commencement; the exaggeration of convergence has not yet passed into a state of permanent habit and arises only under the influence of a transient act of fixation. In such a case, resting the eyes, the cessation of all work necessitating an excessive effort of accommodation, will sometimes suffice to dispel the deviation which, otherwise, would tend to become persistent. Thanks to these precautionary measures, we shall be able to avert imminent strabismus and preserve binocular vision by imposing upon it only a moderate task.

Nevertheless, in order not to condemn the eyes to absolute inaction, especially in the case of children who are commencing their studies, we shall have recourse to still another means; the accommodation will be relieved of at least a part of its work by means of CONVEX GLASSES. Thus it may be that wearing glasses which correct the manifest hyperopia often prevents all strabismic deviation in young hyperopes, or suffices to cure it when it has just commenced.

It is clear that, if the hyperopia is accompanied by astigmatism, the latter should be corrected at the same time. In this way the ciliary muscle will be doubly relieved—first, by being spared the correction of a portion of the optical defect; and, again, by procuring for the person the ability to hold his work farther away, thanks to the increase in visual acuteness afforded by the cylindric glass. The constant wearing of the spectacles chosen is, in such a case, obligatory.

If we have to deal with a young hyperope, affected with a *declared and permanent convergent strabismus*, the glasses which correct the manifest hyperopia will not suffice to dispel the deviation. One would naturally be led to think, in such a case, of excluding even a greater amount of accommodation by correcting the total hyperopia. Such a conclusion would be perfectly correct and logical. But if, for such a patient, we simply prescribe glasses correcting the entire amount of his hyperopia, he will soon return them, or else put them aside, for the simple reason that he is not able to relax his accommodation sufficiently. He will with these glasses see but very poorly at a distance and, for distinct vision near at hand, he will be forced to approach the object excessively. In the first case, consequently, our glasses will have no influence upon the strabismus; and, in the latter, the very opposite result will be obtained from that desired, since every effort at convergence tends to increase, or to confirm, the strabismus.

Nevertheless, in such cases of hyperopic convergent strabismus, the endeavour to relax the accommodation as much as possible, and even entirely so, is more than justifiable; only, in order to gain our object, we must have recourse to MYDRIATICS. Indeed, as soon as we have to deal with a young hyperope, especially with a child whose strabismus has become permanent, we submit him, first of all, to a course of treatment with atropine. We instil into the eyes once or twice daily, according to the age of the patient, one drop of a solution of the drug (1:4000 or 1:2000). It is advisable to keep the eyes closed for five minutes after each instillation, in order, if possible, to localise the effect. If the age of the patient permits, we immediately give him glasses which correct the total hyperopia. In anisometropia each eye will, therefore, be fitted with its correcting lens. Moreover, for protection against dazzling, it is advisable to select smoked and large glasses.

Very young children, in whose cases the wearing of glasses is evidently impracticable, must be protected from the dazzling light by a veil or shade. Older children, in whom acute vision is not a matter of necessity, and whose hyperopia is not very great, may wear smoked coquilles, which protect the eyes better than ordinary glasses. But in no case do we allow the patient, in the beginning of treatment, to use his eyes for looking at near objects. One might think that, where there is a complete paralysis of accommodation, vision for near objects, with suitable convex glasses would be no more injurious than distant vision. But this is not so. One has frequent opportunities for observing that patients, whose eyes are perfectly well directed when looking at distant objects, begin to squint as soon as they fix near ones, and that, too, notwithstanding the fact that they use lenses which adapt them to the near point.

This fact goes to prove that, in such cases, the impulse to convergence, of itself, provokes the strabismus. We try, therefore, to avoid the latter, not only by forbidding the patient to make any attempt at near vision, but by even rendering it impossible for him to do so. If, after a few weeks' properly continued treatment, it appears that, with the required glasses, the strabismus for distance nor that for near vision returns, we may then order, for studious persons, spectacles for working purposes. But we must, at the same time, emphatically advise them not to abuse our permission to return to their work, and to take it up in the beginning for but a few hours each day, and with frequent intermissions. Thus it is often possible to continue a course of atropine treatment for months, without serious detriment to a child's studies. By and by we may try diminishing the dose of the mydriatic, or even to discontinue it altogether. If this succeeds without the patient again relapsing into a squint, we then change the glasses which have hitherto been worn for the strongest convex lenses with which he is able to see at a distance. In anisometropia and astigmatism we follow the rules given in the chapter devoted to their treatment. The patient must now wear these correcting-glasses constantly; but he no longer requires special glasses for working, as his accommodation has again attained its full power.

The strabismus is therefore considered cured, and we may congratulate ourselves upon the result obtained, even after taking into account the annoyance from wearing spectacles. Sensible patients, as a rule, do not complain of this, especially when they are made to understand that without glasses they will again begin to squint, and possibly even need to undergo a surgical operation.

If, after some little time has expired, there has been no recurrence of the strabismus, it may be desirable for the patient to try occasionally to do without spectacles—for instance, when out of doors—but being careful always to wear them while at work. It may also happen that, where the hyperopia is of a moderate degree, the squint but slight, the visual acuteness good and the constitution robust, the muscles of the eye may, in time, be able to resume their proper functions without the aid of glasses. Such cases, which are not very rarely met with, constitute real triumphs of the healing art.

Greater difficulties have to be surmounted in the treatment of convergent strabismus, where the visual acuteness of one eye is very slight. As has been said, and as can be readily understood, it is always the latter which is the deviating eye. Besides the course of treatment above indicated, we seek, in such cases, to restore the weak and deviating eye to the accomplishment of its normal functions. Therefore errors of refraction will have to be most carefully corrected, and we must try to augment its visual power by special training. Such training must, however, always be gone through with while the accommodation of both eyes is excluded, and must, moreover, never be continued until fatigue sets in; otherwise we run the danger of increasing the strabismus. The squint naturally manifests itself in the healthy eye, as long as the weak one fixes, but it immediately returns to the latter as soon as the former is allowed its liberty. This is the reason why, in such cases, we always add to the training of each eye, by itself, that with a stereoscope, in which both eyes take part at the same time. We shall consider them in detail further on, in the chapter devoted to the "Orthoptic Treatment of Strabismus."

It has been sought to accomplish the same end by the use of a remedy whose action is the exact opposite of that of atropine—that is to say, ESERINE.[1] In this case, the therapeutic effect of the two medicaments ought to be the same: atropine, by paralysing the ciliary muscle, gives the patient a feeling of the uselessness of any effort of accommodation; eserine, by contracting the same muscle, exempts it from any voluntary effort. The result must be that the accommodative impulsion is suppressed, and indirectly, also, the impulsion to convergence.

Our experience with the eserine treatment of hyperopic convergent strabismus has not been extended, nor has it, on the other hand, been disappointing; however, we decidedly prefer mydriatics to myotics. Nevertheless, it is justifiable, after an unsuccessful atropine treatment, to try the use of eserine for a time.

We must not flatter ourselves, however, that we shall always succeed in curing the convergent strabismus of hyperopes by the means above pointed out. Many a deviated eye resists all mild measures and bids defiance to all theories. It is then necessary to resort to an OPERATION; *setting back the tendon of the internal rectus, advancing that of the external rectus, double tenotomy of the internal*, or, finally, a *combination of tenotomy with advancement.*

We perform tenotomy in the following manner:—The eye being rendered anæsthetic by cocaine, and kept open by means of a speculum, we seize, with the forceps, a fold of the conjunctiva at the level of the insertion of the tendon to be set back. The fold may be either vertical or horizontal, according as to whether the incision is to be made horizontally or vertically. We prefer a horizontal incision at about the middle of the breadth of the tendon.

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[1] Ulrich, *Die Aetiologie des Strab. converg. hypermetrop.*, &c., Kassel, 1881.

The incision being made of a length of about 4 millimetres, its edges are successively seized with the forceps, and the conjunctival membrane is loosened to a greater or lesser extent, according to the effect which it is desired to produce from the capsule of Tenon, which is subjacent to it. The closed forceps is then, according to the advice of v. Arlt,[1] introduced into the wound, and is passed along the surface of the globe as far as 3 millimetres behind the insertion; then, giving it a direction perpendicular to the globe, if it be an ordinary dissecting-forceps, it is opened widely and the muscle grasped. In raising the part held between the jaws of the forceps, it is easy to entirely detach the tendon, at its scleral insertion, with one cut of the scissors. In order to do so, it is necessary to introduce but one blade of the latter under the muscle, to push it towards its insertion, and then to close the blades. A pair of scissors with curved blades, as shown in Fig. 127, is peculiarly adapted to the performance of this operation.

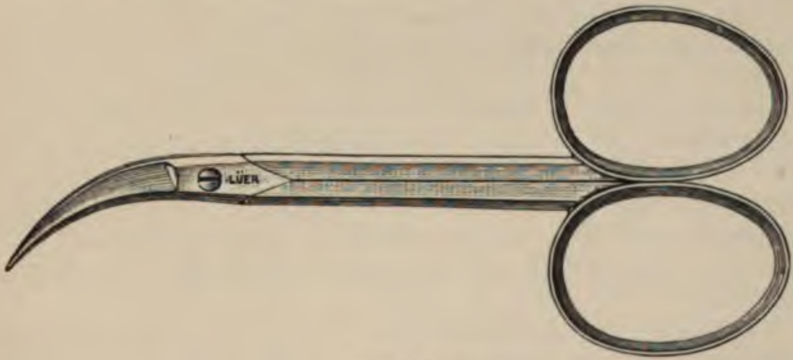

FIG. 127.

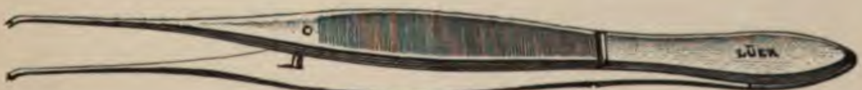

FIG. 128.

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[1] V. Arlt, in Graefe und Saemisch, *Handb. der Ges. Augenheilk.*, vol. iii., p. 399.

The grasping of the muscles to be tenotomised is remarkably facilitated by the use of a forceps, which we have had constructed for the purpose (Fig. 128), the teeth of which, being directed obliquely at an angle of 45°, allow the muscle to be seized without much erection of the forceps.[1]

When it is desired to obtain a greater effect, an ordinary muscle-hook is introduced under the superior and inferior folds of the capsule of Tenon, and the latter is more or less incised.

Some operators prefer to introduce the hook under the tendon of the muscle, instead of seizing it directly with the forceps. This manœuvre is somewhat more complicated and painful than the simple use of the forceps.

We prefer the horizontal incision of the conjunctiva to the vertical. It has the advantage over the latter of preventing the gaping of the wound, which necessitates a conjunctival suture, in order not to leave the sclerotic exposed, and to avoid the sinking back of the caruncule, which gives to the eye a very strange and unpleasant appearance.

Our investigations have demonstrated that a simple tenotomy of the internal rectus, with free detachments, may suffice to correct a deviation of about 15°. This figure may serve as a guide in the dosing of the effect and in the choice of the operation to be undertaken for the correction of the strabismus. If the deviation only slightly exceeds the above amount, tenotomy of the internus of one eye will generally suffice, especially if the strabismus is alternating, or if, at least, the acuteness of vision is good in both eyes. Atropine, convex-glasses, orthoptic training, and also time, help, in such cases, to complete our work.

According as to whether the muscle is more or less separated from the conjunctiva, and above all from the capsule of Tenon, it is possible to obtain, by a simple tenotomy, any desired degree of the effect, from 0 to as high as 15°. If the result of the operation happens to be excessive for the particular case, it is easy to lessen it by means of a conjunctival suture.

The conjunctiva, in fact, still clings to the capsule of Tenon, which yet has a partial connection with the muscle. The deeper the suture the more it will embrace the sub-conjunctival tissues, especially the capsule and also the divided tendon, and the more considerable will be its restricting action. It is thus possible, when necessary, to limit the effect of the tenotomy so as to render it insignificant.

The suture can even be applied several days after the operation, as long as the new insertion is not yet firm and can be detached by means of the hook.

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[1] Landolt, *Ophthalmologische Gesellschaft*, Heidelberg, 1885.

When the convergent strabismus exceeds 20°, the method to be followed varies according to the age of the patient, the localisation of the strabismus (alternating or unilateral), the visual acuteness of the deviating eye, the more or less marked tendency to binocular vision and the condition of its muscles, such as it presents itself upon examination of the field of fixation.

If the case is that of a child, for instance, whose strabismus varies between 20 and 35 degrees after correction of the total hyperopia, and whose two eyes do not show a great difference in visual acuteness, it is necessary to wait several weeks, or months, for the final effect produced by a primary tenotomy. We may endeavour to increase it by the use of *louchettes*, a kind of opaque spectacles, in which, for each eye, is only a small hole, which makes it impossible to look otherwise than in a certain direction.[1] In thus placing the eye at divergence a few days after the operation, it is possible to obtain an amount of correction above 20°. When no further results can be expected from the employment of *louchettes*, after the new insertion has become firmly attached, orthoptic exercises are again to be resorted to for some length of time, with the stereoscope, for instance, according to the indications to be given further on.

The effect of a tenotomy can also be augmented by the aid of v. Graefe's suture[2] or by those of Knapp. The former consists of a sort of capsular advancement. A threaded needle is passed through the conjunctiva of the external part of the globe, at the edge of the cornea, passing through the episcleral tissue for a distance of several millimetres, and tied together more or less tightly.

As to Knapp's ligatures, the author describes them in the following manner: To increase the effect of the tenotomy, a thread, armed with two needles, is passed through the conjunctiva and the superficial layers of the sclerotic, close to the corneal margin, and through the palpebral commissure opposite the divided muscle. When one needle is passed through the conjunctiva and the superficial layers of the sclerotic, the tissue over it is tied by a single knot, then both needles are thrust through the commissure, from the conjunctival to the cutaneous side, and united on the latter. The effect of the suture may be varied considerably by passing the needles more or less deeply through the commissure, drawing the eyeball forcibly toward it, and tying the threads firmly on the skin after properly interposing a small piece of wood or rolled up paper. If only a slight effect is desired, a thread armed with a single needle is passed through the episcleral tissue without tying it, and then through the lid commissure, upon which its ends are united, thus drawing the cornea as much toward the commissure as is desired. This simple procedure is commonly sufficient after a tenotomy of the internal rectus, whereas, for the external, the procedure first described is almost always required. It is important to pass the thread through the fibres of the sclerotic, for simple conjunctival sutures, on account of the elasticity of this membrane, have little or no effect. Both eyes are kept bandaged for twenty-four hours, then the thread is cut.

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[1] It was formerly thought possible to cure strabismus without an operation, by the use of such *louchettes* only. With our present knowledge of the etiology of concomitant strabismus, and of the movement associated in the deviations of an eye, we see how chimerical this hope was.

[2] V. Graefe, *Klin. Monatsbl.*, p. 225, 1869.

If all these means prove insufficient, tenotomy of the internal rectus of the other eye will have to be decided upon. It is to be managed according to the deviation still remaining, and according to the rules we have just given.

When the deviation exceeds 25° and is unilateral and constant, when it affects an amblyopic eye or one much inferior to the other in visual acuteness, and when binocular vision is lost, and its restoration impossible, we combine at once the setting back of the internal, with the advancement of the external rectus.

The muscular advancement we perform in the following manner: After having made the tenotomy of the antagonist, we begin with a vertical incision in the conjunctiva, close to the cornea. The conjunctiva is then detached for some distance beyond the insertion of the tendon. The latter is then taken up on the hook, and drawn into the wound; now, making sure that we have hold of all the fibres, the muscle is delicately loosened along its edges with the blunt points of the closed scissors. An assistant holds it while the operator places the sutures in position. The latter are applied in the most simple manner. A threaded needle is passed through each of the edges of the muscle, more or less behind its attachment, according to the effect desired. Again taking the hook, the surgeon divides the tendon at its insertion. The threads are then passed under the conjunctiva close to the superior and inferior borders of the cornea, and, when necessary, as far as its vertical diameter. It only remains to tightly knot each of the threads together, in order to see the extremity of the tendon appear at the corneal margin. It is advisable, during this manœuvre, to have an assistant turn the eyeball in the direction of the muscle to be advanced.

If, after having tied the threads, the muscle is found to lie upon the cornea itself, where it evidently cannot become attached, a small longitudinal incision, at its divided end, allows the muscle to spread itself upon the tissue of the limbus and to take its insertion exactly at the corneal margin.

The effect of an advancement may be still further increased by resecting a part of the tendon. This practice may become necessary in the case of an extreme strabismus or when examination of the field of fixation shows a weakness of the muscle to be advanced. We perform this resection in the following manner:—

The muscle being taken up on the hook, its line of insertion is carefully noted. Then it is divided, at a greater or lesser distance, behind and parallel to the latter, in order to obtain a clean section. The part of the tendon still attached to the sclerotic is then removed with the scissors.

After the operation, the strabismus ought to be perfectly corrected. We then instil atropine into both eyes, to which we apply an antiseptic dressing. This treatment is renewed twice daily for about four days. Then the stitches are removed and the eye not operated upon may be left at liberty. It is better to keep the bandage on the other for some days longer. The atropisation is continued and the correcting-glasses must be worn as soon as one eye is liberated. We continue this way for several weeks, unless the strabismus should prove to be a little over-corrected.

It remains for us to speak of a method, of curing strabismus, which has been indicated especially with a view to avoiding an operation, but which likewise renders very great service as a complement to surgical treatment. We refer to ORTHOPTIC TRAINING.

We know of what capital importance binocular vision is for the maintenance of the normal direction of the lines of fixation.

When once the exaggerated convergence is reduced by the effect of an operation, it is possible that the eyes will recommence to work together in a normal way, on condition, of course, that their correction is sufficient and that their visual acuteness and refraction are not too unequal. But we observe, in the majority of such cases, a persistent abolition of binocular vision. The person operated upon continues to disregard the images of one of his eyes, or else he projects them wrong, and may be troubled with diplopia.

It is especially in these cases that we may hope to effect a cure of the strabismus by means of orthoptic treatment, with the aid of stereoscopic exercise. This treatment is based on the following considerations.

We recognise two principal causes of the strabismus in question:

1. The depreciation of one eye, which suppresses the desire for binocular vision and the necessity for the normal direction of the eyes.

2. The lack of equilibrium between convergence and accommodation.

In order to remedy these anomalies, we have to:

1. Increase the value of the weaker eye, by the correction of its ametropia, isolated training, &c.

2. Render it conscious of its improper direction, by producing diplopia, and to revive, in the individual, with the aid of the stereoscope, a notion of, and desire for, binocular vision.

These are, in great part, the principles laid down by M. Javal,[1] who has used, in carrying them out, Wheatstone's stereoscope.[2]

But it is evident that, in using the stereoscope, we are not concerned solely with the convergence, *i.e.*, with the *direction* of the eyes, but also with their *accommodation*. We do not hope, as has been tried, to succeed in curing strabismus by placing, in front of each eye, a distinct object or, what amounts to the same, its reflected image, and by varying the separation of the two objects, without taking into account the accommodation of the eyes. In order to obtain binocular and distinct vision by this method, the eyes would be obliged to vary their convergence without changing their accommodation. It would be necessary, then, to entirely destroy the harmony between these two functions. This result would not only be attained with difficulty, but especially contrary to the aim of our treatment, which is to cure the strabismus by the re-establishment of an intimate collaboration of the functions in question, based upon binocular vision.

We have seen, in fact, that the alteration in the relations between convergence and accommodation plays an essential part in the dissociation of the lines of fixation; that is to say, in the production of strabismus.

Why is the mechanism of binocular vision compromised in ametropes? It is because the relation between accommodation and convergence has been developed for eyes that are emmetropic, or nearly so, *i.e.*, for the majority of eyes that exist in nature. This delicate mechanism could not accommodate itself to an ametropia exceeding the limits that we have indicated; beyond them, it is either forced or destroyed.

Out of this grows the important principle, that if, in the case of a strabismic person, whether he has been operated upon or not, we wish to re-establish binocular vision, we must first render the hyperopic eyes emmetropic, or, otherwise expressed, the ametropia must first be corrected.

This being understood, we proceed in the following manner:—

Taking an ordinary box-stereoscope, we introduce, in the place of "views," two objects of some very simple shape, two vertical lines

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[1] Javal, *Ann. d'ocul.*, t. liv., lv., and lxvi.

[2] Landolt, *Ophthalmotropométrie*, vol. i. of Wecker et Landolt's *Traité complet d'Ophthalmologie*, p. 914, Fig. 239.

(Fig. 129), for instance, one above and the other below the same horizontal line. These two lines, which may be brought toward, or removed farther from, each other at will, are placed at a reciprocal

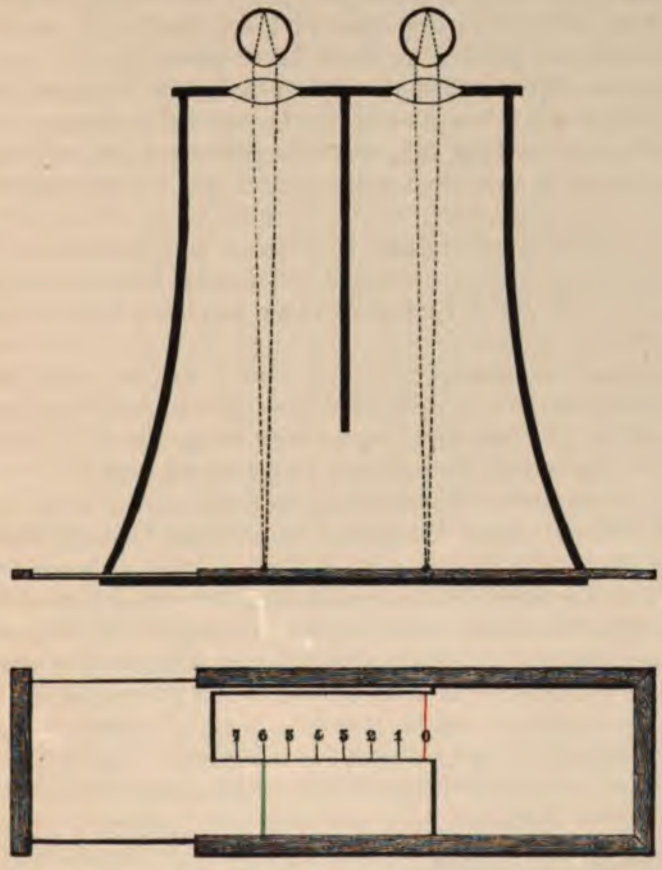

FIG. 129.

distance, about equal to that between the eyes. Under such circumstances, their fusion into one single vertical line necessitates parallelism of the lines of fixation. Knowing that this parallelism is generally possible only in the absence of any accommodative impulsion, we shall provide the patient's eyes, or the sight-holes of the stereoscope, with glasses, which permit him to see at the distance of the objects without any effort of his dynamic refraction.

Ordinary stereoscopes being generally 166 millimetres $(\frac{1^{\text{m.}}}{6})$ long, emmetropic eyes require convex 6 to fulfil this condition. If we have to do with a hyperope of 4 D, we shall give him the convex 10 (4 D to correct his hyperopia, and 6 D more to adapt him for 166 millimetres. For a myope of 2 D, on the contrary, $6-2=4$ D would be sufficient.

It will be noticed, however, that, under such circumstances, the majority of patients do not succeed in fusing the images when their eyes are directed parallelly. These latter generally show a certain convergence. We then help the patient to find the distance between the objects, which is requisite for the fusion of their images. When he has succeeded in doing this, we shall gradually separate the objects more and more in successive sittings, until fusion is effected without the least convergence.

When once binocular vision is obtained, with parallelism of the lines of fixation, which is equivalent to binocular vision at a distance, we shall try to realise it for a point which requires a certain degree of convergence.

To provoke a convergence of one metre-angle, we shall have to bring the objects toward each other through a distance varying with the length of the base-line, the average being about 1 centimetre. With this amount of convergence ought to be associated, physiologically, an equivalent amount of accommodation, that is to say, one dioptry. We will make the patient furnish this by diminishing the strength of his convex glasses one dioptry, in order to associate with the desired convergence, the corresponding effort of accommodation.

We continue in this way until the two objects are both brought on one vertical line. At this moment they require, for their binocular fixation, a convergence of 6 metre-angles and an accommodation of 6 D. An emmetrope would, therefore, have to remove the glasses from the stereoscope, and see with the naked eye. The hyperope, of our example, would see with convex 4, and the myope with concave 2.

Instead of lines, we may use geometrical figures representing solids of some simple form. Stereoscopic fusion is then facilitated by the tendency to a perception of the *relief.* The former have the advantage of allowing the oculist to easily observe whether the patient fixes with both eyes or not.

It may be asked: But why not take any object, hold it before the eyes, on the median line, and carry it a greater or less distance from the eyes, the hyperope having on his correcting-glasses? Are not convergence and accommodation then stimulated to the same degree as by the stereoscopic exercises? Of course they are, but it is impossible for us to ascertain, by this means, to what extent they respond to this stimulation, and to what degree they harmonise. The patient may obstinately affirm that he sees single and with both eyes, while really he sees with only one eye, and neglects the image received by the other. The stereoscope furnishes us an absolutely accurate test of the truth of the patient's assertions.

But let us return to the latter, whom we have left sufficiently trained to be able to see binocularly and single at a distance of 166 millimetres. He is still armed with his spectacles of 4 D. Would it not be possible to let him profit, now, by the relative laxity which exists in the relations between his convergence and accommodation? In other words, could we not accustom him to accommodate more and converge less? To do this, we gradually diminish the strength of his correcting-glasses, so that, with a given amount of convergence, he will be obliged to associate a progressively greater effort of accommodation. We sometimes succeed, by this means, in disengaging a certain amount of accommodation, and doing away with the necessity for wearing glasses in low degrees of hyperopia. But the experiment will always be more or less hazardous, since, by forcing it a little, we might make the patient fall back into his former habit of exaggerated convergence.

Indeed, the exercises which we have been discussing ought to be repeated, generally, for a very long time. Now, as years go by, the accommodation is declining; and by diminishing the strength of the convex glasses, we would impose upon this function an excessive effort. Moreover, from the moment the patient needs spectacles, it will be as well to let him have those from which he will derive the most benefit, as much from a point of view of ease in working, as from that of the the stability of the cure of his strabismus.[1]

## ATYPIC HYPEROPIA.

### *Atypic Axial Hyperopia*, $H^a$.

Hyperopia is, as we have seen, associated with smallness of the ocular globe, notably with insufficiency in length of its axis.

*Axial hyperopia* may be produced in still another way: thus, *retro-bulbar tumours* may exert such a pressure on the posterior pole that the region of the macula is pushed in front of the principal focus, and the eye thus becomes hyperopic.

The same fact has been observed as a consequence of a *partial detachment of the retina* in the region of the macula, by tumours, sub-retinal extravasations, or any other cause capable of detaching the retina and pushing it forward. The nervous membrane of the eye, though displaced, sometimes retains its sensibility, and thus we see eyes, which were formerly myopic or emmetropic, suddenly acquire a certain degree of hyperopia. Myopes, to whom this happens, are surprised and charmed to see at a distance without spectacles. Unfortunately, this happiness is of but short duration; the sight, after having been carried to so great a distance, takes flight, never to return, because, as a rule, the partial detachment of the retina is soon followed by the total separation of this membrane from the subjacent tissues. There are, however, happy exceptions.

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[1] Compare Landolt, article "Strabisme" in the *Dictionnaire Encyclop. des Sciences Médicales*, by Dechambre, p. 281.

We shall always remember a case, which we saw with Professor Horner, at Zurich, in 1868; it was that of a professor of theology, sixty-one years old, myopic from his earliest infancy, and having always worn number 8·7 D ($4\frac{1}{2}$ of the old system). We saw him arrive at the office, one hot summer's day, all out of breath and much excited, with some ill-defined visual troubles. The left eye, myopic by 12 D, having always been the weaker, had only $\frac{1}{6}$ of normal visual acuteness. The right eye proved to be affected by a very limited detachment of the retina, in the upper and outer region of the fundus. Its myopia, at that time, amounted to 8 D. But, little by little, the retinal detachment became more marked, the myopia diminished, and, four months after the first examination, this eye was hyperopic by more than a dioptry (old number 36). Its visual acuteness was equal to $\frac{1}{4}$ of the normal.[1] Our friend Horner informs us that he has since followed the history of the case, and, what is most remarkable and extraordinary, the detachment, instead of invading the entire retina, has gradually diminished. At the same time the myopia has reappeared in this eye, so that, in 1873, it amounted to 6 D, with an acuteness of vision of $\frac{1}{3}$, and, in 1884, the patient still used this eye in preference to its fellow.

Von Jaeger speaks of having observed a characteristic phenomenon in the vision of these individuals who have become hyperopic by the partial advancement of the retina. The objects whose images are received upon the detached portion seem smaller and farther away than those in the rest of the visual field.

The hyperopia, which is determined with the ophthalmoscope, in *optic neuritis* and infiltration of the retina, is also an axial hyperopia, and its degree, compared with the refractive condition before the disease appeared, may serve in estimating how much the retina is pushed forward (see the table on p. 140).

We sometimes see hyperopia arise in the course of *debilitating diseases*, especially when these are accompanied by considerable losses of the nutritive juices. Professor Horner attributes it to a diminution of the entire volume of the globe, which would, therefore, likewise make it an axial hyperopia.

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[1] Horner, *Klin. Monatsbl. f. Augenheilk*, p. 489, 1873.

### *Curvature-Hyperopia*, $H^c$.

We have seen that, generally, the curvature of the cornea, in the hyperopic eye, is not below the average, that it is often even above it, and that the form of the crystalline does not seem to vary much. Exceptionally, however, there have been found, in hyperopic eyes, radii of corneal curvature which were sufficiently great so that a part, if not the whole, of the ametropia might be ascribed to the relative flatness of the first refracting surface. These eyes may have the same length as the average emmetropic eye, notwithstanding their hyperopia, which is produced by the weakness of their dioptric system.

When the contents of the ocular globe are subjected to a very high pressure, the form of the eye must, necessarily, approach that of a sphere, and the cornea will be more or less flattened. Thus is explained the hyperopia or, at least, the diminution in static refraction, which is sometimes observable in *glaucoma*.

Mauthner cites, in this connection, a very instructive observation.[1] The intra-ocular tension was very high, in consequence of the swelling of a traumatic cataract. The radius of curvature of the cornea was 8·5 millimetres. After the extraction of the cataractous masses, and the consecutive lowering of tension, the radius diminished in length to 7·73 millimetres.

Certain affections of the cornea, as *keratomalacia*, or *central corneal ulcers*, may bring about a notable flattening of this membrane and, consequently, a true curvature hyperopia. It must, however, be admitted that the morbid process almost always causes such an alteration of the corneal tissue that clear vision through the flattened portion is speedily abolished. Transparent facets of the cornea, capable of producing hyperopia, are extremely rare.

As for the flattening of the crystalline, which might likewise cause curvature hyperopia, we lack, as yet, any incontestable observations on this subject.

The hyperopia which manifests itself in the course of a *paralysis of accommodation*, whether diphtheritic or of whatever other nature, may be explained by the abolition of a certain spasm of the ciliary muscle, which existed before the inception of the disease, and masked, by increasing it, the true state of the eye's refraction. We have already seen how difficult optometry may be made by this spasm of accommodation, especially in young persons. We shall meet it again under the head of myopia.

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[1] Mauthner, *loc. cit.*, p. 223.

## APHAKIA.

The crystalline (φακός) possesses, *in situ*, an optical action nearly equal to that of a convex lens of eleven dioptries, placed in front of the eye, near the cornea.

Deprived of the crystalline (*aphake*, whence we derive the substantive *aphakia*), the eye is, therefore, deprived of the help of an equivalent lens. If it was emmetropic before, it is hyperopic by 11 D afterwards. If it was previously hyperopic by 2 D, it will, in a state of aphakia, be hyperopic by $11+2=13$ D; if, before, it were myopic by 6 D, it will be hyperopic by $11-6=5$ D.

The only eyes that do not become hyperopic upon losing the crystalline lens, are those which are myopic by 11 D or more, *i.e.*, eyes which, before this loss, already have at least eleven dioptries of refraction in excess. The removal of the crystalline will render a myope of 11 D exactly emmetropic, while a myopia of 20 D would be changed by it into a myopia of $20-11=9$ D.

Since eyes affected with a myopia higher than 11 D form a small minority of eyes in general, aphakia nearly always brings about hyperopia. This cause of hyperopia is even very frequent; there is aphakia every time the crystalline is outside the pupillary domain. Thus luxation, which removes this organ from the path of luminous rays, deprives the eye of the dioptric action of the lens. The same phenomenon is produced as a consequence of the resorption of the crystalline (whether accidental or the result of an operation), oftener still under the influence of the extraction of a cataractous crystalline.

The absence of the crystalline is generally manifested by an increase in the depth of the anterior chamber. The iris, deprived of its support, no longer appears bulged forward, as in the healthy eye; it trembles (*iridodonesis*) with each sudden movement of the eye, unless it be rigid or adherent to the remains of the crystalline or its capsule.

The most peremptory symptom of aphakia is the absence of the crystalline reflexes, of Purkinje's images (p. 161). Both surfaces of the crystalline, the anterior as well as the posterior, give reflexes which are easily seen by oblique illumination, if the cornea is transparent and the pupil not too small. They disappear, necessarily, with the crystalline which produces them.

A very high degree of hyperopia and a considerably weaker refraction in one eye than in the other, render probable the existence of aphakia. Moreover, the fact that an eye can still decipher letters at a short distance is not a proof against the existence of aphakia,

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[1] Compare pp. 11-28.

because the eye may have been strongly myopic before losing its crystalline. The absence of the latter is established only if the eye sees better with strong convex glasses, the number 16 for instance. An eye possessing its entire optical apparatus would see much less under such circumstances.

In aphakia the dioptric system of the eye finds itself reduced to the simplest possible form, *i.e.*, a single refractive surface, the cornea. The aqueous humor and vitreous body, having a nearly identical index of refraction, may be regarded as forming only one single medium.

The aphakic eye is thus directly comparable with the *reduced normal eye*, which was described on p. 98 *et seq.* It has the same length as the normal eye, but the curvature of its cornea is much weaker than that of the refractive surface, which, in the reduced emmetropic eye, represents the dioptric action of the entire refractive apparatus. That is the reason why hyperopia from aphakia may be regarded as *curvature-hyperopia.*

The nodal point of the aphakic eye corresponds with the centre of curvature of its cornea. It advances considerably when the high degree of hyperopia is corrected by means of strong convex glasses, which are, necessarily, placed at a certain distance from the cornea. Hence it is that these eyes receive very large retinal images, not only much larger than if they could adapt themselves by means of their own dioptric system (which would again make the images equal to those of the normal eye), but much greater than those of axial hyperopes of the same degree, corrected under the same circumstances.

The removal of the crystalline entails, for the eye, not only the loss of a considerable portion of its static refraction, but also deprives it entirely of its dynamic refraction, accommodation being a function of the crystalline. Hence the aphakic eye can see clearly only at the distance for which it is adapted by the length of its axis and the curvature of its cornea, with or without the aid of spectacle-glasses.

But, since aphakia produces, generally, a very high degree of hyperopia, the correcting-glasses are very strong, and the least change of distance between them and the eye determines a notable alteration in their refractive action. Thus, a hyperope who is corrected for infinity by the convex 13, placed 1 centimetre from the cornea, would need only to carry this glass 15 millimetres farther away, and it would act like a convex 16 and adapt him for 35 centimetres.

Variations in the distance of the correcting-glass can, therefore, up to a certain point, supply the place of the accommodation, the hand, in this case, assuming the *rôle* of the ciliary muscle. Let us hasten to say, however, that the patient is never given a single correcting-glass. We should always give one for distant, and one for near vision, the latter being especially charged with providing for adaptation to the different distances of ocular work.

The influence of the variations in position of these strong convex lenses does not make itself felt solely in the enlargement of images and the optical adaptation of the eye. It is exerted from all the points of view that we have considered in connection with the action of convex glasses. The least inclination produces at once an astigmatism, and the least displacement in the plane of the glasses gives rise to a diplopia or a notable apparent displacement of the objects looked at. Hence it is never of more importance to be careful about the adjustment of the correcting-glass than in aphakia, especially that consecutive to a cataract operation. For the same variations in the action of the glass, which can cause the most disagreeable inconveniences, may be very profitably used in the interest of clear and convenient vision.

We have already said that persons operated upon for cataract generally present a certain degree of regular astigmatism. If this is very great, it is, of course, to be corrected by giving one surface of the glass a cylindric and the other a spherical curvature. But if the astigmatism is slight, its correction is more simply obtained by the inclination of the convex lens. This is especially easy in the case under consideration, because, the incision being made at the upper or lower corneal margin, the vertical meridian is generally the least refractive; it therefore suffices to turn the glass slightly about its horizontal axis, in order to increase its action in the vertical meridian. We thus avoid the inconvenience of too heavy and too strongly curved glasses.

Indeed, if it is necessary that one surface alone should exert all of the spherical action necessary to correct the high degree of hyperopia, this surface will, evidently, require a curvature twice as great as that of either surface of the biconvex glass. The surfaces of the number 16, which, when bispherical, have each a radius of 66 millimetres, must be replaced by one surface described with a radius of only 33 millimetres, when the other surface is devoted to correction of astigmatism. Hence spherical and chromatic aberration, prismatic action, &c., as well as the weight of the lens, are increased when the glass, for the same diameter, becomes thicker.

As to the prismatic action of strong convex lenses, this is so pronounced, that a person hyperopic by ten dioptries or more in each eye could never see near at hand without diplopia, unless the glasses were turned, decentered or combined with prisms. This is a point extremely important to consider for persons who are aphakic in both eyes. The glasses must be more decentered in proportion as they are stronger, or they must be combined with prisms whose apices are directed toward the temples.

The same attention is demanded with respect to the direction of the line of fixation relatively to the plane of the glass. We frequently hear patients who have undergone cataract operations complain of not being able to go up and down stairs. This is simply because, when looking obliquely through their glasses, they see the steps displaced.

Hence it is a poor plan to give a person who is aphakic in one eye, spectacles containing one glass for distant vision, the other to be used for near work, and the pair intended to be turned around the antero-posterior axis. In this way both glasses can never have a proper position in front of the eye. The mountings made by the optician Hunter, according to Dr. Noyes' directions, are far preferable.[1] They, too, contain the two glasses for the same eye, but, to change them, they are turned about the *vertical* axis, so that the bridge always occupies the same position on the nose. This movement is rendered possible by a hinge which permits the temple-pieces to be turned forward as well as backward.

But we must stop without having exhausted the series of resources that an aphakic person possesses in his spectacles, for the correction of his defect, provided he possesses also the necessary intelligence, willingness and patience. Unfortunately these qualities are too often lacking in many who consult us—so wrongly called *patients.* And these advantages even frequently become a source of interminable annoyance to the operator who has been successful in the extraction of the cataract. My satisfaction is always mingled with great apprehension when I see come back to me a fine black pupil which I have delivered of its thick veil; the cataract, removed from the eye, too often is changed into a torrent of complaints against the operator. And we much oftener see a look of gratitude in the empty orbit of him who has suffered an enucleation, than in the quondam cataractous eye, to which we have restored its sight. Enucleation simplifies so many things. Aphakic vision is so complicated. At the very moment when I am writing these lines, I receive a visit (how many times have I had this pleasure during the last seven years) from a lady upon whom I operated for a cataract. With a combination of the spherical and cylinder, she has a visual acuteness of 1·5, markedly exceeding, therefore, the normal, the highest that I have ever met with after cataract extraction.

But there are always new recriminations:—"Doctor, I can't tell the time by the clock, and I don't recognise any one coming in, when

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[1] Dr. Noyes, "Improvements in Spectacle Frames" (*Transactions of the American Ophthalmological Society*, p. 356, 1875). The same optician (Hunter) has sought to avoid the inconveniences of the spherical aberration of strong convex lenses, by superposing two plano-convex glasses by their curved surfaces.

I raise my eyes from my work, and I am always in danger of pouring my wine at one side of, rather than into, the glass," &c. &c. She is not very old, however, and might be expected to have retained some little degree of intelligence. But she has not succeeded in substituting manual accommodation for the ciliary accommodative mechanism, nor in adapting herself to the exigencies of her new sight.

Instances of this kind are abundant, and I am sure of finding an echo in the hearts of my colleagues when I say that the instruction of aphakic persons often presents greater difficulties than the cataract operation itself.

### *Hyperopia due to alteration in the Index of Refraction,* $H^1$.

*Diminution of the index of refraction* of the *aqueous humor,* or of that of the *crystalline,* necessarily lessens the refractive power of the eye, and may thus cause hyperopia. Well observed cases of this kind of hyperopia are, however, as yet lacking.

*Equalisation of the indices of refraction* of the different layers of the *crystalline* must have the same effect. This follows directly from what we have shown on pages 114 and 115, when discussing the influence of the peculiar structure of the crystalline upon its optical action. The hyperopia which replaces, as age advances, emmetropia or even slight myopia (see the curve $rr$ of the diagram on page 174), is attributable, at least in part, to an alteration of this kind in the indices of refraction of the lens. Those of the peripheral layers increase relatively to that of the nucleus. The crystalline becomes more nearly homogeneous and, consequently, less refractive. The very marked reflex which we obtain from the anterior surface of senile, although not cataractous, crystalline lenses, proves, as we have remarked, that the index of refraction of their peripheral layers has increased relatively to that of the aqueous humor.[1]

It is possible that still other factors contribute to this almost physiological hyperopia. Thus Donders and Mauthner think that, in old age, the dimensions of the eyeball diminish somewhat, which would necessarily bring the retina nearer to the dioptric system. It is also possible, although not proved, that the crystalline becomes somewhat flattened with age.

Finally, *increase in the index of refraction of the vitreous body* must likewise lessen the refraction of the eye, because this body presents a concave surface to incident rays, and because the rays, on leaving the crystalline, are evidently more strongly deviated in proportion as they pass into a less refractive medium.

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[1] Donders, *loc. cit.*, p. 206.

In this way has been explained the hyperopia which is sometimes seen to develop itself in the course of *diabetes.*

We have observed, among others, a most interesting case of this kind. A lady had a fall and became diabetic as a consequence of this traumatism, although her general condition was not much affected by it. At the same time she commenced to no longer see well at a distance, and found the spectacles, that she had previously worn on account of her presbyopia, to be insufficient. The refraction could be determined, in this case, with the utmost accuracy, not only because the patient was very intelligent, but because she had reached an age at which the accommodation is almost *nil.* I made out a hyperopia of 0·5 D, which certainly had not previously existed. It increased and then diminished according to the amount of sugar excreted, and finally disappeared entirely. So that, when this traumatic diabetes was cured, the patient could dispense with her distance-spectacles, and substitute, when reading, her former spectacles for the stronger glasses which she had been obliged to resort to. I have seen the patient for several years, and it has been extremely interesting to note that each little recurrence of diabetes has announced itself at once by a diminution of refraction, to such a point that the curve of hyperopia was, so to say, parallel with that of the quantity of sugar eliminated with the urine.

Although direct proofs are wanting, it seems more natural to explain these rapid variations, in the refraction of the eye, by an increase of the index of refraction of the vitreous body, saturated with glucose, than by changes in the volume of the eyeball, which could hardly be produced with such promptness.

## MYOPIA.

Let us again, first of all, recall our definition of this form of ametropia, page 125: *Myopia is present whenever the retina is situated behind the focus of the dioptric system of the eye.*

Very numerous measurements of the curvature of the cornea have demonstrated that this surface, the most important of the dioptric system of the eye, is not more convex, but that it is often even less so, in myopes than in the normal eye.

The crystalline and its indices of refraction do not seem to differ much, either, in this form of ametropia, from those of emmetropia and hyperopia.

Myopia is, therefore, in the great majority of cases, attributable to an *elongation of the optic axis.* The excess in length of the myopic eye is, moreover, striking in high degrees, and suffices to explain the latter without the intervention of an alteration in the optic apparatus. If the hyperopic eye is too short, the *myopic eye would, therefore, be too* *long*, or it would represent, *relatively to the position of the retina*, an *excess of refraction*. Only, the refractive defect in hyperopia constitutes a true anomaly, inasmuch as, as it is, the hyperopic eye is not adapted to any real distance, and since, in order to be in a condition to see distinctly, it is absolutely necessary that its ametropia be corrected, whether by the aid of its accommodation or by means of optical instruments. The hyperopic eye is, therefore, *absolutely* too short. The myopic eye, on the contrary, is too long only for great distances, while it is perfectly adapted for near vision.

## TYPICAL MYOPIA.

Myopia, *of itself*, if it is not of such a degree that the ordinary working-distance is greater than that of its *punctum remotum*, does not, therefore, constitute a troublesome anomaly of the eye. It may even be advantageous under certain circumstances, and, if hyperopia represents an arrest of development, *typical* myopia may be considered as a state created by nature with the aim of adapting the organ to the functions of a superior race. It is, indeed, interesting to observe that, if all animals are hyperopes, myopia seems to be peculiar to the human race and, moreover, that it is much more frequently met with among civilised nations than among savages.

The *disposition* to myopia is, therefore, found in the development of the human race, and the *determining cause* in what is more particularly called civilisation.

The conformation of the body, that of the cranium in general and that of the orbits in particular, undergoes such changes that a change in the conformation of the eye offers nothing that is not very natural. Moreover, intellectual development requires of this organ a much more perfect structure than is to be found in the higher orders of animals. For the latter the visual organ has no greater value, often even less, than those of hearing and of smell. In man, on the contrary, vision acquires an absolutely predominant importance. The strongly hyperopic eye, perhaps even badly accommodated in former times, adapts itself perfectly to a great distance, either by its form, in becoming emmetropic, or with the help of its dynamic refraction. All the savages examined have indeed proved, by the high degree of their acuteness of vision, that if they are hyperopic because of the peculiar structure of their eyes, they can remedy this defect perfectly by means of the ciliary muscle.

When, still in the midst of a semi-barbarous condition, in which the principal occupations were hunting, fishing, tilling the soil, and the like, civilisation began to dawn, it required of the eye, hitherto employed exclusively in seeing afar off, a much more circumstantial work at a short distance. Long before it became a question of reading or writing, men were given to the manufacture of textile fabrics, for example, and to a thousand other occupations which necessitate distinct and continued vision of near objects. Is it matter for surprise that Nature, which has known so well how to modify the form and functions of our bodies according to the exigencies of the race, should have attempted to do something analogous for the eye, by elongating it somewhat, in order to render it better qualified to fulfil its new destination? I mean to say, in a general way: The tendency to myopia may be regarded as a phenomenon of adaptation to the exigencies of the development of the species.

The eye, however, never being exclusively applied to work at a short distance, nor even the greater part of the time, nor by all the individuals of the same nation, it is not astonishing that the myopic conformation has nowhere become the rule and probably never will.

This way of thinking is, moreover, confirmed by the analogy which is frequently met with, between the conformation of the myopic eye and that of the cranium in one of whose cavities it is contained. Dolicocephaly, the very pronounced development of the bones of the face, of the root of the nose and the surroundings of the orbit, and especially the depth of the latter, have very often been pointed out as accompanying typical myopia. These facts seem to indicate Nature's intention to create a myopic type, in view of which not only the eye, but also the cranium itself is modified.

Although we may regard the myopic type as instituted in the interest of intellectual progress, as consecutive to civilisation, it would be wrong to suppose the development of the myopic eye to be more nearly perfect than that of the emmetropic, that its refractive surfaces were more regular or better centered, the accommodation more rigorous, the retina more sensitive or the motor apparatus at once more powerful and more supple.

No, the average emmetropic eye is not yet surpassed in its workings by the average myopic eye. Those of the latter kind always bear the imprint of a forced evolution. They are the type of hypertrophy, rather than of perfection. Although this hypertrophy is of service to them under certain circumstances, they are not, on the whole, superior to emmetropic eyes. But there exists, nevertheless, a TYPICAL MYOPIA, which, like typical hyperopia, is met with in absolutely normal eyes.

*Low degrees* of this form of myopia are hardly distinguishable, externally, from emmetropia. However, eyes affected by a myopia of one dioptry, or of even only half a dioptry, see notably less, at a distance, than emmetropic eyes, and acquire the same acuteness of vision only with the help of the correcting-glass. This could not be otherwise, since there is no negative accommodation capable of diminishing the static refraction of the eye. Notwithstanding this fact, myopes are able to go about and to attend to their affairs perfectly well, without glasses; they may even practise target-shooting and become successful huntsmen. Only the occupations which require a maximum of visual acuteness at a great distance—as certain naval duties, for instance—are closed to them, unless they avail themselves of the aid of correcting-glasses.

Myopes of the *medium degrees* could not dispense with glasses in ordinary life; but, with these, they generally have perfect acuteness of vision. Moreover, the vision of each eye separately, at a short distance, is markedly better than that of the emmetropic eye, and this for two reasons: first, because the retinal images of the axial myope are larger than those of the emmetrope accommodated for the same distance (see Fig. 119); again, because the myope sees, at this distance, with a minimum effort of accommodation, or even without its help, when the object is at his *punctum remotum*. Hence he fatigues his ciliary muscle less than the emmetrope.

But the too slight accommodative work, required by the myope for near vision, may also have its disadvantages. We are already acquainted with the difficulties which result, for binocular vision, in the case of the medium hyperope, from the disproportion of the efforts imposed upon the accommodation and convergence.

In myopia of medium degree something similar is produced, but in the contrary sense. Not taking into account the distance between the eyes, convergence depends, for all eyes, solely upon the distance of the object. This is the same for the myope as for the emmetrope. Now we have seen that the physiological relation, which has been established between the two functions, takes the state of emmetropia as its basis, so that accommodation and convergence are always sensibly equal. Hence it is not astonishing if the myope experiences difficulty, greater in proportion to the amount of his ametropia, in associating, with the slight amount of accommodation required by his ametropia, the necessary degree of convergence. He finds himself in a position similar to that of the hyperope who, inversely, produces only with great difficulty the amount of accommodation which must exceed that of his convergence.

Thus it is that we meet with troubles of binocular vision in the medium degrees of both forms of ametropia. Only, if the hyperope easily falls into an excess of convergence, with the myope we oftener find convergence defective. A *tendency to divergence* manifests itself whenever the eyes are not attentively directed toward the object of fixation. This tendency becomes more and more difficult to overcome. Divergence, at first *latent*, or manifest only for a very short distance, becomes *absolute* and shows itself for all distances. The *divergent strabismus*, which before existed only *in posse*, now becomes an established fact. Thus it is that myopia, of itself, without other alteration of the eye—*typical myopia*—may become a cause of asthenopia and destroy the harmony in the simultaneous working of both eyes. Let us remark, in fact, that this tendency to divergence, which has been improperly confounded with insufficiency of the internal recti muscles, may be brought about simply by a discord between accommodation and convergence, in spite of the perfectly normal condition of the motor apparatus of each eye. We have, indeed, found, in myopes of this kind, that their field of fixation is as extended, and of the same form, as in emmetropes.[1]

One may easily perceive this *virtual divergence* when the myope is made to fix some small object, such as a letter or the point of a needle. As soon as the eye is excluded from vision, it turns more or less outward from its primitive position, diminishing its convergence or even diverging absolutely. It is not necessary, for this purpose, to cover the eye entirely. It suffices that the object be hidden from it by the interposition of the hand or a small screen. In this way one has also the advantage of being able to watch directly the position of the eye. As soon as the diaphragm is taken away, the eye resumes its normal direction, at least in the lower degrees, and when the patient is required to look at the object.

On the contrary, when the tendency to binocular vision has already been given up, the eye persists in its vicious direction, because the diplopia, which must necessarily be produced, no longer troubles the myope, since he has learned to ignore the retinal impression of one or the other eye. In order to obtain binocular fixation, the object must, in this case, be carried farther away, to a distance at which fixation is accomplished with greater facility. By bringing it gradually nearer, along the median line, binocular vision, thus re-established, may be maintained until the object is very near the eyes.

When first one, and then the other eye is successively covered and uncovered, the phenomenon is produced alternately in each of them, and the straightening movement which they execute, in directing themselves toward the object, resembles perfectly that of true strabismus. When the observer commences by covering one eye, and then causes the object to be fixed, bringing it gradually nearer, it is noticed that the excluded eye follows but imperfectly the movement of convergence of the other, and that at a certain instant it stops, seems to hesitate, and finally diverges outright.

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[1] Landolt, "Les mouvements des yeux et leurs anomalies," *Arch. d'Opht.*, i., p. 586, 1881.

The greater the myopia is, the more pronounced is this phenomenon. Whenever one of the eyes of a myope is covered, or otherwise excluded from binocular vision, and he fixes an object situated at or beyond his *punctum remotum*, the excluded eye has no reason for directing itself toward the object of fixation. The effort of accommodation, equal for both eyes, which informs the emmetrope and hyperope as to the distance of the object, is lacking in the myope under the circumstances mentioned. The complete relaxation of the accommodation, required by the vision of any object situated at or beyond the *punctum remotum*, would rather render the eye immobile in a direction parallel with that of the other; both eyes will consequently be placed in divergence relatively to the object of fixation. Hence this divergence is not necessarily due to muscular weakness. It shows us, nevertheless, that the myope will fall into divergent strabismus more easily than the emmetrope or hyperope, especially if other causes favour this deviation. Indeed, a true insufficiency of the internal recti is more often found in the cases of myopes than in other refractive conditions.

Relative insufficiency, and especially the absolute weakness of the muscles charged with convergence, which often succeeds it, may give rise to very pronounced symptoms of asthenopia. The eyes are quickly fatigued in near vision, and attentive persons perceive that they are troubled with diplopia when they persist in fixation. Pains in the head and even vertigo are the penalties inflicted upon those who attempt to overcome the difficulties which oppose themselves to near work; we have seen young persons, who were perfectly well, and whose eyes were otherwise absolutely healthy, obliged to give up their studies, solely on account of this asthenopia. The sound condition of their eyes, far from being useful to them, becomes even disadvantageous in such cases; giving to each of them the same value and the same right to vision causes the patient to persist in the desire for binocular vision. In such a case, the difficulties encountered by this co-operation of both eyes become a cause of fatigue, while an inferior eye is soon vanquished in the struggle and takes refuge in divergent strabismus. As soon as the latter is established the asthenopia ceases and the patient thinks himself cured; but, in reality, he has sacrificed one eye, and has paid, for the ability to work in peace with one eye only, the price of stereoscopic vision.

Aside from the necessity of using concave glasses in order to see at a distance, the difficulties of binocular vision at a short distance, the asthenopia or the divergent strabismus which may result from it, are, therefore, the principal inherent disadvantages of typical myopia of medium degree.

The visual troubles are, necessarily, more pronounced in the *high degrees* of myopia. Concave glasses are indispensable, even for work at moderate distances, as, for instance, playing the piano, painting, writing, &c. Binocular vision is very rare, often impossible, and divergent strabismus is the rule.

Indeed, to the destruction of the equilibrium between the functions essential to binocular vision, is joined, in these cases, still another cause of divergence: this is the *shape*, itself, of the eye. In order to realise a high degree of myopia, the eye must be notably elongated; but this ectasis does not limit itself solely to the posterior pole of the eye; the entire globe participates in it.

The strongly myopic eye strikes one at first sight by its form. It is voluminous, even enormous, especially in its antero-posterior dimension. It affects the form of an ellipsoid of revolution. When the lids are separated and pushed back, and the eye is caused to look toward the nose, the observer notices at once, that the equatorial region presents a much less curvature than the poles—just the reverse of the hyperopic eye—and that the eyeball seems to be hindered in its excursions. This form and length of the eyeball have, in fact, an unfavourable influence upon its movements. The field of fixation is less extended than that of the emmetrope, or, especially, that of the small and so mobile eye of the hyperope of low degree. But it is especially limited at the inner, nasal side. This is quite natural: the ellipsoid formed by the myopic eye is necessarily placed with its major axis parallel with that of the orbit. The axes of the orbits diverging markedly, the eyes diverge likewise, and can change this position only with difficulty. There is all the more reason why they should never realise such a convergence as is demanded by the proximity of the *punctum remotum* of these strongly myopic eyes. Finally, the muscles themselves seem to lack strength and "grip" on the eye, as if, by an excess of tension, they had lost their elasticity and contractile power.

The extreme cases of ectasis of the globe and myopia do not, moreover, belong to the typical form that we have been considering. They are almost always accompanied by so profound and multiple alterations in the organ that the ametropia represents only a secondary symptom in the *ensemble* of this morbid condition.

Indeed, besides this typical myopia, there exists a form of *atypic myopia* almost as common as the former, but immoderate in its development and baleful in its consequences. We are about to enter upon a more detailed consideration of it.

## ATYPIC MYOPIA,

### *Malignant, Progressive Myopia.*

The atypic myopia that we have in view is also an axial ametropia, but is a bastard form of typical myopia. The elongation of the eye, which produces the refractive anomaly, is accompanied, in this case, by a series of morbid phenomena, which go on increasing, multiplying and invading the ocular membranes most essential to vision, not sparing even the accessory organs, such as the ciliary muscle and the motor apparatus of the eyes. At the same time, the enlargement of the eyeball seems to know no bounds, and far exceeds the degree desirable for adaptation to work.

It is for this reason that the name *progressive, pernicious, malignant* myopia has been given to this atypic form. At their beginning, these two species of myopia are difficult to distinguish from each other. The ametropia is often the only appreciable symptom, the fundus of the eye presenting no anomaly.

Sometimes, in the medium degrees, the optic papilla is bordered, on the temporal side, by a lighter, or perfectly white, *crescent* (P N, Fig. 130), which is, itself, limited by a pigmentated contour. But this is not a certain sign of the malignity of the myopia. The crescent, indeed, is sometimes met with in hyperopic and emmetropic eyes, and very often in typical myopia which has remained stationary from the beginning, or which has made but moderate progress. On the other hand it may be lacking, although this is very exceptional, in malignant myopia.[1]

What is most important to note, is that, at its *début*, the optic papilla often looks red and congested. At the same time that irregularities of pigmentation and the crescent are noticed appearing at the external edge of the papilla, the entrance of the vessels *seems* to be nearer to this same border. But this phenomenon is only apparent, for what one is tempted to regard, as seen with the ophthalmoscope, as the nasal border of the papilla (S, Fig. 130), is only the retina, which is, as it were, drawn over the internal edge of it. The attentive observer discovers, under this prominent retina, the real outline of the optic nerve at its normal place, and nearly completing the circle formed by the generally pigmentated border of the crescent (Fig. 130).

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[1] Loring (*Trans. of the Internat. Med. Congress, Philadelphia*, 1876) finds the crescent in 20·5 per cent. of myopic eyes, in 3·33 per cent. of emmetropic ones, and in 3·49 per cent. of hyperopic ones; Cohn (*Untersuchungen, &c.*, p. 61) in 20 per cent.; Conrad (Leipsic Thesis, 1875), in 28·17 per cent. (3·41 per cent. of emmetropic, 1·6 per cent. of hyperopic); Schnabel (*Arch. f. Ophth.*, xx.. 2, p. 1) finds, in 135 crescents, 73 per cent. of myopes, 13 per cent. of emmetropes and 13 per cent. of hyperopes. M. Dor (*Etudés sur l'hygiène oculaire au lycée de Lyon*, 1878) finds, in 890 eyes, 21 per cent. of crescents, and 83 per cent. of the crescents in myopic eyes. According to von Graefe (*Arch. f. Ophth.*, i., p. 294, 1854), the crescent is met with in as many as 90 per cent. of myopes of high degree and Donders says that its frequency is such that "Myopia and Staphyloma posticum have thus become nearly synonymous." *Loc. cit.*, p. 354.

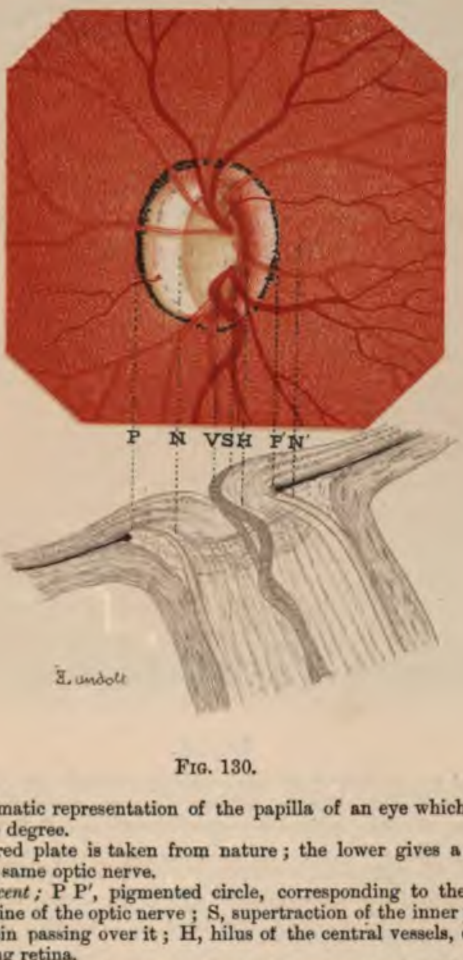

FIG. 130.

Semi-schematic representation of the papilla of an eye which is myopic in a moderate degree.

The coloured plate is taken from nature; the lower gives a schematic section of the same optic nerve.

P N, *crescent;* P P', pigmented circle, corresponding to the choroidal ring; N, outline of the optic nerve; S, supertraction of the inner part of the retina; V, vein passing over it; H, hilus of the central vessels, covered by the overlapping retina.

The inner and outer sheaths of the optic nerve are noticeably separated from each other.

In the highest degrees of this myopia, these phenomena are much more marked. The hyperæmia may be accompanied by exudation. The outline of the papilla becomes indistinct. Later, the hyperæmia and the inflammatory symptoms generally disappear, leaving, as a trace of their passage, the external half, or even the entire papilla partially atrophied. The extent of the crescent increases and sometimes entirely surrounds the papilla, like a ring. But the latter extends especially towards the posterior pole, where it becomes frequently confused with a white patch of pigmentary and choroidal atrophy.

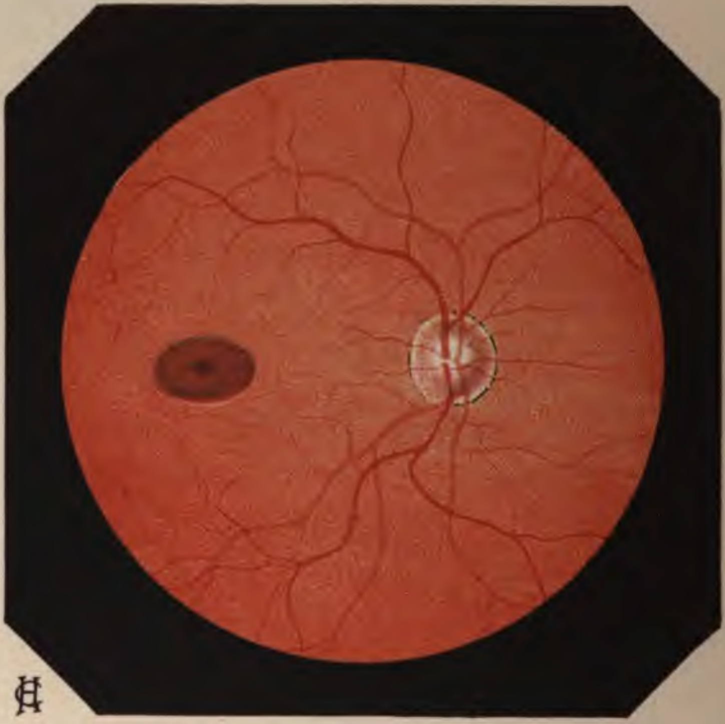

FIG. 131.

Fundus of a normal eye; the papilla being surrounded by its clear *sclerotic*, and its pigmented *choroidal* ring. The macula is seen as a dark oval, limited by a light outline, the *fossa centralis* as a brownish-dark spot in its centre.

Notable alterations in the pigmentated membranes of the *fundus oculi* are added, in fact, to those of the papilla. The epithelial layer is the one that first loses its pigment, its nuclei and even its cells in the vicinity of the optic nerve, but especially over an extent which may reach from the posterior pole to the equator. So that this portion of the fundus looks all white, and choroidal vessels are here seen through the epithelial layer deprived of its pigment (Fig. 132).

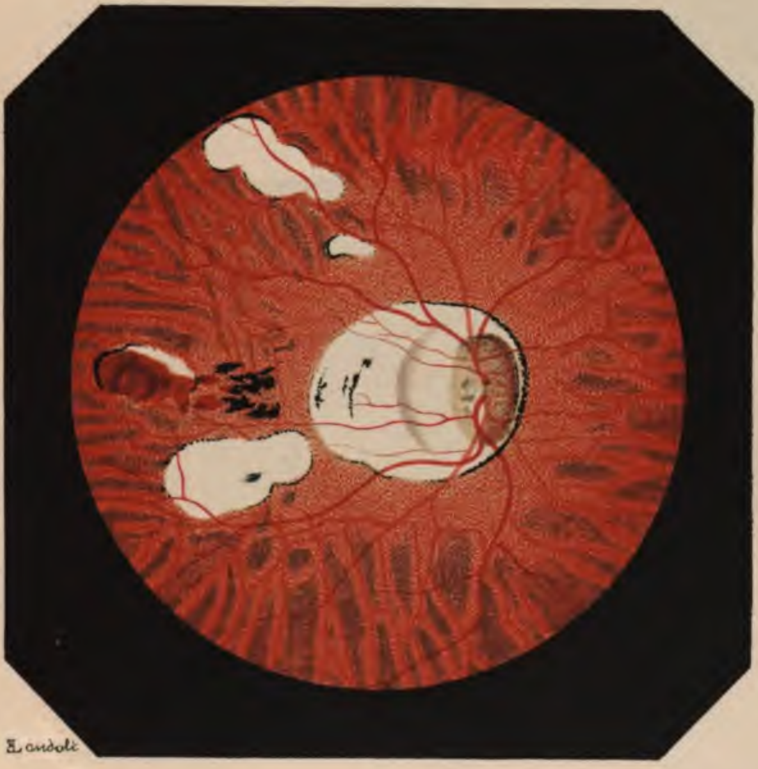

FIG. 132.

Fundus of a strongly myopic eye of the malignant type. Deep staphyloma; choroiditis, especially of the posterior pole; large white atrophic patches; black spots of accumulated pigment; smaller and larger hæmorrhages; a very large one in the macula; the clear halo which limits the hæmorrhage indicates that the resorption of the blood is beginning.

The latter accumulates, on the contrary, at the macula, where it forms a dark patch, which increases in extent and in intensity. Frequently hæmorrhages are added to this, and make it appear as if swimming in a pool of blood. Later, the extravasation is resorbed; the dark patch may clear up at its centre, in a stage of atrophy, and leave only a blackish ring surrounding a few scattered particles of pigment. The posterior polar region of the eye whitens more and more in consequence of the disappearance of its colouring matter. It becomes ectasic, and forms what has been called, since Scarpa's[1] time, a *pos-*

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[1] Scarpa, *Trattato delle principali malattie degli occhi*, v. ii., p. 146, 1856.

*terior staphyloma*—a sort of bulging projection, which adds itself to the eyeball, already generally hyperplastic, to the outer side of the optic nerve.

The latter is drawn, in part at least, into the staphyloma. The papilla turns itself more and more toward the temporal side, and, as we shall see, its fibres, with their closely associated envelope, seem torn from the external sheath of the nerve.

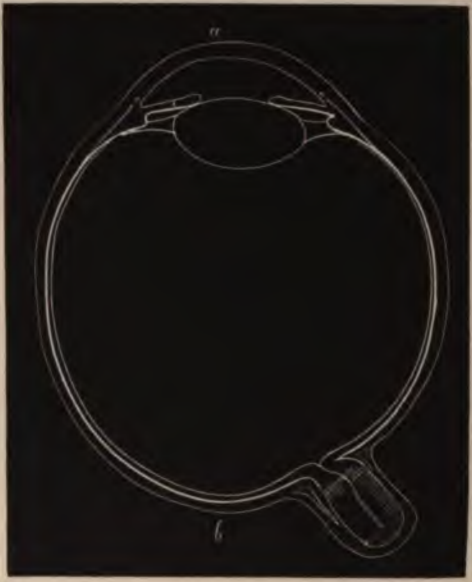

FIG. 133.—Schematic section of a strongly myopic left eye, seen from above.

The choroid opposite this ectasia becomes thinned, the vessels of the chorio-capillaris atrophy, and those of the stroma, enlarged in places at the beginning and giving rise to exudations, become smaller and smaller and may be even entirely obliterated.

The sclerotic, especially at the location of the staphyloma, shows itself very thin, to such a point that it becomes transparent and that the contents of the globe are visible through this membrane, as in the protuberant ectasis sometimes met with in the ciliary region as a consequence of chronic irido-cyclitis. Its tissue is profoundly altered in its structure: its fibres appear separated, distended and stretched.

The above-considered alterations in the envelopes of the eye may extend far forward toward the equator. Even the ciliary muscle manifests its participation in the morbid process—at least in pronounced cases. It appears elongated in the direction of the optic axis; its circular fibres are lacking or are only feebly represented.[1]

The vitreous body, more or less liquefied, contains elements which are derived from the choroid or retina. It is separated from the latter over a considerable extent, opposite the staphyloma. The space thus formed is traversed by membranous septa, is excessively thin and partially perforated, and filled with a lymphoid liquid.[2] Similar cavities are likewise found in other parts of the vitreous body and seem to be in communication with the lymphatic space claimed, by Henle and Merkel, to exist between the internal limiting membrane and the layer of nervous fibres of the retina.

The nutrition of the crystalline may suffer. The anterior and posterior poles become dimmed and the microscope reveals, under a thin layer of crystalline fibres, round globules which seem due to the coagulation of a homogeneous mass.[3] Even the suspensory ligament of the crystalline, the zone of Zinn, seems partially atrophied.

We shall not go farther into the pathological anatomy of the myopic eye. It has been the subject of many publications since Donders gave his classical description of it, as long ago as in 1864.[4] But there is one point upon which we must be permitted to dwell more in detail, because it is peculiarly adapted to clear up the mode of production of myopia. This is the condition of the optic nerve and its anatomical relations.

E. von Jaeger[5] was the first to observe that the ophthalmoscopic appearance of the papilla in the myopic eye, such as we have just sketched it, was due especially to a sort of stretching, in the direction of the posterior pole, which the nerve and retina undergo. Microscopical investigation, undertaken by Weiss at Nagel's instigation, has confirmed Jaeger's opinion and given it a very peculiar importance. The following are the facts as they result from the minute study of several eyes affected with weak and strong myopia.[6]

The intra-bulbar portion of the optic nerve has not only assumed an oblique position; it seems also to have been subjected to a traction which has partially separated it from its sheath, and drawn or pushed it toward the posterior region of the eye (Fig. 131).

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[1] See (Fig. 69) the ciliary muscle of a myope, after Iwanoff, and compare it with that of the emmetrope (Fig. 68), and that of the hyperope (Fig. 70), pp. 154-156.

[2] Duke Charles Theodore, *Arch. f. Ophth.*, xxv., iii., p. 111, 1879. Leber, *Ophthalmologische Gesellschaft. Heidelberg. Bericht*, p. 18, 1882. Weiss, *Mittheilungen aus der ophthalmiatrischen Klinik in Tübingen*, pp. 100-101.

[3] Weiss, *loc. cit.*, pp. 63-117, 1882. This alteration seems to us to be identical with that which we made out, described and figured in an article on pigmentary retinitis (Landolt, *Arch. f. Ophth.*, xviii., p. 25, 1872, and *Ann. d'ocul.*, 1872).

[4] Donders, *loc. cit.*, p. 367.

[5] E. von Jaeger, *Ueber die Einstellung des dioptrischen Apparates des menschlichen Auges*, Vienna, 1861. Mauthner, *loc. cit.*, p. 436.

[6] Weiss, *loc. cit.*, pp. 67-69, 78-80, and 82-88.

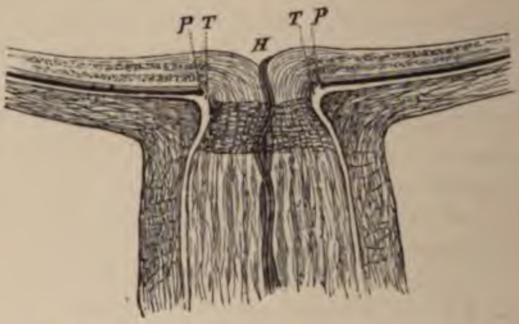

FIG. 134.—Normal entrance of the optic nerve.

The optic nerve, before spreading out to form the retina, has to pass through two openings, that of the sclerotic (T T, Fig. 134), which gives rise to the white tendinous ring surrounding the papilla (Fig. 131) and that of the choroid, P P, which corresponds to the black circle around the disc. The aspect of the fundus of the normal eye denotes an exact correspondence of these two orifices.

In the myopic eye this is no longer the case. The choroidal ring (c c′ Fig. 135) appears to have been displaced, relatively to the sclerotic ring (s s′, Fig. 135), backward and outward, and the papilla presents, in section, a characteristic deformity due to this phenomenon.

Indeed, if a section of the intra-bulbar portion of the optic nerve be examined, the *internal* limit of the choroid (c′ P′, Fig. 135), accentuated by the darker colour of its pigmentated epithelium, is seen to considerably overlap the corresponding border of the sclerotic ring. At this point, all the layers of the choroid are represented, but they grow progressively thinner and constitute a sort of corner, around the summit of which the nervous fibres curve. Hence the latter present, on the nasal side, a double curvature. At first rectilinear, when they enter the scleral opening, they bend outward to pass over the displaced choroidal limit. This point once passed, they curve in at an acute angle to go and form the anterior layers of the inner portion of the retina.

On the *outer* side it is quite different. Here, the choroidal limit is farther from the fibres than the scleral limit. The nervous fibres, which normally present a very pronounced curvature at the point of their distribution into the retina, are here bent at an obtuse angle and seem subjected to a traction which has a tendency to straighten them.

To what shall we attribute these deformities? To the fact that the *lamina cribrosa,* which constitutes the support of the papilla, is not composed solely of scleral fibres, but also of supports given off from the superficial and deep layers of the choroid. These supports place in close dependence the tissues of the papilla and choroid; they prevent the continuity of this membrane being, in reality, broken by the passage of the optic nerve through it, so that, if the choroid undergoes a sliding movement toward the posterior pole of the eye, it executes this movement as a whole. The entire choroidal orifice is displaced and tends to drag the papilla with it; but this, fixed as it is by its choroid connections, yields only in part; and is penetrated by the sharp internal edge of the choroid. Its resistance is such that, at its outer side, we observe the rupture of the deep choroidal supports which go to make up the *lamina cribrosa.*

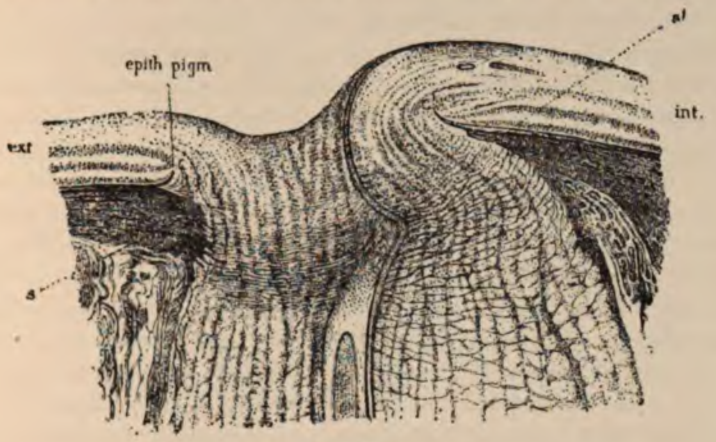

FIG. 135

**We borrow this figure from the work of Nagel and Weiss. It is the entrance of the optic nerve of a myopic (left) eye, seen from above. In the posterior portion of the nerve the optic fibres are still directed normally. After having passed through the *lamina cribrosa,* some go, slightly curved, toward the *outer* side. The others are obliged to abruptly change their direction twice, to pass around the *inner* edge c′ of the choroidal ring, which, with the sclerotic, has sunk down as a corner in the optic nerve. The violence which the latter undergoes is visible in the angular course of the central artery. On the *outer* side, connective tissue fibres in relation with the *lamina cribrosa,* and even optic fibres, are, as it were, pushed into the choroid and retina. They have removed the external layers of the latter from the sclerotic border and lifted up the pigmentated epithelium. In considering the lack of correspondence between the positions of the outer and inner borders of the sclerotic and choroidal rings, and the peculiar disposition of the *lamina cribrosa,* we have exactly the impression that the optic nerve has been partially torn from its sheath, and that the retina has undergone a sort of *traction* over the edge of the papilla. Nagel, in fact, calls this phenomenon *supertraction.***

The same alterations are produced in strong myopia; only, in this case, there is added a broadening of the choroidal ring, due to partial atrophy of its external contour. It is to this traction, exerted upon the optic nerve, that the detachment of its sheath is due, a fact observed by Donders (Fig. 130).

Thus are explained the *changes* which are observed, ophthalmoscopically, in the myopic eye (Fig. 130). The inner pigmentated limit of the choroidal ring has encroached upon the papilla, so that the vessels are nearer to it. The dark coloration of the choroid scarcely permits the real inner contour (N′) of the papilla to be perceived; however, the concealed portion of the latter is sometimes indistinctly seen through the choroid in the shape of a crescent, whose lighter coloration renders it faintly visible upon the pigmentated background. As to the external limit of the papilla, it is bare behind the retina. The sclerotic border, uncovered by the slipping of the choroid, shines with a pearly lustre through a greater or lesser extent, which constitutes the white crescent, so long accredited to an arrest of development, to a coloboma or to the atrophy or tearing of the choroid.

These facts, so well observed and described by Weiss, are evidently of the highest importance for the etiology of myopia. They constitute not only a new proof in support of the opinion which seeks the cause of this ametropia in the elongation of the ocular axis, but they also point out very clearly how this elongation is produced.

It is due to a force which acts especially upon the posterior polar region of the eye, pushing this portion backward with sufficient vigour to distend the membranes of the eye, carrying along the optic nerve, which becomes obliquely placed and separated from its sheaths.

What is this force? It could be nothing else than the contents of the eyeball, whose pressure is exerted, under certain conditions, especially upon the posterior pole of the eye.

But these contents being semi-liquid, the pressure exerted by them ought to make itself felt equally in all directions. In order that it shall subject some special region, as the posterior pole, in preference to any other, to its action, this part of the eye must offer less resistance or be more peculiarly exposed to the effect of the tension of its contents. The latter need not be raised above the normal in order to bring about ectasis of this weak part of the eyeball; but it is evident that the effect will be more pronounced and the ectasis deeper, in proportion as the pressure is stronger.

The real protective membrane of the globe, and the one which determines its form, is the sclerotic. This it is which must give way, in case a partial or total ectasis is produced. In the normal eye, however, this membrane presents its greatest thickness and resistance at the very place where the staphyloma is formed in the myopic eye.

In order, then, to understand the production of myopia, we have, in the first place, to find the causes capable of diminishing the resistance, especially that of the posterior pole.

The first idea which presents itself is, necessarily, to ascertain what connection there may be between the ectasis of the globe and the changes, in the membranes, which we have just been considering. The latter are, evidently, of a congestive or inflammatory nature. They always consist, essentially, of a choroiditis, which may attain more or less advanced degrees, from a simple hyperæmia up to alterations which are not second in importance even to those of *choroiditis disseminata.* It is an inflammation which may localise itself in a limited region of the *fundus oculi,* or may extend to the entire uveal tract, and which above all may affect the contiguous and adjacent membranes, the retina and sclerotic, and even other tissues of the visual organ. But is this choroiditis the effect or the cause of the ectasis of the globe?

There is no doubt that, if the sclerotic is primitively attenuated, it will not only yield to the intra-ocular tension, but will no longer properly protect the inner membranes of the globe. The choroid, deprived of its natural support, and subjected to distention by the contents of the eyeball, will suffer in its nutrition. The mechanical conditions will become favourable to its inflammation. Circulatory changes will soon be the consequence of its ectasis; a stasis of blood in these posterior regions will bring about an increase of intra-ocular pressure; and finally the anatomical elements of this membrane will respond, by phlogosis, to their constant stretching.

Hence a congenital weakness of the sclerotic, at the posterior pole, has been admitted; a weakening which is accidental or consecutive, either to an arrest of development,—imperfect closure of Ammon's fissure,[1] *funiculus scleroticæ* of Hannover;[2]—or to the separation of the sheaths of the optic nerve. We have already mentioned the latter; it is supposed to be due to exaggerated efforts of convergence, and to relative shortness of the optic nerve. The external sheath, in becoming detached, is presumed to drag with it a great part of the sclerotic.

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[1] Von Ammon (*Arch. f. Ophth.*, iv., i., p. 1, 1858) advances the proposition that the sclerotic, during the first fœtal epoch, presents, below, an opening, a pyriform fissure, whose broader extremity is toward the optic nerve, while the smaller extremity is directed forward. At the time when this fissure becomes closed, the globe is supposed to execute a movement of rotation from within outward, and to retain, for some time, on the outer side of its posterior pole, a protuberance covered by a thin membrane which closes the sclerotic fissure.

[2] The *funiculus scleroticæ*, discovered by Hannover (Copenhagen, 1876), presents itself as a filiform cord planted obliquely in the posterior pole of the eye, in the region of the *fovea centralis.* The anterior, or internal, extremity is attached to the choroid. The external extremity spreads out at the surface of the sclerotic. Both these extremities are broadened out in the form of membranes, so that the sclerotic may be rendered thin at this point. The whole space included between the optic nerve and the macula, and even beyond this, may present a furrow, and the sclerotic may be open along the whole extent of the fœtal fissure.

Rothholz (*Arch. f. Ophth.*, xxvii., 2, p. 45, 1881) thinks, from examinations of pigs'

The persistence of a fœtal fissure in myopic eyes, however, demands proof, and the widening of the intervaginal space of the optic nerve never extends as far as the staphyloma, not even to one of medium size. But the separation of the sheaths would indicate, at least, the stretching to which the nerve, its envelopes, and the sclerotic have been subjected at that point.

Without falling back upon an arrest of development of the sclerotic the cause of the lack of resistance may be found in the form, itself, of the eyeball. Suppose the latter, instead of being spherical, or nearly so, takes the form of ellipsoid or ovoid, whose strongest curvature corresponds to its posterior pole. If the intra-ocular pressure increases, its effect will make itself felt—conformably to the laws of hydrostatics—at the place which offers the most surface over the least space, *i.e.*, at the posterior pole, which is most curved. Hence an oblong conformation of the eye predisposes to myopia, under the influence of intra-ocular pressure.

According to all of these theories, the ectasis of the ocular globe would, therefore, be the primitive fact, and the affection of the intra-ocular membranes would be only a consequence of it.

It is not impossible that this is what happens, at least in certain cases; but the inverse hypothesis, which looks upon the inflammation of the choroid as the cause of the trouble, is no less probable. The choroiditis, which, as we have said, nearly always accompanies malignant myopia, communicates itself very easily to the sclerotic, renders it less resistant, and thus brings about the ectasis of the ocular globe. We have seen the proof of this, already cited, in the bunchy prominences which are produced at the circumference of the cornea, in the ciliary region, when the latter is a seat of an inflammation of the uveal tract. At times this entire region presents a blackish tint, due to the thinning of the sclerotic, and the anterior segment of the globe seems to form but one single large staphyloma. The external aspect of these anterior staphylomas is exactly the same as that of the posterior staphyloma of myopes, and histological examination confirms the identity of their nature.

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eyes, that the sclerotic fissure corresponds to the choroidal and retinal fissure. The sclerotic remains gaping at the point where the mesodermum, which formed the vitreous body, entered the eye. It is the pedicle of the embryonic vitreous body which becomes the funiculus, and the latter may cause the predisposition to sclerotic ectasis, when its entrance-orifice is not tightly closed.

The choroiditis is communicated, on the other hand, to the optic nerve and retina, which always participate in the inflammation of the vascular membrane, whether this be followed by myopia or not.

Exudations and hæmorrhages, which are produced in the retina, under this membrane, and in the vitreous body, and which may give rise to all sorts of visual troubles—to detachment of the retina and liquefaction of the vitreous body, alterations in the crystalline and its suspensory ligament, in short, all the objective symptoms that we have above cited, and all the subjective symptoms with which we shall hereafter become acquainted—find their natural explanation in an inflammation of the nutritive membrane of the eye. As a matter of fact, neither the ophthalmoscope nor the microscope reveal any difference between the choroiditis of progressive myopia and that of any other kind, unless it be that the former occupies by preference the posterior pole of the eye.

Not only are all the symptoms of pernicious myopia and the softening of the sclerotic perfectly explained in this way, but, too, the progression of the ectasis and, consequently, that of the myopia offer nothing that is not perfectly natural. Indeed, the exudation furnished by the choroid must contribute to increase the intra-ocular tension, to which the sclerotic yields where it is least resistant, *i.e.*, at the posterior pole, in the case of a posterior polar choroiditis, or *posterior sclerotico-choroiditis*, as it has been called since Graefe's time. There is, moreover, no doubt that a choroiditis may become the direct cause of myopia. With regard to this, we cite a very conclusive observation of Schœler's:[1]

An emmetrope nineteen years old acquires a myopia of 6·5 D in four weeks, as a result of acute posterior sclerotico-choroiditis. His visual acuteness diminishes to $\frac{14}{40}$ and $\frac{14}{70}$. With the cure of the inflammation, acuteness of vision becomes normal again, but a myopia of 8·7 D persists in one of his eyes, and a myopia of 8 D in the other.

It might, however, be asked why the posterior pole so often becomes the seat of morbid changes. The answer to this does not seem to be difficult to find. The posterior pole of the eye is almost constantly exposed to light, and possesses a preponderant importance in vision. Being the most active and the most fatigued, it must likewise be the most threatened, and its membranes will easily become inflamed if they be charged with excessive work or if, as sometimes happens, they present the least morbid disposition.

It should not be forgotten that the act of vision, indeed, although still a secret to us, necessarily implies physiological work, and a consumption of nutritive substance which must be constantly reproduced. The choroid, as well as the retina, participates in this process. The frequency of alterations about the macula is certainly significant, as is also the fatigue experienced by eyes affected by chorio-retinitis in this region, when they analyse distinct retinal images. We very often see myopes, complaining of no fatigue as long as they are not adapted, experience the most violent asthenopia as soon as they use a correcting-glass (to the exclusion, of course, of all effort of accommodation), or if they try to work with the naked eye, at the distance of the *punctum remotum*. This fact seems to indicate that (which is, moreover, quite natural) distinct vision requires more energetic work, and, consequently, causes prompter and greater fatigue than does inattentive, vague, indistinct vision.

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[1] Schœler, *Deutsche Klinik.*, p. 11, 1874.

On the other hand, von Arlt points out the fact that the external rectus and inferior oblique muscles must exert, under certain circumstances, a pressure on the posterior ciliary vessels. He attributes the softening of the sclerotic, at the posterior pole of the eye, to a venous stasis. The same cause could certainly provoke a hyperæmia and an inflammation of the choroid in this region.[1]

Whatever it may be which diminishes the resistance of the protective membrane of the eye, the latter must yield the sooner, and the ectasis become the deeper and more extended, in proportion as the intra-ocular tension is higher. The latter may increase, as we have said, in consequence of the exudation arising from the stasis of blood, and from the inflammation of the membranes at the fundus of the eye.

On the other hand, *compression of the globe by the extrinsic muscles* may have, directly, the same effect.

The external rectus, among others, on account of the obliquity of its course, curves along the temporal portion of the eyeball. It is in contact with a greater extent of the sclerotic in proportion as the convergence is stronger, or as the globe is more voluminous. The eye, clasped between the contracted internal rectus and the stretched external rectus, and compressed, too, by the oblique muscles and the superior and inferior recti, must have a tendency to become elongated antero-posteriorly. When one examines the horizontal section of a myopic eye, one cannot help feeling that this muscular pressure must have contributed largely to give it its elongated shape and the prominent curvature at the outer side of the papilla. The eye seems to want to escape the compression by becoming elongated in the region comprised between the optic nerve and the point where the external rectus enters into contact with the eyeball (Fig. 133).

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[1] The latest and most valuable investigations of Prof. Fuchs (*Arch. f. Ophth.*, xxx., 4, p. 1, 1884), speak in favour of v. Arlt's theory. They show that one of the *venæ vorticosæ* is found below the inferior oblique, and must be compressed during the effort of convergence.

Ectasis of the posterior pole of the eye will seem still less surprising when we recall the fact that this part is not protected, like the rest of the globe, by Tenon's capsule and by the muscles which envelop it. The pressure exerted by the muscles upon the eyeball can be directly perceived by delicately palpating a strongly myopic eye. It feels slightly flattened in the portions corresponding to the recti muscles.

It cannot be objected to this way of thinking, that intra-ocular tension is not generally exaggerated in myopes. It is not necessary that it should be, in order to produce ectasis of a sclerotic whose resistance is already diminished; and if the hardness of a myopic eye is not perceptible on palpation, it is for this very reason, that its envelope has already yielded to the pressure of its contents. Neither can it be said that we are in disagreement with the usual characteristic facts of increase of intra-ocular tension: the papillary excavation, among others, which, in glaucoma, is not accompanied by elongation of the optic axis. It must be borne in mind, indeed, that glaucoma arises under quite other circumstances and in conditions of resistance of the sclerotic which are entirely different, as they present themselves in eyes that are generally hyperopic and at a more or less advanced age. We may well suppose, with Professor Laqueur, that the effects of an increase of intra-ocular tension are different in the various periods of life; that, in infancy and youth, it produces a distention of the sclerotic and, when the latter has become rigid with age, a pushing back of the *lamina cribrosa*—that is to say, a glaucomatous excavation of the papilla.

Each of the hypotheses that we have mentioned, concerning the production of malignant myopia, has been in turn attacked as inadequate, but not one of them has been entirely overthrown. All have remained standing beside the adversaries that they have seen raised against them. It is more than probable, in fact, that multiple causes contribute to beget this form of ametropia. It is even possible that their *rôles* become inverted among themselves, that one begets the other or takes its rise from it, as we have seen in the case of posterior polar choroiditis and lack of resistance of the sclerotic. It is also more than possible that, to these known causes, there are added still others which have escaped us up to this time, and which will give a more satisfactory explanation, of the production of myopia, than that which we possess.

However this may be, clinical observation confirms our way of thinking, by proving that all circumstances capable of diminishing the resistance of the sclerotic, or of producing or maintaining choroiditis, and compression of the eyeball by the extrinsic muscles, *may* bring about or cause the growth of malignant myopia.

## ETIOLOGY OF MALIGNANT MYOPIA.

Malignant myopia, like typical myopia, is very rarely congenital.

The observation of von Jaeger, who claims to have found a considerable number of myopes among newly born children, is almost isolated. Ely[1] finds only 18 per cent. of them, Horstmann[2] 10 per cent.; Schleich[3] has found no cases of myopia among the numerous newly born children that he has examined. Koenigstein[4] and Schleich, in a new series of observations,[5] have found the refraction of the newly born exclusively hyperopic. They have constantly had recourse, in their examinations, to the atropinisation of their subjects, which they regard as indispensable. They even think that if preceding writers on the subject have found myopes among the newly born, it is because they have neglected to take this precaution.

Up to the age of eight years, myopia seems to be very rare.

Cohn[6] did not find even 1 per cent. of myopes in 240 children between the ages of six and eight years. None of the children, eight years old, that Kotelmann[7] has examined, were myopic; Callan[8] has met with no cases of myopia among negro children under the age of ten, nor did Koppe[9] find any among the young pupils, in a Kindergarten, whom he examined.

It is from this age on that it commences to manifest itself, and it develops especially and makes greatest progress between the ages of twelve and eighteen years.

Let us cite, as one example in a thousand, an observation of Dr. H. Derby's: A young man, whose father and mother were both myopic, and whose refraction was as in the following table:—

| | Left. | Right. |
|---|---|---|
| At the age of 10 years: | M 0·75 | Emmetropia. |
| " " 12 " | M 0·75 | M 0·75 |
| " " 15 " | M 2·75 | M 2·25 |
| " " 17 " | M 4·5 | M 4·5 and posterior staphyloma. |
| " " 19 " | M 5·5 | M 5·5 |

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[1] Ely, *Archives of Ophth.*, vol. ix., 4, p. 431, 1880.

[2] Horstmann, *Danziger Naturf. Vers.*, 1880.

[3] Schleich, *Mittheilungen aus der ophthalm. Klinik in Tübingen*, 1882.

[4] Koenigstein, *Wien. med. Jahrb.*, 1881, p. 47 *et seq.*

[5] Schleich, *loc. cit.*, 1884, p. 44 *et seq.*

[6] Cohn, *Klin. Monatsblatt*, p. 460, 1871, and *Arch. f. Ophth.*, xvii., 2, p. 305, 1871.

[7] Kotelmann, *Programme du Johanneum à Hambourg*, 1877, and *Jahns Jahrbücher für Philologie u. Pædagogik*, 1877.

[8] Callan, *Amer. Journ. of Med. Sciences*, vol. lxix., p. 331, 1875.

[9] Koppe, Dorpat Thesis, 1876.

*Per-centage proportion of Myopes according to School-Classes or Age.*

| AGE. | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21-23 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1871 Erismann[1] (St. Petersburgh), . . | | 10·2 | 14 | 13 | 20·6 | 21·2 | 28·6 | 32·6 | 39·3 | 43·2 | 40·4 | 47·2 | 40 | 50 | |
| Koppe[2] (Dorpat), . | | | 11·4 | | | | 21·2 | | | 43·1 | | 61·1 | | | |
| 1876 Pflüger[3] (Luzerne), | 0 | 2 | 3 | 6 | 6·5 | 6 | 10 | 14·5 | 26 | 30 | 43 | 55 | 56 | 40 | 61·5 |
| Florschütz[4] (Coburg), according to classes, . . . | | 7 | 7·8 | 8·5 | 15·8 | 22 | 24·4 | 30 | 33 | 50 | 59 | | | | |

| AGE. | 6-8 | 8-9 | 9-10 | 10-11 | 11-12 | 12-13 | 13-14 | 14-15 | 15-16 | 16-17 | 17-18 | 18-19 | 19-20 | 20-21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1875 Conrad[5] (Germany), | 11·1 | | 15 | 20 | | | 25 | | 40 | 45 | 50 | 55 | 60 | 62·2 ✓ |
| 1876 Loring and Derby[6] (New York), . . | 3·55 | | 5 | | 10 | 11·7 | 13·4 | 13·4 | 15 | | 20 | 25 | | 26·8 |

| AGE. | 8<br>7-9 | 10·5<br>8-13 | 11·5<br>10-13 | 14·5<br>10-19 | 12·5<br>10-15 | 14<br>13-15 | 14·5<br>13-16 | 15<br>11-19 | 16<br>15-18 | 17<br>15-19 | 18<br>17-19 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1883 Nordenson[7] (Paris), | 6·6 | 6 | 6·4 | 8·3 | 13 | 4·7<br>7 | 23 | 2·5 | 25 | 50 | 75 |

This fact is especially evident from numerous statistics gathered in nearly all civilised countries, and which already extend to more than 70,000 individuals. The higher one goes in the classes in schools, the more frequent is myopia and the higher are its degrees. The foregoing table shows the results of observations in different countries.

The statistics of examinations that have been repeated from year to year, on the same persons, are still more instructive than those already mentioned.

The reader may compare, in this connection, the following statistics, which we borrow from Cohn,[8] von Reuss,[9] H. Derby,[10] Seggel,[11] Ott,[12] Reich[13] and Erismann[14]:—

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[1] Erismann, *Arch. f. Ophth.*, xvii., i., p. 1, 1871.

[2] Koppe, Dorpat Thesis, 1876.

[3] Pflüger, *Arch. f. Ophth.*, xxii., 4, p. 63, 1876.

[4] Florschütz, *Die Kurzsichtigkeit in den Coburger Schulen*, Coburg, 1880.

[5] Conrad, *Die Refraction von* 3036 *Augen von Schulkindern*, Leipzig, 1875.

[6] Loring, *Internat. med. Congress*, Philadelphia, 1876.

[7] Nordenson, *Ann. d'ocul.*, p. 110, March 1883.

[8] Cohn, *Annual Report of the Friedrichs-Gymnasium at Breslau*, 1877.

[9] Von Reuss, *Arch. f. Ophth.*, xxii., 1, p. 211, 1876, and *Hygiene des Auges*, p. 64, 1883.

[10] H. Derby, *Boston Medical and Surgical Journal*, p. 337, 1877.

[11] Seggel, *Bair. aerztl. Intelligenzblatt*, p. 33, 1878.

[12] Ott, *Correspondenzbl. f. Schweizer-Aerzte*, viii., Nos. 15 and 16, 1878.

[13] Reich, *Arch. f. Ophth.*, xxix., 2, p. 303, 1883.

[14] Erismann, *Handbuch der Hygiene*, &c., v. Pettenkofer u. Ziemssen, ii., 2, p. 30, 1882.

### 1.—*Cohn's Statistics.*

In a Breslau gymnasium, the examination could be repeated on 138 pupils, of whom 84 had been emmetropes and 54 myopes.

At the end of eighteen months only 70 had remained emmetropes; 14, *i.e.*, 16 per cent., had become myopic by from 0·8 to 2 dioptries; 28 myopes, that is to say, 52 per cent., have had their myopia increase. It has not diminished in any case.

### 2.—*Von Reuss' Statistics.*

| General variations in refraction. | Increase. | Stationary. | Diminution. |
|---|---|---|---|
| At end of one year in . . | 47·7 % | 42·1 % | 10·5 % |
| " two years in . . | 50·8 | 37·5 | 11·7 |
| " three years in . . | 61 | 28·4 | 10·0 |
| Variations of Myopia. | | | |
| At end of one year in . . | 57·7 % | 29·6 % | 10·7 % |
| " two years in . . | 68·1 | 19·1 | 12·8 |
| " three years in . . | 77·5 | 12·3 | 14·2 |

(The diminution of refraction found by von Reuss is, moreover, very slight as compared with the increase. It scarcely exceeds 1·5 D, and might well be ascribed solely to the dynamic refraction, inasmuch as, in 76 eyes examined with the ophthalmoscope, 42·1 per cent. remained stationary, 57·9 per cent. increased, while none of them diminished, in refractive power).

### 3.—*Derby's Statistics.*

Within a period of three years, 10·6 per cent. of the emmetropes in Harvard College became myopic; 21·2 per cent. of the myopes became more so; 16·6 remained in the same refractive condition.

### 4.—*Seggel's Statistics.*

| | In a military school. | In a gymnasium. |
|---|---|---|
| Increase of myopia in three years, . | 13·3 % | 28 % |

### 5.—*Ott's Statistics.*

At the end of three years, of 132 eyes, 66·6 per cent. had undergone an increase of refraction, which was divided as follows, according to the different refractive conditions:

| | |
|---|---|
| Of the Hyperopes, . . . . | 52 % |
| Emmetropes, . . . . | 51·5 |
| Myopes, . . . . . | 99 |

### 6.—*Reich's Statistics* (85 school children).

| General refractive variations. | Increase. | Stationary. | Decrease. |
|---|---|---|---|
| At the end of six years in . . | 71 % | 25 % | 3·5 % |
| *Variations in myopia*, . . . | 80 | 5 | 14 |
| *Do. in emmetropia*, . . | 44 | 56 | — |
| *Do. in hyperopia*, . . | 90 | 10 | — |

14 per cent. of the hyperopes became myopic. Of the cases of emmetropia which changed into myopia, in only two did the latter reach a degree of 2·5 D.

The myopes, on the contrary, had this anomaly increase, in six years, from 2·75 (maximum) to 5·5 and 6 D.

### 7.—*Erismann's Statistics.*

This author again examined, in 1876, 350 eyes whose refraction he had already determined six years before. In 23 per cent. of the cases the refraction *had not changed.*

what were they

In 67 per cent. it had *increased.*
Hyperopia had diminished in 7 per cent. of the cases.
  Do. had changed into emmetropia in 8 per. cent.
  Do. „ „ myopia in 13 „
Emmetropia „ „ „ in 16 „
Myopia had increased . . . in 25 „
Refraction had *diminished* in 9 per cent. of the eyes.
Hyperopia had increased in 3 per cent.
Emmetropia had changed into hyperopia in 5 per cent.
Myopia had diminished in 0·5 per cent.
Myopia had changed into emmetropia in 0·5 per cent.

This same writer very justly calls attention to the fact that the few cases of diminution of refraction are attributable to the disappearance of an accommodative spasm, which, before, had made the refractive power appear greater than it really was.

The same facts result from all these statistics—that is to say, that among young people refraction has a tendency to increase as they grow older, but that *this tendency is not only more general, but also more pronounced, in the case of myopes than with those of any other refractive condition.*

At the same time that the number of myopes is increasing, that of hyperopes diminishes. That of emmetropes is difficult to estimate, because writers vary considerably in their definitions of emmetropia. Some hold strictly to the classical definition, and, as a natural consequence, find very few emmetropes; others extend the limits to one or even several dioptries above and below zero, and, naturally, meet with many more normal eyes.

This, however, is a matter of little importance. The fact is, that static refraction increases in a great number of eyes during childhood. After this period it remains nearly stationary, not only for hyperopes and emmetropes, but also for a great many myopes. The examination of adults has nowhere given a number of myopes greater than that found in the higher classes in school, but we find higher degrees of myopia.

It seems, therefore, to be between the ages of twelve and eighteen years that the eye acquires its final form and, consequently, its abiding static refraction. The latter is, as we have said, for the majority of them, higher than that of newly born children, but the difference is is not very great, being between 0 and about 3 D.

This evolution must be regarded as normal. What is pathological is, that in a certain number of eyes the refraction does not remain stationary, but increases constantly and with rapidity, being accompanied by the changes at the *fundus oculi* which we have pointed out, and by functional troubles which may considerably hinder the use of the organ.

Hence normal myopia and the pathological myopia are developed at the same period of life. There is nothing surprising about this. It is quite natural, on the one hand, that the eye should assume its final shape at the same time with the rest of the body, and, on the other hand, that the deviations from normal evolution should be produced at a period of life when the tissues are comparatively non-resistant and, so to say, in a condition of germination. For the visual organ this is less astonishing, since it is at this very period that it enters into its full activity, and in a way which is as sudden as it is severe, in the case of persons who have to submit to the exigencies of instruction or apprenticeship.

But what is more difficult to explain is the *why* and *how* of the production of this pathological myopia ?

The fact that certain eyes, exposed to absolutely the same external influences, are developed perfectly normally, to end in hyperopia, emmetropia, or even in physiological myopia, while others step aside from this path to walk in that of pernicious myopia, leads us to admit two orders of causes of myopia: *first*, a *predisposition* inherent in the person, or, at least, in his visual organ; in the *second place*, one or more *determining* causes, which, on this prepared soil, make this vice of formation of the eye thrive.

As to the predisposition, we have seen that it consists essentially in circumstances capable of diminishing the resistance of the sclerotic, of increasing the intra-ocular tension, of favouring the compression of the globe, especially in a certain direction, or perhaps the separation of the sheaths of the optical nerve.

Among the first, we have mentioned as probable, if not proved, arrests of development of the sclerotic, a peculiar conformation of the globe, but, above all, choroiditis.

Now, choroidal lesions often result from the general systemic condition. Faulty nutrition and a lymphatic or scrofulous constitution doubtless offer a soil favourable to choroiditis. Then, too, progressive changes are frequently met with among debilitated people, and among the populations who, inhabitating infertile countries, have always lived under unfavourable conditions or have had to maintain a particularly hard struggle for existence. Thus it is that the most characteristic specimens and the highest degrees of malignant myopia are often met with among peasants, and in the midst of illiterate people, who expose their eyes to none of the injurious causes, of which we shall hereafter speak as being capable of leading to myopia.[1]

We are thus able to understand that myopia may become endemic, and the *disposition* to acquire it *hereditary*, because the constitution may be transmitted from father to son.

It is, however, possible for this disposition to exist without becoming apparent. But it will show itself as soon as *determining circumstances* join themselves to the conditional causes.

As to the compression of the globe, and the stretching of the optic nerve, it must be produced especially during *convergence.* It will be more energetic in proportion as the convergence is stronger or more sustained,—that is to say, the more the external rectus encircles the eyeball,—and as the optic nerve is shorter. The insertion of the externus nearer to the cornea, or an angle gamma, which is very small or even negative, will act in the same way. In this last case, the line of fixation passing outside of the axis of the eye, binocular vision requires a more extended rotation of the eyeball inward, and, consequently, exposes the latter more to the pressure of the external rectus.

Let us say, before going further, that accurate data concerning the muscular insertion, and the length of the optic nerve, relative to the refraction of eyes, are still lacking, but that Donders' measurements, and those of other observers, prove that, generally, the angle gamma is smaller in myopic eyes than in others, and is sometimes even *nil* or negative.

Our own experiments,[2] like those of Dobrowolsky,[3] relative to the distance between the *fovea centralis* and the optic papilla, are in accord with this fact discovered by Donders; they indicate an approach of these two points toward each other, in the myopic eye.

A still more potent cause, capable of increasing the degree of muscular pressure during convergence, ought to be found in the *distance between the eyes.*[4] The angle of convergence, for a given point, is necessarily greater for each eye, in proportion as the centres of rotation are farther from each other, *i.e.*, as the base-line is longer. Now, the distance between the eyes evidently depends upon the *conformation of the cranium.* Hence there has been sought, in this latter, a preponderant cause of myopia, the sequel of an increase in the length of the base-line, which may be the consequence of it.

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[1] Landolt, "On Myopia," *London Ophth. Hosp. Report*, Dec. 1874.

[2] Landolt, *Annali di Ottalmologia*, ii., 1, January 1872.

[3] Dobrowolsky, *Klin. Monatsbl.* and *Ann d'ocul.*, December 1871.

[4] Mannhardt, *Klin. Monatsbl.*, p. 425, 1875; *Arch. f. Ophth.*, xvii., p. 69, 1871.

We have already seen that the form of the cranium presents still another connection with myopia; we have shown, indeed, that a *dolicocephalic* cranium often contains eyes formed like itself, *i.e.*, elongated antero-posteriorly.

If we knew Nature's secret, the delicate way of working by which she succeeds in creating typical myopia, there is no doubt that we should see clearly into the etiology of malignant myopia. In their relations to the conformation of the cranium the two forms are very like each other. But, if we are disposed to think it very natural that the ellipsoidal globe of the typical myopic eye should be formed in a dolicocephalic orbit, as in a mould, it seems, however, that this result is not brought about so simply in all cases, especially in that of progressive myopia.

Indeed, we often meet with crania of the most pronounced dolicocephalic type, with myopic eyes whose form is absolutely typical. And we say—Here are eyes which Nature must have created, in conformity with the cranium, directly for myopia, just as she destines others, which she lodges in shorter orbits, to emmetropia; one kind must be as healthy as the other; the sole difference consists, without doubt, in their form, as this results from the development of their antero-posterior axes: the first are myopic only in consequence of excessive length. But, when we come to examine such eyes with the ophthalmoscope, we are quite astonished to find very frequently the same alterations as in other myopic eyes: a crescent near the papilla, sometimes deep staphylomas, obliquity of the papilla with overdrawing of the retina, and even most extended choroidal changes.

These phenomena afford ground for reflection, especially in cases of anisometropia with asymmetry of the cranium. One side of the head is here visibly smaller, the forehead receding, the prominence of the cheek effaced and the chin rather small. It contains an emmetropic eye. The other half, on the contrary, shows a forehead and orbital margins strongly developed, a projecting cheek-bone and a larger chin. It would not be at all astonishing if the eye on this side were longer and, consequently, myopic; but what is surprising is, that here again the myopic form of the eyeball is not merely a copy of that of the cranium, but also the result of a pathological ectasis of its posterior pole, of a staphyloma, with the changes that accompany it.

Eyes thus deviating from the normal myopic type are not at all rare. They seem to indicate that the dolicocephalic form of the orbit may sometimes also represent the germ of malignant myopia. It may be that an eye, originally ellipsoidal, is disposed to posterior ectasis, for the reasons given above; or, in its endeavour to adapt itself to the development of the half of the face and the orbit, it may meet with obstructions which it can overcome only by the sacrifice of its normal structure, as, for instance, the traction exerted upon the posterior pole, by a relatively short optic nerve. Or, the peculiar form of the orbit may be associated with such an insertion of the muscles as predisposes to the myopic development of the eyeball. Moreover, these cases of malignant myopia with dolicocephaly show that, if choroiditis may become the cause of ectasis of the globe, the former may also often be the consequence of the latter.

However this may be, we know, up to the present time, two principal factors which may give rise to myopia: *disturbances of nutrition of the membranes of the eye*, and the *conformation of the cranium*. These two factors may be combined. Now both of them are eminently *hereditary*. We have already shown this in respect to the first, and nobody will deny it with regard to the second.

If, therefore, it is possible that typical myopia is transmitted from generation to generation, without, however, being constantly and inevitably reproduced, for the reason given on p. 421, the disposition to malignant myopia is no less hereditary. But it is not necessary that it should be developed. The individual may escape it, thanks to an exceptionally fortunate constitution, and especially when the determining circumstances are wanting.

If we consider the relationship of typical and progressive myopia, it will not be astonishing either, if, among the descendants of a family or tribe of typical myopes, a case of malignant myopia is occasionally met with.

From the point of view of heredity, we may, therefore, distinguish the following forms of myopia:

1. *Hereditary and congenital myopia.*

2. *Myopia* which is *hereditary* as regards the disposition, but *acquired* in consequence of circumstances which promote its production.

If we admit that, as is doubtless the case, an individual may present, spontaneously and without heredity, the physical peculiarities of which myopia is the consequence, we shall have—

3. *A congenital, non-hereditary myopia.*

Finally, a second form of non-hereditary myopia is that which we have seen produced as a consequence of affections of the membranes of the *fundus oculi*, whether accidental or attributable to debilitating causes. It is—

4. *Acquired, non-hereditary myopia.*

We thus find ourselves in perfect agreement with Horner,[1] who was the first to clearly establish the four preceding categories of myopia. We rejoice in this agreement, for one rarely sees a more favourable field for the study of myopia than is offered by the clinic at Zürich, presided over by so competent an observer as the master whom we have just cited.

This scheme requires only one further explanation. It is that of the way in which acquired myopia may be developed. This is a study of the causes which generate myopia, and make it progress on soil already prepared for it by heredity or by a spontaneous predisposition.

## DETERMINING CAUSES OF MYOPIA.

Among these causes must be mentioned, first of all, *ocular work at a short distance.*

This idea very naturally presents itself. We have considered even normal myopia as a phenomenon of adaptation of the organ to near work. If, in the succession of centuries, normal myopia can be produced under the influence of near work, it is not astonishing that a bastard form of it should arise, in the individual, under the same circumstances, provided always that the predisposition exists.

Moreover, near work requires of the organ much greater efforts than does distant vision. The simplest experiment suffices to convince the most sceptical person of this, by proving how soon fatigue is produced by fixation of a near object. Hence there is nothing illogical about attributing to this kind of vision a morbific influence upon the organ.

Finally, statistics *seem*, at least, to add to this kind of testimony, by showing a greater number of myopes among persons who work with near objects and continuously, than among others.

We have already mentioned the ever-increasing number of myopes found as one goes from the lower to the higher classes in schools. This progression is easily explained by the fact of an increase of age, which, as we have seen, is accompanied by a general tendency to increase of refraction. But it is also proportionate to the increase in the number of working hours. This has been established by Erismann in his statistics. He found:

Among scholars occupied 2 hours per day, 17 per cent. of myopes.
„ „ 4 „ 29 „ „
„ „ 6 „ 40 „ „

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[1] Horner, *Klin. Monatsbl.*, p. 458, 1875.

Seggel,[1] in examining 1600 soldiers, found:

| | | |
|---|---|---|
| Among the peasants, | 2 | per cent. of myopes. |
| " day-labourers, | 4 | " " |
| " mechanics, | 9 | " " |
| " merchants, printers, &c. | 44 | " " |
| " one-year volunteers, | 58 | " " |

Those of the last class are recruited, as is well known, among young men who have attended the higher schools or, at least, have studied longer than the others.

Tscherning[2] finds myopia, among the male population of the same age, distributed as follows :—

| | Total. | Myopes. | Per cent. |
|---|---|---|---|
| Day-labourers, peasants and sailors, | 2326 | 57 | 2·45 |
| Mechanics of various kinds, | 2861 | 150 | 5·24 |
| " engaged with near work, | 566 | 66 | 11·66 |
| Artists, engineers and architects, | 270 | 36 | 13·33 |
| Merchants, | 1009 | 159 | 15·76 |
| Professional men, | 491 | 159 | 32·38 |

Buschbeck[3] has studied the action of visual labour at short distance, among the children employed in threading needles in knitting-mills. These children, who, at the same time, go to school, furnish a contingent greater by 5·4 per cent. (boys and girls) than that of their school-fellows who are not engaged in the mills. The number of myopes among them increases proportionately to the length of time they have followed this occupation. The author is careful to call attention to the fact that the trade is not inimical to hygiene in any other respect.

In support of this hypothesis might also, perhaps, be cited the fact laid down by all observers, that myopia is of commoner occurrence in *lycées* than in industrial schools, the former generally requiring more prolonged work than the latter. Cohn counts 30 to 55 per cent. of myopes in the Gymnasium, and but from 20 to 40 per cent. in industrial schools (*Realschulen*).

Near vision may, in various ways, provoke the production and favour the progress of myopia in eyes predisposed to it: first, by the *convergence* which it necessitates and the muscular pressure produced by the latter. This point is not in doubt, for, in a state of absolute rest, under the influence of an anæsthetic, of syncope, or after death, the eyes diverge. It is not alone the internal rectus that contracts and presses on the eye when it is turned toward the median line; we have seen that the external rectus and the inferior oblique, being rendered tense at the same time, likewise compress it. Moreover, there can be no doubt that the muscles of the eye scarcely ever act separately. All of them certainly contract more or less, even when the resultant of their combined action is the contrary of that of their isolated action. This simultaneous contraction is, moreover, necessary, in order that the eye, maintained in position solely by the muscles and Tenon's capsule, with its attachments, may turn, as is demonstrated, about an immoveable centre. Another proof of this exists in the depression found in strongly myopic eyeballs, opposite the superior and inferior recti, and in the relative shortness of the vertical diameter of the eye, which has been pointed out by Donders. We have explained, above, how compression of the globe may bring about posterior staphyloma and the alterations of the *fundus oculi* which accompany it.

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[1] Seggel, *Bair. aerztl. Intelligenzblatt*, p. 33, 1878.

[2] Tscherning, *Arch. f. Ophth.*, xxix., 1, p. 201, 1883.

[3] Buschbeck, *Vierteljahrschr. f. gerichtl. Medicin, &c.*, January 1881.

Another factor which it is important to consider in reference to near work, is *accommodation.* The part it plays in the production of myopia has been exaggerated by some and not sufficiently appreciated by others. After having shown that most young myopes are affected with a spasm of accommodation, in the sense that their refraction, as subjectively determined, was perceptibly higher than it was in reality, it has been sought to find, in the accommodation, the principal agent in the production of myopia. This is certainly a mistake, and the proof in support of it, that has been drawn from statistics, was especially untenable. For all eyes, and not alone those that are myopic, or which afterward become so, have, in youth, a tendency to exaggerate their refraction by a spasmodic contraction of the ciliary muscle. Moreover, this spasm is not more intense in myopes than with other persons. It diminishes and finally disappears with age, and it may exist for several years without bringing about the slightest trace of myopia.

But although this almost physiological spasm is well borne by normally constituted eyes, it may be admitted that, in eyes already predisposed, it may become one of the determining causes of myopia.[1]

It is, indeed, incontestable that the effort of accommodation very often becomes, of itself, a cause of fatigue and asthenopia. Why should it not be able to bring about changes capable of producing myopia in eyes containing the germ of it, or contribute, at least, to increase it? Those who have denied to accommodation any influence in the production of myopia have, therefore, certainly been wrong. Furthermore, the reasoning by which it has been thought possible to reduce this hypothesis to an absurdity is utterly misleading. It has been said that, rather than increase refraction in myopic eyes, accommodation ought to render hyperopes myopic, since they fatigue accommodation much more than short-sighted persons, who have little or no need of it.

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[1] Schiess, Hosch, Chisolm, and others.

In the first place, it is, in our opinion, a great mistake to suppose that eyes always adapt, or try to adapt, themselves to the objects toward which they are directed. Very often they do not *look*—that is to say, they do not fix attentively; in short, do not try to obtain distinct retinal images. This is especially true of what is commonly known as "an absent look." But many occupations, although they cannot dispense with the intervention of the visual organ, do not, however, require absolutely faultless vision and are certainly often carried on in spite of imperfect adaptation. Any hyperope or emmetrope can detect himself at any moment, in a state of inexact adaptation, if he will give the matter his attention. And, indeed, if my neighbour, who, on account of opacities of his dioptric media or changes in his nervous apparatus, has only half as great an acuteness of vision as I, can read and write all day long, why should I use, for the same work, the degree of accommodation which gives me my maximum of visual acuteness, if this accommodation costs any effort?

Moreover, proofs of this ocular work without complete adaptation are abundant.[1] Thus it is that we see, every day, presbyopes reading the newspaper at a distance for which it is utterly impossible for them to have distinct vision. I am convinced, too, that, while most hyperopes, when not fixing any object attentively, relax a great part, if not all, of that accommodation, they exert more than is necessary during an examination of their refraction and visual acuteness. At all events, the fact that a person "needs" a certain amount of accommodation by no means proves that he brings it into play.

One thing, moreover, seems to be always forgotten: a hyperope of one dioptry, who wants to see distinctly at a distance of 33 centimetres $(\frac{1^{\text{m.}}}{3})$, needs no more accommodation than a myope of one dioptry who, for any reason, brings the object, at which he looks, to a distance of 20 centimetres $(\frac{1^{\text{m.}}}{5})$ from his eyes. If the myopia is of higher degree—of 14 D, for instance—it suffices to bring the object 16 millimetres nearer than the *punctum remotum*, in order that vision shall require the same effort of accommodation as that of the hyperope above cited. In short, the less the distance between the eyes and its *punctum remotum*, the more the accommodation is brought into play by the approach of the point of fixation from the *punctum remotum* toward the eye.

The influence of the contraction of the ciliary muscle in the production of myopia is explained, without difficulty, by the compression of the choroidal vessels, by the venous hyperæmia that it produces at the *fundus oculi*, as well as in the ciliary processes,[1]—a hyperæmia which leads to an inflammation of the uveal tract.

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[1] Landolt, "L'état actuel de la question de la myopie" (*Arch. d'Ophth.*, p. 47, 1884).

But it is not alone in high degrees and long duration of accommodation that one of the causes of myopia is to be sought,—it is rather in its *too often repeated variations* at short intervals. We have seen, in fact, on pages 160 and 161, that each contraction of the ciliary muscle causes the choroid to glide forward, while this membrane retires to its normal position with the relaxation of the accommodation. These frequent displacements must, of course, favour its inflammation; and we called attention, in the place just cited, to the fact that persons affected with choroiditis and retinitis are often relieved by using atropine, which forces the ciliary muscle and, consequently, the ocular membranes, to assume a state of rest.

Now, these changes in accommodation are nowhere more frequent than in very high degrees of myopia, where the least variation in the distance of the object immediately entails a notable change in the state of adaptation of the eye. If a seamstress or embroiderer, myopic by 10 dioptries, whose work cannot be done without perfect adaptation of the eye, holds her work 3 centimetres too close to her eyes, she at once needs three more dioptries of accommodation; if the work is 5 centimetres too near, even 10 dioptries are requisite.

In accordance with Von Graefe's opinion,[2] it has been thought that great importance should be attributed, in the production of myopia, to what is termed *insufficiency of the internal recti.* By this name has been designated the tendency to divergence which is manifested when the object fixed comes within a certain limit, or when one eye is suddenly excluded from binocular vision by a diaphragm, or, again, when binocular vision is suspended by means of a prism which produces vertical diplopia.

This phenomenon is frequently met with in myopia.[3] It has been thought that the latter might be the consequence of the insufficiency, in the sense that the excess of innervation of the muscles, necessitated by the imperfect development of their tissue, might be accompanied by an excess of accommodation. Thus the latter is held to exert the injurious influence which has been pointed out above.

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[1] Coccius, *Heilanstalt für arme Augenkranke*, Leipsic, 1870. (See page 162 of this work.)

[2] Von Graefe, *Arch. f. Ophth.*, ii., 1, p. 174, 1855; iii., p. 308, 1857; viii., 2, p. 314, 1861.

[3] Pflueger (*Luzerner Schulkinder, &c.*) has found functional difficulties of the internal recti in 49·2 per cent. of the myopes examined.

Schleich (*loc. cit.*, p. 27 *et seq.*) finds, among 578 myopes, 7·0 per cent. of concomitant strabismus; 16·4 per cent. presented latent disturbances of muscular equilibrium, viz.—4·7 per cent. latent divergence during near fixation, 3·9 per cent. latent divergence for any distance, 2·0 per cent. manifest divergence for near, 1·6 per cent. manifest divergence for near and latent divergence for distant fixation; 0·4 per cent. latent divergence for near and latent convergence during distant fixation; 0·1 per cent. manifest divergence for a short, and latent divergence for infinite distance.

Here, again, one has gone too far both in accepting and in refusing to accept this doctrine. Indeed, as we shall see, a phenomenon which did not have this significance at all has often been mistaken for a symptom of insufficiency, and the operations on the muscles of the eyes, which have been carried to a considerable excess, have only exceptionally at once rendered convergence easier and the myopia stationary. They have, on the contrary, very often given rise to complaints from the victims, which were as bitter as they were justifiable.[1]

But, if surgical intervention be limited to cases of genuine insufficiency, it may have this happy result. This, then, as well as the favourable influence of other means of diminishing the convergence effort, speaks in favour of the opinion which attributes to this function a certain importance in the production of myopia.

There is one fact which may be interpreted as a demonstration of the influence exerted by convergence, as well as by accommodation, upon the form of ametropia under consideration: this is the small number of myopes found among watchmakers and goldsmiths.[2] Their work requires to be held very near the eyes. But what distinguishes these artisans from others, whose working-distance is also a very short one, is the fact that they fix with only one eye, which, moreover, looks through a magnifying glass. The distance at which they work is so short, that convergence for it would be impossible, at least for any length of time. Moreover, the magnifying-glass relieves them from the necessity for accommodation, so that there is, on this account, no impulsion to convergence.

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[1] Von Graefe, *Klin. Monatsbl.*, p. 221, 1869; Von Oettinger, *Die ophthalmol. Klinik Dorpats*, p. 86, 1871; Rossander, *Hygiea*, 1870; Wicherkiewicz, *Compte rendu de la Clinique oculaire de Posen*, 1882; Mooren, *Fuenf Lustren, &c.*, 1882; Abadie, *Ann. d'oc.*, t. lxxxv., p. 64, 1880; Tscherning, *Arch. f. Ophth.*, xxix., 1, p. 244, 1883.

Let us cite an instructive observation of Schneller's, which seems to speak in favour of the influence of convergence on the progression of myopia. Having done the operation of discission of soft cataracts in the case of a child about ten years old, he afterwards made out a hyperopia of 5·57 D. (The child had, therefore, been myopic before the operation.)

The hyperopia diminished, in ten years, to 4·5 in one eye and 2·5 in the other, this being equivalent to the increase of myopia which would have been noticed if the patient had not been aphakic. At the same time, the patient had strong asthenopia, with diminution of visual acuteness, hyperæmia of the optic nerves and insufficiency of the internal recti.

After a tenotomy of the external recti, the asthenopia ceased and there was no further decrease of the hyperopia. The author considers this a striking instance, tending to prove the influence exerted by difficulties of convergence upon the development of myopia (Schneller, *Tagebl. d. Vers. deutscher Naturf. u. Aerzte zu Danzig*, p. 250, 1881).

[2] Dr. Cohn (*Centralbl. f. pract. Augenheilkunde*, April 1877) finds only 12 per cent. of myopes among jewellers and watchmakers, while he counts 45 per cent. of them among lithographers. Dor also speaks of the scarcity of myopes among watchmakers.

Convergence and accommodation are, however, not the only conditions which must be taken into account among the factors which might favour the production of myopia in work at a short distance. There is still *the position of the head and of the entire body.* In the vast majority of cases, near work is done while the one performing it is in a sitting posture, the body being inclined forward and the head bent over the object looked at. This attitude, when maintained all day long, and this for years together, must interfere considerably with circulation. The development of the thoracic and abdominal organs suffers from it, digestion is interfered with, constipation becomes the rule, the extremities are cold and the head congested. If this active and passive hyperæmia has troublesome consequences for the most robust constitutions, it becomes deleterious for eyes already disposed to choroiditis and ectasis of the globe,—*i.e.*, to malignant myopia.

The influence of near work may manifest itself in different ways. It will be noticed even that the three principal modes of production of myopia that we have mentioned, are so intimately related to each other that one of them necessarily suggests the others. Exaggeration of convergence leads to that of accommodation. The latter, of itself alone, and also by the excessive nearness of the object, which it necessitates, and the position which gives rise to cephalic hyperæmia, favours the production of choroiditis. The affection of the membranes of the *fundus oculi* entails a diminution of the acuteness of vision, which, in its turn, makes the gradual approach of the object and exaggeration of convergence obligatory.

Sometimes this vicious circle will be opened, on the contrary, by diminution of the acuteness of vision, and, at other times, by a spasm of accommodation. But at whatever point this wheel of misfortune takes up the victim, he must go round with it, and will have to inexorably pass through the series of injurious influences, which reinforce each other to aggravate the evil.

When we take into consideration this linking together of harmful causes, we are no longer surprised at the rapid and constant progress made by myopia in an eye which it has once attacked, especially when the latter has been, from birth, disposed to it, or is deprived of the ability to resist it. This fact is rendered especially evident by statistics, which show that, if refraction generally increases for all eyes, it progresses in a much more marked manner with myopes. Thus it is more than probable that, if one had not only counted the number of myopes in whose cases the error had been progressive, but if more attention had been paid to their *constitution,* it would have been found that those who show the most marked type of pernicious myopia are those whose refraction has undergone the greatest increase.

Now, the influence of near work on myopia being incontestable, it goes without saying that any condition which tends to bring the work nearer may become, *indirectly*, a cause of myopia.

Such conditions are multiple.

To them belong, in the first place, those which entail a *diminution of the acuteness of vision.* By bringing the object nearer, the eye seeks to make up for the indistinctness of the retinal image by its enlargement. Opacities of the cornea, optical defects and affections of the fundus, of any kind, may thus play an accessory part which is, however, of some importance in the history of myopia.[1]

It seems to us that it is in this way only that astigmatism can be capable of producing myopia. Certain very competent authors would like to attribute to it a more direct action, the explanation of which appears to us to be open to criticism.[2]

Besides the causes inherent in the organ itself, there are others, external and no less numerous, which lead to the exaggerated approach of the object.

Let us mention, among others, *a bad arrangement of seats and tables;* for instance, too low seats, or those which are, at least relatively, too low for horizontal tables; *the excessive fineness of certain kinds of work*—as engraving, embroidery, fine stitching, mathematical draughting and the reading of too fine print; *lack of clearness of the object,* due to a faulty typography. Pale ink and poor paper render

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[1] Nicati, *La myopie dans les écoles de Marseille,* "Assoc. franc. pour l'avancem. des Sciences," Montpellier, 1879, in statistics comprising 3434 individuals, finds the greatest number of persons with opacities of the cornea, astigmatism, lesions of the *fundus oculi,* among myopes.

| | |
|---|---|
| Myopes with opacities, | 29 |
| ,, well-marked astigmatism, | 72 |
| ,, hemeralopia, pigmentary retinitis, | 4 (out of 6) |

Total 105, or 20 per 100 of the total number of myopes.

[2] As to the frequency of myopic astigmatism, Green finds 54 per cent. of myopes (examined only subjectively) astigmatic, Nordenson, among thirty-three myopes, an equal number of cases of astigmatism, and Snellen, in 1273 astigmatic eyes, 59 per cent. of myopes. Horner, in examining a large number of children, before their beginning to go to school, finds from 2 to 4 per cent. of myopes, who were, almost without exception, affected with myopic astigmatism, so that he is almost inclined to hold that congenital myopia always consists of myopic astigmatism, and that it would, consequently, be attributable to the shape of the cranium (communication in a letter).—Green (J.), *Transact. Amer. Ophth. Soc.*, p. 105, 1871; Thomson, *Transact. Amer. Ophth. Soc.*, p. 310, 1871; *Amer. Journ. of Med. Sciences*, vol. lxx., p. 383, 1875; Javal, *Société de méd. prat.*, 27th Oct. 1880; Risley, *Philadel. Med. Times*, p. 673, 1881; Fuchs, *Arch. f. Ophth.*, vol. xxviii., p. 139, 1882; Prouff, *Rev. clin. d'Oc.*, 1883; Szili, *Centralbl. f. pract. Augenheilkunde*, p. 358, Dec. 1883.

vision difficult, by diminishing the difference between the object and the background. This difference is too slight, for instance, in the case of writing on a slate. *Insufficient illumination,* too, is doubtless often an indirect but powerful cause of myopia.

It would be at once impossible and useless to enumerate all the circumstances which may have a similar tendency. They are easy to discover, and the ever-increasing work that is required of the eyes, in civilised countries, tends to augment still more their number.

It is self-evident, too, that if we admit that *congestion* of the head, due to a stooping position, plays a preponderant part in the production of myopia, we must accord the same importance to any other cause capable of begetting a similar congestion. Thus, working in overheated, poorly ventilated or crowded apartments, the heat of lamps or other sources of light that are too near the head, and circulatory anomalies of any kind, may, and often do, determine hyperæmia of the head and visual organs which is as injurious as that brought about by a wrong position of the body.

The same may be said of circumstances, such as a weak constitution or hereditary dispositions, which may provoke those alterations of the ocular membranes which we have pointed out as being favourable to the production of myopia.

## VISUAL TROUBLES IN MALIGNANT MYOPIA.

We have already passed in review the visual troubles due to the optical defect—the ametropia properly so called—which necessarily accompanies all myopia; the impossibility of seeing at a distance; in certain cases, the discord between convergence and accommodation, &c.

To these troubles are added, in malignant myopia, the infinitely more serious calamities which result from the same cause as does this form of myopia itself.

The hyperæmia of the *fundus oculi* often gives rise, even at the beginning and in low degrees of myopia, to all sorts of *phenomena of asthenopia:* photopsies, scintillations, and fatigue of the eyes. The myope often experiences at the bottom of the orbit, or even in the entire forehead, a sensation of pressure which may become a genuine cephalalgia. The pain becomes more and more intense and more and more frequent, sometimes even constant, and often takes on the character of a neuralgia. Indeed it is not rare to see these phenomena, as also asthenopia generally, mistaken for neuralgias dependent upon some affection of the nervous system, and treated accordingly, without, of course, any success, and to the detriment of the general health.

The illusion of dark patches dancing before the eyes is scarcely ever lacking in such cases. These are the shadows, projected on the retina, of corpuscles suspended in the vitreous body; these apparitions are designated by the collective name of *muscæ volitantes.* Every eye, even though normal, contains similar corpuscles. They doubtless are cellular *débris,* nutritive waste of the vitreous body or other irregularities of this transparent tissue. They are not perceived under ordinary circumstances, because their shadows are not sufficiently intense, and the retina has become accustomed to them; but they are easily made visible by looking at some uniformly lighted surface, as the sky, a thick, well-lighted fog, the ocean, a dusty street, a wall, or simply a white paper reflecting solar light. They will still more surely appear if one look through a pin-hole in a diaphragm held very near the eye, the other eye being closed. The pin-hole forms a solitary and very small source of light, for which the eye is not adapted, and which makes the shadows more distinct and more intense.

The myope sees these phenomena with greater ease, because he is seldom adapted to the source of light. Moreover, when the myopia is pernicious, the sensitiveness of the retina undergoes, from the beginning, a pathological exaggeration. Later on, to these almost physiological corpuscles are added others which are due to the retinal and choroidal exudation. The latter are now more numerous, larger and more troublesome and disquieting to the patient. He ascribes to them all kinds of shapes, and never wearies of their description, which he willingly accompanies with a faithful sketch. This is a proof of the torments to which they subject him and of the anxiety with which he observes them. This observation itself places him in a vicious circle of action and reaction, for the more he pursues these phantoms, the more he is harassed by them, nothing being more fatiguing than the observation of such entoptic phenomena. Thus it is that they become a cause, both direct and indirect, of the weakening of the eyesight.

This weakening is complained of in an alarming manner, especially when, to the disturbances of the circulation, are added anatomical changes of the *fundus oculi.* At the same time that the ophthalmoscope reveals a growth of the staphyloma and an ever-increasing irregularity in the disposition of the choroidal pigment, and principally its accumulation in the macula, the acuteness of vision gradually diminishes.

Very often the patients complain of certain deformities of the object fixed: straight vertical and horizontal lines seem crooked in certain places. This *metamorphopsia* is attributable to the dislocation and to the differences of level undergone by the photesthetic elements of the retina, in consequence of the exudations and the immigration of the pigment from the subjacent membranes.

Little by little, or sometimes suddenly, the situation may become still worse; direct vision, over a greater or less extent, disappears; the affection of the macula has increased, or a hæmorrhage has been added to it (Fig. 132). The functions of this portion of the retina are abolished and central vision destroyed. *Scotomata* of the same kind may manifest themselves at still other places in the field of vision, as a consequence of choroiditis. The resorption of the blood, when it does take place, is effected very slowly and almost always leaves behind it a white patch of pigmentary atrophy, with the retinal functions annihilated or considerably diminished. Moreover, these hæmorrhages are easily renewed; relapses are almost the rule, and we may even rejoice if they do not take place *behind* the retina, where they may produce a detachment of this membrane.

*Detachment of the retina* is, in fact, an accident to which strongly myopic eyes are very much exposed. Its etiology is not yet thoroughly understood. It has been attributed to rupture of the subretinal vessels, to choroidal exudations or to a previous tearing of the retina (Leber). Horner has found but very few cases which speak in favour of this last hypothesis; detachment of the retina being produced especially in old age, between fifty and seventy years, it seems to him more natural to refer it to atheromatous degeneration of the vessels. It is, at all events, due to the profound disturbances of nutrition which have brought about, with ectasis of the eyeball, alterations of all its parts, and finally reduce them to this almost atrophic state, in which the tissues lose their power of resisting and their connections are relaxed.

Thus we have seen that the vitreous body becomes liquefied, a circumstance which ordinarily facilitates the detachment of the retina, that the crystalline becomes opaque and that its suspensory ligament partially atrophies. These are new circumstances which singularly aggravate the condition of such myopes. *Cataract*, almost always polar, adds itself to all the other causes of amblyopia and metamorphopsia, generally with a slowness which one might be tempted to call disheartening, if its removal offered the certainty of good success. But here again we encounter new difficulties. The liquefaction of the vitreous and the slight resistance offered by the hyaloid membrane and the zone of Zinn, complicate the cataract operation in myopes, from the danger of the escape of the vitreous and secondary detachment of the retina. The removal of the cortical remains, which are generally very adherent after the nucleus is out, is as difficult as it is dangerous, and the extraction in the capsule is more than hazardous, on account of the bad condition of the vitreous body. Moreover, the least inflammation of the iris and ciliary body may rekindle the former foci of choroiditis, and for ever compromise the vision, already very precarious, of a retina which performs its functions only as if by a miracle. Even after having happily escaped both Scylla and Charybdis, after the wound has successfully healed, the sight evidently cannot be very good, because of the changes at the fundus, especially in the region of the macula.[1]

Although detachment of the retina and the dangers of cataract are threatening only in the higher degrees of myopia and at an advanced age, the *phenomena of irritation of the intra-ocular membranes,* of which we have spoken at the beginning of this section, belong to every period of life, and may accompany any degree of malignant myopia. Even at a period when one would suppose the fundus of the eye to be no longer capable of being irritated, when the sclerotic shines with a greenish-white lustre through the thinned membranes which are atrophied, deprived of vessels and pigment, and when the pallor of the papilla indicates the gradual disappearance of the optic nerve's vitality, even then the unhappy myopes are not protected against the torments arising from an exaggerated irritability of the few nervous elements which remain to them; a moderately strong light hurts them, and they seem unable even to bear the distinct retinal images that their own skill and the oculist's art succeed in procuring for them. By placing the light in a certain way, and seeking the most favourable position of the object, they succeed in making its image fall upon the least altered part of the retina; they render the image distinct, by bringing the object sufficiently near, or else an appropriate concave glass, properly placed and inclined, furnishes them perfectly distinct images of distant objects. But, at the end of an instant, they become fatigued by this, abandon with a sigh the book or lorgnette and re-enter the chaos of their diffuse vision ; or even, by closing the eyelids, return to complete obscurity. They draw the curtain upon a scene which costs them more discomforts than it presents attractions.

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[1] We would not, however, exaggerate the dangers of the cataract operation as performed on myopes. In reality, we have never lost an eye under such circumstances; but it is none the less true that in such cases special precautions are necessary. It is well, for instance, to perform the iridectomy several weeks beforehand. There comes to us by chance, while we are writing these lines, an ecclesiastic, still young, formerly myopic by 6 and 10 D, and affected with a very pronounced choroiditis disseminata. We have operated upon him with entire success, without anæsthesia, by performing on one eye the extraction of the crystalline with its capsule, and on the other without the capsule. He recovered in this way, and still possesses sufficiently good vision, which enables him to devote himself to the most fatiguing mathematical studies. Unfortunately his sight has a tendency to diminish, because the choroiditis is making noticeable progress. Another patient, on the contrary, upon whom we operated three weeks ago, sees exceedingly well at a distance with a simple convex cylinder, so great was her myopia before, and she will probably always see well, because her myopia was not complicated by alterations of the fundus.

Still this renunciation does not always afford them rest ; the optic nerve, however much its normal working may be damaged, is the prey of a pathological over-excitement which finds expression in the most distressing luminous sensations. These phenomena present themselves before the myope with variable intensity ; they torment him more or less according to their extent, duration, and the interpretation he makes of them. Sometimes he sees, independently of the surrounding light or darkness, a brilliantly illuminated white surface which dazzles him ; objects appear indistinct, discoloured, or are entirely effaced. Sometimes, on the other hand, the imaginary field of vision becomes obscured, and in the darkness appear little floating flames, rockets, suns, stars, plants, &c. Again, at times, he sees less disagreeable images, as landscapes or human faces, but which, by their persistency and their morbid character, become as unbearable as the subjective sensations from the auditory nerve in affections of the inner ear.

All these phenomena are evidently attributable to the atrophic process which is going on in the nerve of the eye, since they are met with in atrophy due to any other cause. It must be said that, very fortunately, not all myopic eyes, even those in the highest degree ametropic, are doomed to this extremity, because the atrophy of the optic nerve only exceptionally attains so advanced a degree. But we have seen that, besides this, there remain, for the myope, enough causes for uneasiness and complaint, and, while the ametropia, of itself, represents an unenviable condition, its complication with morbid changes in the eye makes malignant myopia one of the most formidable of diseases.

## DIAGNOSIS OF MALIGNANT MYOPIA.

The diagnosis of malignant myopia consists essentially in the differential diagnosis between it and typical myopia. This follows directly from what we have just been discussing.

In the lower degrees and at the beginning of the ametropia, this diagnosis will almost always be difficult to establish, inasmuch as the two forms of myopia bear a striking resemblance to each other. Each is developed in childhood ; either may be accompanied by the ophthalmoscopic symptom to which we have referred as the crescent. In both we meet with the spasm of accommodation and the tendency to bring objects too near the eye. Moreover, these last three phenomena are manifested in eyes that are not, and are never to become, myopic.

The existence of malignant myopia may be presumed in infancy only when the father or mother is affected with this disease, or when it is common in the family. It becomes more than probable when the increase of refraction makes rapid progress, when asthenopic troubles are added to it, and especially when the crescent exceeds the form from which it derives its name, as well as the moderate and clearly defined limits which it assumes in healthy eyes, and when it spreads and becomes complicated with various pigmentary changes. The hyperæmia of the nerve and the signs of its overtraction are scarcely ever wanting, and the symptoms, taken collectively, leave no doubt as to the malignancy of the affection, even before the acuteness of vision is sensibly altered.

These symptoms do not always manifest themselves at the same period of life nor in the same succession. We meet with children five years old who already present a relatively high degree of myopia, 5 and 6 D, with staphyloma and profound alterations of the membranes of the fundus, without there being any asthenopia.

At other times, in the cases of young persons of from sixteen to eighteen years, asthenopia accompanies even moderate myopia. The ophthalmoscope does not reveal very extended crescents, though some hyperæmia of the papilla; and the ametropia, however moderate it may be, is of rather rapid production. The myopia is to be regarded as malignant in both these cases. It evidently is always so when its progress and the accompanying morbid phenomena take on the proportions that we have outlined in the preceding section.

In general, it will be easier to tell if a case of myopia is malignant than if it is not. Indeed, we rarely see a myopia, which has commenced with pathological symptoms, stop and change its character, while we frequently see cases of myopia, which we had supposed absolutely inoffensive, become progressive and pernicious. It is, in fact, in the immoderate progression, as well as in the affections of the ocular membranes, that the difference between the two forms of myopia lies. If such affections of the membranes are not very pronounced, it is, therefore, impossible, after a single examination, to pass judgment upon the nature of the ametropia. A series of examinations, repeated at regular intervals, will then be necessary, whose *ensemble* may serve to establish a curve representing the tendency to progress of the myopia.

## THE THERAPY OF MYOPIA.

If, as Donders has shown, static refraction gradually diminishes after a certain age, so that a formerly emmetropic eye is hyperopic at the age of sixty years, then myopia, which represents an excess of static refraction, must diminish, or even entirely disappear, provided it be slight enough and the person affected by it attains the necessary age.

This diminution of static refraction commences at the age of fifty, and amounts to 2·5 D at the age of eighty.[1]

Hence we must have a great deal of patience and very little myopia in order to derive any appreciable benefit from this lucky event. Only the myope whose ametropia does not exceed 2·5 D will be entirely cured of it—at the age of eighty! He whose ametropia does not reach this figure has even a prospect of becoming hyperopic.

On this slight hope myopes have erected a whole monument of illusions, to which they seem to cling all the more tenaciously, the less they have seen them realised. The most extreme myope, who can scarcely move his eyes, and who does not see beyond his nose, perhaps not even with spectacles, serenely asks: "Short-sightedness gets well of itself, in time, doesn't it, Doctor?" This is one of those common and pious beliefs, like the one which holds that, although the sight of myopes is not long, it is at least extraordinarily *strong*. There are even some otherwise logical persons who think that it is strong in direct proportion as it is short. We have seen what is to be thought of the visual power of myopes. We shall not return to this topic unless it be to seek means which may enable us to come to its aid. With the exception of the recovery which Nature, in her remarkable mercy, has placed at the end of a long and virtuous life—and even then, with the correction of the ametropia, she by no means re-establishes the nervous apparatus—with this exception, there is no such thing as spontaneous recovery from myopia, unless we are to regard as a favourable issue the detachment of the retina, which makes the eye emmetropic or even hyperopic. The comparatively rare cures of this sort are generally of very short duration, and followed by entire extinction of direct vision.

A curative process which is somewhat less baleful for the sight, but nevertheless rather dangerous, consists in the spontaneous luxation of the crystalline. If, in the former instance, it is diminution in the length of the axis which produces the change in the refractive condition, in the second case it is diminution in the strength of the dioptric system. It must be admitted, however, that we reach this latter mode of cure more safely by bringing art to Nature's aid and performing discission or extraction of the crystalline. The latter having, as we said when speaking of the production of hyperopia, nearly the same effect as a convex lens of 11 D placed 13 millimetres from the eye, its suppression is equivalent to that of a myopia corrected by a concave glass of the same strength, placed at the same distance.

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[1] See pages 174-178 of this volume.

It will be seen that this operation would cure a very respectable degree of myopia. But, unfortunately, this radical treatment is applicable only when the crystalline permits, or rather *requires* it—that is to say, when it is cataractous and young enough to be resorbed after discission, or ripe enough to allow of its removal by extraction. Attempts to disperse the normal crystalline have not given encouraging results. At all events, the simple existence of myopia by no means justifies the application of operative procedures whose aim is to cause the disappearance of the crystalline. Moreover, this lens having approximately the same optical value in all eyes, its suppression would exactly correct only a myopia of the corresponding degree. Myopia of less degree would be over-corrected. Myopes whose error is under 11 D are hyperopic after the extraction of a cataract, those of 1 D being hyperopic by 10 D, &c. And then, again, the crystalline is the organ of accommodation, a faculty which myopes, at least those whose ametropia is of low degree, would not willingly renounce. Hence the use of this means is still very restricted, and it will only exceptionally bring about recovery from myopia.

It might be supposed that this desirable end had been attained in a case in which the ametropia was seen to disappear under the influence of a mydriatic. But, in the great majority of such cases, the ophthalmoscopic examination made before the atropine has been instilled, shows that the myopia is only apparent and due to a spasm of accommodation. Hence it is only this latter that is eliminated, and not myopia, since the latter did not really exist. It is true, however, that it sometimes happens that the refraction, after prolonged atropisation, is less than that previously determined with the ophthalmoscope. But these cases are always open to the objection, or suspicion, that perhaps the examiner did not entirely relax his accommodation, or, at least, that the accommodative spasm of the examined eye was not entirely in abeyance during the ophthalmoscopic examination; so that, in this case, too, the myopia might have been only functional and not real. Even if we admit, as certain authors do, that atropisation, continued for months and years, can finally modify the form of the crystalline, and thus diminish the refractive power of the eye, thereby reducing or checking the myopia, even then this treatment would be susceptible of only a very limited application. Observations of this kind are very rare, the degree of correction obtained is very moderate and the duration of this improvement generally uncertain.

If we cannot cure the disease, let us try to prevent it; if curative therapy is generally impotent, so far as myopia is concerned, the task of *prophylaxis* becomes all the more important.

Finally, here too, we possess, as in other forms of ametropia, *palliative remedies,* notably optical correctives, which are so perfect, so varied and so easy to use, that their effect almost takes the place of a radical cure. Typical myopes relinquish the latter all the more willingly because, while enjoying as good a vision as emmetropes, they retain the advantages which their eyes have over normal visual organs in vision at short distance.

### The Prophylaxis of Myopia.

It would be as irrational as it would be impossible to restrict the application of prophylactic measures solely to eyes which are threatened with malignant myopia, under the pretext that the other form of this ametropia does not constitute a troublesome condition. Nor could it be alleged either that emmetropes stood in no sort of need of it, nor that it might rob hyperopes of the benefits arising from a progressive increase of their refraction. In the first place, as we have said and reiterated, it is very difficult to recognise in advance what kind of refraction will manifest itself in an eye later on, when the visual organ shall have become fully developed, or even to distinguish, at the outset, between typical and pernicious myopia. On the other hand, the preventive means at our disposal are, after all, only ocular hygiene. As such, they cannot be otherwise than advantageous to all eyes, of whatever kind; for the circumstances which may lead to progressive myopia, in eyes which are already somewhat disposed to it, are capable, likewise, of altering the vision of an eye, which has no such tendency to the elongation of its axis. And, since we are far from being able to stop the exaggerated development of an eye, hyperopes have nothing to fear, but everything to hope, from our prophylaxis, which will at least contribute to perfect the workings of their visual organs.

The benefit which eyes of any kind derive from this, is the best answer to the pessimistic assertions of those who would represent the prophylaxis of myopia as impotent and useless. It has been said, indeed, that it could not prevent those who have the germ of myopia in themselves from becoming myopic; however, this assertion still lacks proof. Although Mayweg and Just[1] were not able to make out a diminution in the number of myopes, as a consequence of the improvement in certain educational establishments, it must be borne in mind that the time spent in school represents only a small portion of ocular work. On the other hand, Florschuetz[1] has seen the number of myopes fall from 21 to 15 per cent., three years after the building of the Coburg schools according to hygienic principles, and von Hippel[2] finds only 34·5 per cent. of myopes in the new schools at Giessen, while the average was, for the old school buildings, according to Professor Cohn, 40·5 per cent. But, even though the *number* of myopes remained invariable, we know very well that myopia presents a great diversity, as well from a point of view of its degree as with regard to the accompanying alterations of the nervous apparatus. And, if hygiene had, for its only effect, one which cannot be denied it, *i.e.*, that of preventing a great many eyes from reaching a degree of myopia which would be pathological in itself, or of preserving for them a better acuteness of vision, we might consider this result as an extremely valuable one.

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[1] Mayweg and Just, *Soc. ophth. de Heidelberg,* 1883 (*Compte rendu,* pp. 87-89).

The principles of ocular hygiene capable of preventing myopia, or of diminishing the annoying consequences of it, necessarily result from what we have said concerning the circumstances which may, directly or indirectly, favour its outbreak. Thus, all hygienic conditions calculated to *strengthen the constitution*, to maintain good health, to protect it against debilitating influences, must contribute, at the same time, to make the visual organ more resistant and better able to react against any injurious agent, especially against those that produce myopia.

But it is *ocular work*, especially that at a short distance, which will demand most attention. All agree in attributing to it the preponderant *rôle* in the production of myopia. Again, it cannot be postponed until a period of life when, the body having attained its full development, the tissues of the visual organ shall be less vulnerable. Hence this near work should be made as inoffensive as possible.

The rules which we are about to formulate do not specially apply to "work," properly so called, but to any effort of the eyes. They must be watched, not only at a period when it may be a question of their working, but also, and especially in earliest infancy, one might almost say, from the time when the eyes first commence to functionate together. It is equally important to remember that we are concerned with their training, not exclusively at school, as sometimes seems to be supposed, but with the same care at home, in the workshop,—everywhere, in fact, and, of course, not alone in the case of children but in all cases, indiscriminately.

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[1] Florschuetz, *Die Kurzsichtigkeit in den Coburger Schulen*, Coburg, 1880; in Cohn, *Die Hygiene des Auges in den Schulen*, p. 67.

[2] Von Hippel, Academic Discourse delivered July 1st, 1884, at the anniversary of the foundation of the University of Giessen.

It is certainly unwise to hold bright objects, such as toys, too near the eyes of very young children. It is possible that, in this way, the germ of convergent strabismus may be given to hyperopes, or that that of myopia may be developed in those who are already disposed to it. When children are larger, they ought not to be permitted to occupy themselves with objects which require a too circumstantial vision, such as small engravings, minute playthings, &c. It will be difficult enough to prevent children from bringing them too near the eyes, even when the objects are large. Hence it is better to have them, so far as possible, play games which will necessitate their walking and running out of doors, which will make them vary the distance of adaptation of their eyes and thus doubly favour their development. If they are seated about a table and amusing themselves, the toys should have proper dimensions, and the children's heads should be as high as possible above the table. The body must be supported in such a way that fatigue may not dispose the child to lean forward, or to support himself by his elbows and thus bring the face again nearer the table. In the first place, therefore, the seats should be very high, then the table must not be too thick,—should have no drawer for instance. A drawer diminishes by its entire depth the height to which the child's head can be raised, because his knees must necessarily be passed under the table, in order that he may be comfortably seated. The feet should rest on a board which will permit of their being supported without the thighs being compressed by the edge of the chair or the knees raised above the horizontal. The chair should have besides, for very small children, arms destined, in the first place, to prevent their falling, and also to permit their supporting themselves and resting in various positions. The seat should never be without a back. This is of special importance when the child sits long at a time, as at school or when studying at home.

The proper construction of seats and desks and their reciprocal relations have been made the subject of most extensive investigation and publication. It is not possible for us to mention all of these.[1] We content ourselves with quoting the words of Prof. Esmarch, who very ably sums up the principles which should govern the construction of a sensible seat. This erudite surgeon says:—"School children become *scoliotic* and *myopic*, owing to the bent position of the body, when they are seated on badly constructed, old-fashioned benches. The trunk is bent when the bench is too far from the desk, when the former is too low relatively to the latter, or when it is without a back. So seats are inoffensive only when the child can sit upright upon them while reading or writing, and can keep this position long at a time without fatigue. In order that this requirement may be met, it is necessary—1. That the bench be raised above the floor (or the step for the feet) the length of the child's legs (measured from the popliteal space to the sole of the foot); 2. it ought to be as broad as the thighs are long (the distance from the popliteal space to the back); 3. the front rounded edge of the bench ought to project 2 or 3 centimetres beyond the inner edge of the table; 4. the seat should be sufficiently high so that the child can, when writing, conveniently rest the fore-arms on the table, without raising the shoulder and without bending the neck or leaning forward; 5. the lower part of the child's back should be properly supported, when the scholar is reading, by means of a short chair-back. These conditions vary with the growth of the child and the above measurements should be repeated, and the scholars assigned suitable seats, at least as often as once in six months."[1]

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[1] Fahrner, *Das Kind und der Schultisch*, Zürich, fifth edition, 1865 ; Parow, *Berliner Schulzeitung*, 1863 ; Cohn, "Die Schulhäuser auf der Pariser Weltausstellung.," *Berl. klin. Woch.*, p. 41, 1867 ; Cohn, *Die Schulhäuser und Schultische an der Wiener Weltausstellung*, Breslau, 1876 ; Buchner, *Zur Schulgesundheitspflege*, *Niederrh. Corr. Bl.*, 1873 ; Baginsky, *Handbuch d. Schulhygiene*, Berlin, 1877 ; Cohn, *Die Schulhygiene an der Pariser Weltausstellung*, Breslau, 1878 ; Cardot, *Traité du mobilier scolaire*, Paris, 1881 ; Snellen, *Le Banc d'Ecole*, Le mouvement hygiénique, No. 9, p. 358, 1885, and many others.

The feet should be advanced as much as possible under the table, and not bent backward under the bench. The first precaution renders the lowering of the head unnecessary, and the second makes it almost impossible. Another condition essential to the avoidance of the too near approach of objects of fixation, and especially a position capable of bringing about and maintaining the congestion of the face, is that the book to be read and, more especially, the paper to be written on, shall be placed on an *inclined plane*. This is obtained either by means of a desk, or by a board and a piece of wood slid a greater or less distance under it, according to the amount of inclination desired. This inclination will be greater for reading than for writing, because the construction of our pens puts a limit to it. Reading will be still better carried on if the back is supported and turned toward the light, while the book is held in both hands. In this way the reader will always have a tendency to hold the book farther away, on account of the fatigue of the arms, while he approaches, involuntarily, nearer and nearer to it when it lies before him on the table.

It is, however, easier to make rules than to secure obedience to them. Children have an extraordinary tendency to bring objects too near their eyes. Hence all sorts of *straighteners* have been invented;[1] chin-rests or a frame, in which the face is fixed, supported by a standard which is screwed to the table. A strap, attached to the high back of a chair, is intended to be passed around the forehead, and so keep the head more or less erect. It is not difficult to get apparatus of this kind—of which the simplest is always the best—and so to keep the eyes at a distance of at least 30 centimetres from the table.

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[1] Prof. Esmarch, *Zur Belehrung über das Sitzen der Schulkinder*, Kiel, 1883.

But reading and writing are not the only occupations that are carried on at a short distance. Although the painter enjoys the enviable privilege of being able to stand while at work, to remain at arm's length from his canvas and much farther yet from his model, many kinds of work necessitate a much shorter distance and, what is still more troublesome, an unfavourable position. We know that the former of these inconveniences may be rendered less injurious by means of convex glasses, which lighten the burden of the accommodation. For the latter there is, unfortunately, no other remedy than *frequent interruption of the work.* This precaution is, moreover, required for any application of the eyes, and all the more imperatively in proportion as the individuals concerned are younger, weaker, and their eyes more threatened with progressive myopia. It is indispensable, we repeat, for any person who works at a short distance, to let his eyes and the muscles of his back rest from time to time, by straightening up the back and looking at a distance at some dark or shaded place, or even by closing the eyelids. To do this it is not necessary to take one's mind off the work. The thoughts having, on the contrary, leisure to rise somewhat above the pre-occupations of detail, this bird's-eye view cannot but be advantageous to the work, of whatever kind it may be.

For eyes which are otherwise sound and strong, such interruptions need not exceed a few minutes in duration, nor need they be repeated oftener than once every half-hour, for example. Weak persons and children, especially if they are affected with progressive myopia, are to be advised to rest their eyes oftener and longer at a time, except in cases where they must be forbidden to work at all, which sometimes is necessary.

Another essential point to consider is the *illumination.* The light must be of good quality, neither too strong nor too dim, and must be properly directed.

The best light is evidently that of the sun, in the midst of which our visual organs have developed and with reference to which they were created. No adequate substitute for it will ever be found, any more than we shall ever be able to replace, for our lungs, atmospheric air with anything else. Consequently, any artificial source will be more or less prejudicial, especially if, while its defective light is being used, the eyes are doing work which, of itself, is fatiguing. Hence it is of capital interest for any one who wants to preserve his eyesight, for children whom it is desired to save from myopia, and for myopes who do not wish to increase their trouble, to profit for work by the light which Providence lights for us every morning. There are few countries which, for six months of the year, are not sufficiently lighted by the sun, so that work may be carried on by this light for at least ten hours a day. There are few, too, in which most people, by a more rational distribution of their time, could not gain two hours of solar light per day during the rest of the year. It is a question simply of beginning one's work at least as early as sunrise in winter. In this way, the night can be cut short with the greatest profit to the eyes.

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[1] Schreber, *Aerztlicher Blick in das Schulwesen*, p. 22, Leipzig, 1838. Cohn, *Die Hygiene des Auges in den Schulen*, pp. 105-112, Wien u. Leipzig, 1883.

Artificial light, such as that of candles, petroleum or gas, is in every respect inferior to that of the sun. It is strongly coloured; it is not bright or is very hot, so that, if its lack of intensity necessitates a very near approach to the source of the illumination, the heat which it gives off is a serious disadvantage, not only for the eye, but also for the entire system. This disadvantage increases if several persons are working in the same room, for, in this case, the number of sources of light and heat must be multiplied. It becomes complicated by still another disadvantage, which is inherent in a multiplicity of luminous sources: these dim the perception of relief, and, consequently, annoy the person in his work. Finally, they deprive his eyes of all means of repose. Moreover, these flames necessarily consume a vast amount of oxygen; many of them spread about in the atmosphere the injurious and malodorous products of their combustion, all of which contribute to vitiate the air of artificially lighted apartments.

The electric light certainly has immense advantages over any other artificial light that has been used before it. In point of colour, it more nearly approaches daylight than any of its rivals. Its great strength admits of raising the luminous focus far above the eyes, and of lighting, at will, from one single point, or by a small number of lights, a vast work-room. Moreover, it does not vitiate the atmosphere. But it still has the drawback of not being sufficiently constant, and the sudden variations in intensity, with which we may also, in a certain measure, reproach gas-light and that of certain flickering lamps, are very prejudical to the sight. The small incandescent burners, which are more constant, still give a strongly coloured light. However, we live in the hope that the fixity desired in the electric light will one of these days be obtained, and that some cheaper mode of producing it will be discovered. Then mankind will be blessed with an artificial light far superior to any of its predecessors.

As to the *intensity of the illumination*, it is easy enough to protect one's eyes from it when it is excessive. One can always find, for work, a place upon which the sunlight does not directly fall; shade is easily made, or the protection may be secured by means of lighter or darker coloured glasses. These glasses being destined to lessen the radiations of light, it is evidently desirable that they present a colour the opposite of that predominating in the light. Daylight being white, black or smoke glasses will be used to weaken its intensity. Artificial coloured light, on the contrary, will be neutralised by blue glasses, which absorb the yellow, orange and red rays predominating in it. The shade of black or smoke glasses, as also that of cobalt blue ones or, again, of those of intermediate colours, will evidently depend upon the intensity of the light against which it is desired to protect the eyes, as well as upon the sensitiveness of these organs.

What is infinitely more difficult, is always to procure for the eyes a sufficient amount of light by which to work. As we declared before the Congress of Hygiene, held at Paris in 1878, the topic of good and bad illumination will be eternally discussed, without any mutual understanding being arrived at, so long as a precise definition of good illumination is wanting, not alone with reference to the *quality* of the light, but especially with regard to its *intensity*.

The minimum intensity with which the eye can safely work, the light below which the illumination of no school or workshop ought ever to fall, has not yet been accurately fixed. We have proposed, in order to find this, and to easily verify the degree of illumination of any place where ocular work is to be performed, a primitive photometer.[1] It consists essentially of two pieces of board so joined by a hinge that they may be brought nearer to each other or separated at pleasure. One of them is placed directly on the place whose illumination is to be determined, whether it be a wall or a table, a horizontal or an inclined surface. It has on it, as test-objects, black points on a white ground. The other piece carries a mirror, which reflects the test-objects into the observer's eye. A coiled tape-measure serves to measure the distance at which the objects are recognised. Having given the maximum distance D at which the observer counts the points under the most favourable circumstances

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[1] Landolt, "Une méthode simple pour determiner l'éclairage des salles d'écoles" (*Compte rendu du Congres international d'hygiène tenu à Paris*, 1878, t. ii., pp. 122-125).

(the paper being exposed to bright daylight), the illumination $e$ of the place in question is expressed by the formula—

$$\log. e = \frac{d}{D}$$

in which $d$ is the distance at which the points can be counted.

When the fraction $\frac{d}{D}$ equals 1, *i.e.*, when the acuteness is at its maximum, $e$ is equal to 10, since the unit is the logarithm of 10. It will be seen that, in this way, we make use of the acuteness of vision in estimating the illumination. This method, although uncertain in physical photometry, because the relation between the perception of form and luminous sensation could not be determined with mathematical precision, is perfectly applicable in the present case. In the first place, so great accuracy is unnecessary and even impossible in such a case. Then, again, for the limits between which the quantity of light to be examined oscillates, the acuteness of vision evidently varies directly with the logarithm of the illumination.[1]

The method is, as will readily be seen, as simple as possible. It has, moreover, the advantage of giving the illumination of the very place at which the work is done, and not that of a contiguous wall nor of a vertical plane erected at this place. The illumination is, indeed, very different in the two cases. To convince one's self of this, it is necessary only to lay an open book on a table, and then hold it vertically with the text turned toward the light. The illumination is, necessarily, much weaker in the former case than in the latter.

It seems to us, although this opinion may be modified by more numerous observations, that the illumination ought never to fall below the point of lessening the acuteness of vision beyond three-fifths for reading, writing, drawing, &c. That is to say that, if the observer, under the most favourable circumstances, counts the points at a distance of 5 metres, he ought not to be obliged to come nearer than to a distance of 3 metres from them, when these points are put at a place where the work is done and the experiment is made when the work is in progress. It is hardly necessary to say that the same apparatus may be used for any person, provided his maximum distance D is made out beforehand.[2]

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[1] A. Posch, "Ueber Sehschärfe und Beleuchtung" (Knapp's *Arch. f. Augen- und Ohrenheilk.*, vol. i., p. ii., 1876).

[2] In a recent pamphlet of Cohn's (Cohn, "Tageslicht-Messungen in Schulen," in P. Börner's *Deutsche med. Wochenschr.*, No. 38, 1884) will be found interesting measurements of the diurnal light of schoolrooms, obtained by means of a photometer made by L. Weber, of Breslau. Cohn fixes the minimum amount of light, for a school-boy's place, at a quantity equal to ten candles. Cohn's collaborator, L. Weber, has also invented a very ingenious little apparatus for measuring the *quantity of sky* which a school-boy

After the illumination in general, it is in order to consider the *difference between the visual object and the background* against which it is seen. The greatness of this difference and the facility of vision are in direct ratio. The greatest possible contrast is between white and black. Every one has had opportunity to notice that reading is most easily accomplished when the paper is very white and the letters very black, and that writing is most agreeable when done with black ink on white paper. Nothing is more fatiguing than reading a pale text on poor, greyish paper. Seamstresses would complain much less of asthenopia if it were allowable for them to sew their white cloth with black thread, and especially if the opposite were the case. So, too, it would be a hundred times better to procure for school children, for readers of newspapers, for everybody, in fact, nice white paper and pure black ink, than to manufacture coloured paper in accordance with purely theoretical considerations. We have already said above that white light ought certainly to be more appropriate for our eyes, which develop under its influence, than any other.

It is no less important, for the distinctness of objects, that they should be *clearly defined.*

This observation has especial reference, again, to typography. Defective print is absolutely injurious to the eyes, just as it is infinitely more fatiguing to decipher manuscript than to read printed matter. Simple characters, like the Latin, are, other things being equal, preferable to more complicated ones, such as the Gothic. And, among those derived from the Latin type, the elzevirs and the type commonly used in America and England are superior; the thin strokes are more pronounced than those of the characters generally employed in France; the letter has thus more body and remains more easily recognisable when the test deteriorates or when the light becomes weaker. Careful execution is especially demanded in the case of figures, the reading of which requires one to clearly distinguish all the details, as is likewise true of a proper name, or a word with which one is unacquainted, while the reader accustoms himself to guess at common words from their general aspect, without spelling them letter by letter. A word of this kind is, for him, an entirety of definite shape, and not a composition of distinct letters.

Finally, objects must have *a certain size.*

As regards typography, whose importance is so preponderant in the civilised world, and which, for all the enlightenment it has spread

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**ought to be able to see from his seat. It consists simply of a convex lens, which produces an inverted image of the space, outside the window, which lights up any given part of the room. This image is projected on a graduated screen, which enables one to estimate the size numerically. Compare also Cohn, "Untersuchungen über die Tages und Gasbeleuchtung, &c.," *Berlin. Klin. Wochenschr.*, No. 51, 1885.**

abroad, has nevertheless blinded, in every sense, so many people, the following is the advice of the authors who are experts in this matter:

The height of small letters must not be less than 1·5 millimetres. The thickness of the full strokes should be, at least, 0·25 millimetres. The different letters of the same word should not be nearer together than 0·5 millimetres, and the interline, the space between the tops of the small letters, like *n*, from the bottoms of similar letters in the preceding line, should be, as a minimum, from 3 to 2·5 millimetres. They recommend, too, that the length of a printed line shall not exceed 100 millimetres, on account of the variations of convergence and accommodation which an excess over this may lead to, in the case of persons who hold the text very near their eyes.[1]

We have already discussed typography quite at length, more, perhaps, than is consistent with the scope of this manual. But there still remains a vast number of important things that might be said, and explained, on this topic. We refer the reader to the many publications to which it has lately given rise, and of which we cite the most noteworthy.[2]

Reading and writing not being the only occupations which require near vision, the principles enounced will have to be taken into consideration under many other circumstances. Embroidery, engraving, the making of instruments of precision, &c., which imperatively demand the clear discernment of very minute objects, ought never to be entered upon before the age when the eyes have reached their full development. Persons affected with progressive myopia find such occupations injurious, while those whose myopia remains stationary and has no tendency to be accompanied by changes at the fundus, excel in these callings, owing to their aptitude for seeing small objects near at hand. It will, moreover, often be possible for an intelligent man to facilitate his work by managing the illumination well, by assuming as proper a position as possible, and by taking care that the objects with which he works are against a proper background, &c.

Lately, the *school* has been specially accused of being the provoking cause of myopia and of many other ills besides. And although, if we were to listen to some, we might suppose that it contained the solution of all social problems and the *summum bonum* of mankind, others would make it responsible for all sorts of evils. As is always the case, both sides have exaggerated.

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[1] Cohn, *Die Hygiene des Auges in den Schulen*, p. 148-163, Wien und Leipzig.

[2] V. Arlt, *Die Pflege der Augen im ges. u. kranken Zustande*, Prag, 1865. Javal, *Essai sur la physiologie de la lecture* (*Annales d'ocul.*, t. lxxix., pp. 79 and 240; t. lxxx., pp. 5 and 135, 1878; t. lxxxi., p. 61, 1879, and t. lxxxv., p. 210, 1881). Weber, *Ueber die Augenuntersuchungen in den höheren Schulen zu Darmstadt*, 1881. Blasius, *Die Schulen d. Herzogthums Braunschweig; Deutsche Vierteljahrschr. f. öff. Gesundheitspflege*, Bd. xiii., 3. Schubert, *Die Schulbücher der stædtischen Unterrichtsanstalten Nürnbergs*, p. 99 *et seq.* *Mittheilungen aus dem Verein f. öff. Gesundheitspflege Nürnbergs*, 5 Heft, 1882.

No reasonable person would deny that generalised instruction is indispensable in a civilised country, nor that this instruction of *all* could not be obtained otherwise than by means of schools. It ought to be a matter for rejoicing that the State, or some other powerful association, having taken in hand the establishment and control of schools, it is possible not only to ensure to all young citizens the primary elements of an education, but also to place them, for a large part of the day, amid advantageous hygienic surroundings, which are, at least for the majority of the scholars, far superior to those that they find at home.

But it is just this mission which the school is reproached with fulfilling very imperfectly. Some think that the hygiene of the public schools is deplorable. Nor are they altogether in the wrong. However, it ought not to be lost sight of that a powerful movement is making itself felt throughout the civilised world, which has in view the improvement of schools in every respect. The start taken by France, since 1870, is altogether remarkable. The United States appropriate millions and millions for purposes of public instruction. And this should be borne in mind: the first condition, upon which will depend a proper school-hygiene, is money. Without plenty of money there will be no favourable location, no room, no air, no light, and no good teachers. In certain cities in Germany the happy influence of the "school-palace" upon the diminution of myopia has already been shown by figures. Switzerland, which, although small and not wealthy, has always taken the lead in matters of public instruction, has never relaxed her zeal, and continues to make almost superhuman efforts toward assuring the physical and intellectual well-being of her children.

Nevertheless, many more sacrifices must be made before the schools will have risen to a level with the perfectly just exactions of hygiene. There are already some fortunate exceptions, but most of them still sin against all the principles that we have just been discussing.

Aside from the lack of air and light, which lack, unfortunately, is not so easily made up, the books which some governments, not by any means the poorest, put into the hands of school children, are often of a deplorable and sometimes even revolting execution: dirty paper, small characters and a print inferior to that of the commonest newspaper, which is destined to be thrown into the waste-basket at the end of its ephemeral life. This is a negligence which it is absolutely essential to correct, and that all the more because, while waiting for the completion of the model school-house, the scholar's work ought, in every possible way, to be facilitated in the dark old room where he is still forced to abide.

Another thing with which the school may be reproached, is its excessive claims with reference to children. It is asserted that it claims them while they are yet too young, that it keeps them too great a part of the day, and that it imposes upon them too many tasks which are to be done at home.

As regards the first point, it would certainly be better, so far as the production of myopia is concerned, if ocular work could be put off till after the age of sixteen years. But this is hardly possible. What would children do before that age? And what would become of many of them, already six or seven years old, in cases where the parents, unable to take care of them, can now confide them to the government for a part of the day, which often appears to them too short and, unfortunately, really is so in many cases? Let us not forget that, as a matter of fact, most children are far from finding at home the conditions of physical and moral hygiene which the school can procure for them, when it fulfils its requirements. Do not let us complain, then, if it opens its doors to the earliest youth. Let us only take care that this double hygiene always reigns there, and, as regards our special subject, that children are not put to reading and writing before the age of seven or eight years. The strong ones among them will not suffer from it; the weak ought to be able, on their physician's advice, to postpone their instruction until an age when they have acquired the necessary strength.

Having ascertained the minimum of knowledge which the citizen of a civilised country ought to have acquired by the time he stops going to school, we see no serious objection to healthy children spending twenty-two hours a week at school up to the age of ten, twenty-five to twenty-eight between the ages of ten and thirteen, and even as many as thirty-two hours per week thereafter. It need hardly be said that we do not include, in these figures, the hours devoted to gymnastic exercises. But, in this case, the school ought not to charge the scholars with duties which take as much more of their time at home, where they generally work amidst unfavourable surroundings; otherwise it deprives them of one of the principal advantages which it ought to procure for them. It takes away from them, above all, the possibility of moving about freely, and of taking the exercise which is indispensable to the normal development of both body and mind. Although it is proper and reasonable that the pupil should learn to study by himself and independently of the teacher, such work ought not to occupy more than a fifth of his school hours before he is thirteen years old, and not more than a third of them after that.

I know very well that these requirements are easy to formulate, but I am also convinced that they are easy to fulfil. The secret of their realisation is found in the principle: *non multa sed multum.*

It is incontestable that, when scholars are overworked, both at school and with school-work at home, they are taught a number of things which are of infinitely less value to them than an hour or two of recreation in the open air. They have imposed upon them knowledge which is not indispensable, and which those among them who are to need it will easily acquire afterwards. These are, moreover, subjects which have but little interest for them, of which they do not appreciate the bearing and which, consequently, are forgotten. Hence the time devoted to them is almost wasted. By suppressing them, several hours per week are gained. This reduction in the number of duties would be a good beginning. Later, by offering a sufficient inducement, teachers will be found intelligent enough to teach the majority of the pupils (there will always be some dull ones who should not be permitted to hinder the progress of the others) many things in little time. Their number might be increased, so that fewer pupils would fall to the share of each, and that in this way their teaching would become still more profitable to the scholars.

However evident the imperfections of the school may be, we hold that it affords much less that is prejudicial to children, especially in so far as the condition of their eyes is concerned, than does overwork at home, in workshops and factories, against which childhood cannot be too well protected.

If the observance of the hygienic rules that we have outlined is to be recommended to every one, it is indispensable to people of weak constitution, or to those belonging to families in which myopia has already manifested itself. But when we have already found symptoms which make the imminence of myopia presumable, or when a true myopia has already become established, recourse must be had to more vigorous measures. Thus all near work will be prohibited in the cases of children affected with such a spasm of accommodation that they cannot clearly distinguish distant objects, although they are emmetropic or hyperopic in reality, or when they are unable to see distinctly at the distance of their *punctum remotum,* in case they are myopic. Their going to school is to be delayed, and, if they have already commenced to go, they are to be kept away, at least temporarily, for the treatment of the spasm. This treatment is specially indicated when the exaggeration of refraction is accompanied by symptoms of asthenopia.

Sometimes it suffices to *rest the eyes* for a few weeks, to send the children to the country, to let them run and play in the open air, and to improve, in every way, their *general health.* In cases of photophobia, when the season or the country exposes them to the bright sunlight, they may be advised to wear *smoke-glasses.*

Before having recourse to this means, however, we should ask ourselves whether or not the patient is able to wear glasses *constantly* when out-of-doors. If, as is the case with turbulent children, the glasses are to be taken off when the patient is playing, and put on again presently, they would do more harm than good, on account of the change in the illumination of the retina.

Frequently the spasm of accommodation will not yield to these means, or will reappear with other symptoms—fatigue of the eyes and pains in the head,—as soon as school-work is resumed. In such cases it must be suppressed by the use of *mydriatics.* Once or twice a day, for several weeks, a drop of a solution of atropine or duboisine (1 : 200) is to be instilled, the patient being enjoined to keep the eyes closed for five minutes after each instillation. In such cases it will be noticed that, even at the end of the first day, the refraction has considerably diminished and that the accommodation is almost entirely paralysed. It happens exceptionally, however, that a remnant of the spasm persists and reveals itself by a difference in the refraction, as determined subjectively and objectively. When this occurs, the atropisation must be continued longer, and made more energetic. The dilatation and immobility of the pupils, which this treatment necessarily produces, will require the use of rather dark smoke-glasses for bright light, and lighter ones for the evening. Such glasses should be shell-shaped (coquilles). In the cases of weak myopes or emmetropes, they need not exert any optical action. For myopes whose anomaly is of the higher degrees, these glasses may, at the same time, be concave, but not correcting all the myopia, since he might make use of them for near vision. The hyperope, on the contrary, will take advantage of the opportunity to have the whole of his refractive defect corrected.

At the beginning of the treatment, all near work is to be prohibited. Later, when the symptoms of asthenopia have disappeared, the patient may be permitted to busy himself a few hours a day, with long intervals of rest, and using convex glasses which adapt him to a distance of from 30 to 40 centimetres, when his refraction is less than 2·5 D of myopia ; he may do work without a glass, at the distance of his *punctum remotum,* when the myopia is of greater degree. That of young people rarely exceeds 4 D. If it be stronger, we may give them concave glasses which will permit them to hold objects farther off, and thus diminish the effort of convergence. The accommodation being paralysed, the selection of glasses is too simple to be discussed again.

We have treated at length the appropriate calculation on page 184 *et seq.*

If, at the end of several weeks, in spite of the gradual increase in the amount of work, the patient no longer complains of any fatigue, the dose of the mydriatic may be diminished little by little. The use of spectacles will be continued. The strength of the convex glasses worn will, however, be diminished slowly and in proportion to the reappearance of the accommodation. The medium myope will remain without glasses and those of the myope of high degree will not have their strength increased. At the same time, double attention will be paid to hygiene and especially to the distance at which the patient works. So long as the accommodation was paralysed, he was obliged to hold his book at a certain distance—that for which we had adapted him by means of the spectacle-glasses. If we want to prevent him from relapsing into the spasmodic contraction of his ciliary muscle, it is indispensable that this same distance be maintained. The myope of 4 D ought to be able to read, write, &c., in virtue of his static refraction, at ¼th of a metre (25 centimetres), with or without atropine. One of 5 D, for the same reason, will read at the same distance with the concave 1.

Let us take, on the other hand, an emmetrope, who, owing to a spasm of accommodation, presented an apparent myopia, whose disappearance we have brought about by means of atropine. In order that he might work at 33 centimetres, we should furnish him with convex glasses of 3 D. According as his accommodation is re-established under the influence of the diminution of the action of the mydriatic, we shall give him weaker convex glasses. Our aim is, indeed, not at all to accustom him to fix without accommodation, but to bring as little as possible of it into action; three dioptries for seeing at 33 centimetres, and to relax it entirely for distant vision. The same principles will guide in the case of a hyperope, except that we shall let him enjoy the benefits of his correcting-glasses longer.

But as soon as we perceive that the accommodative spasm tends to reappear, that the myope can no longer read at the distance of his *punctum remotum*, that the corrected emmetrope or hyperope no longer sees well at 33 centimetres, and that they do not possess their full acuteness of vision for a distance, we shall immediately resume the treatment, the atropisation, the auxiliary glasses and, if necessary, absolute rest.

It need hardly be said that this treatment will be prolonged in cases where spasmodic myopia is already of long duration or is associated with real myopia, especially when the latter is progressing, and the ophthalmoscopic examination leaves no doubt as to its malignant character. The hyperæmia of the fundus, the pigmentary changes which we have described and the phenomena of asthenopia which are never wanting in such cases, necessitate here a much more protracted interruption of work and a more scrupulous sparing of the visual organ. It is even indicated to employ a derivative medication, as warm foot-baths, dry cups to the temples, every two days or every day, for twenty minutes in the evening, or light purgatives. At the same time the patients are to be enjoined to avoid, more than ever, staying in overheated and ill-ventilated apartments, to breathe fresh air, to keep the circulation as active as possible—in short, to place themselves in the best hygienic conditions. We shall prescribe for them exercise out-of-doors or, if possible, a sojourn in the mountains or in a wooded country.

The still more profound changes in the organ—the exudations, the hæmorrhages, the hyperplasia and atrophy of pigment — or in its functions, as photopsies, metamorphopsy, diminution of the acuteness of vision, openly take on the character of choroiditis, sclerotico-choroiditis or chorio-retinitis. Hence they call for the same *anti-phlogistic treatment* as inflammation of the membranes of the fundus. They sometimes occur in the cases of young people, but oftener in adult age, which bears a somewhat energetic course of treatment well. The primary condition of the success of this is, that the patient remain in almost absolute darkness for several weeks. If the symptoms require, and the constitution will admit of it, repeated applications of Heurteloup's artificial leech to the temples, and sudation by means of wet sheets—or of pilocarpine—and preparations of iodine internally, will be added to this seclusion in a darkened room.

Unfortunately, the victims of this sclerotico-choroiditis are frequently neither old nor healthy enough to be subjected to a radical course of treatment; for such treatment cannot but be very debilitating, on account of the length of time it must be kept up in order to produce a result worth obtaining, that is to say, a permanent effect. The physician here finds himself in a truly sad position. On the one hand he sees a local affection which threatens to destroy a precious sense and, with this, often the very existence of the patient, and, on the other hand, a constitution incapable of bearing the remedy. What is to be done under such circumstances? When the sufferers are still young, as is generally the case, it is, first of all, necessary to explain to them the condition of affairs; they must be made to understand how weak their eyes are, and how ill adapted to do work that is at all tiresome, so that they may give up all idea of entering upon a calling which requires perfect vision, or to abandon it, if already entered upon. If they cannot tolerate the staying in a dark room, their eyes may at least be protected against the action of light by means of black glasses; if their constitution prevent bleeding, sweating, &c., we must try to supply the place of these with dry cupping, foot-baths, &c. Abstention from work is evidently indicated for a longer period and its resumption ought to be surrounded by all the more minute precautions, as regards both general and ocular hygiene. Thus we shall lighten as much as possible the task of the accommodation and convergence of these young myopes, whose ametropia is generally of medium degree, in which the lack of harmony between the two functions often becomes a cause of asthenopia. We shall point out, hereafter, the principles which should guide us in this intervention, because they are applicable to all myopes and not alone to those whose myopia assumes a malignant character.

When opacities of the vitreous body, or of the crystalline itself, are added to the symptoms above mentioned, *paracenteses* of the anterior chamber, as they have been employed on so vast a scale by Sperino, may be of real service.

They are best made with a little instrument proposed by Graefe —a little stop-needle, 2 millimetres broad by 4 millimetres long; the other end of the handle is provided with a fine, rounded and curved silver spatula. If the point is good, as it should be, the needle can be thrust through the cocainised cornea, into the anterior chamber, without seizing the eye with the forceps and without putting in a speculum. The lids may be held open and the eyeball steadied, at the same time, with the fingers only, as is done for the removal of a foreign body from the cornea. We choose, for the puncture, a point a few millimetres from the edge of the cornea, and carry the point of the needle so that it will not come in contact with the crystalline. By doing the operation delicately, the instrument may be entered and withdrawn without the loss of a single drop of the aqueous humor. This is afterward permitted to escape slowly, drop by drop, by pressing the spatula on one of the lips of the incision. It is well to do this last part of the operation little by little, at intervals of at least a minute, so as not to modify the intra-ocular tension too suddenly. Then a light dressing is put on the eye and retained till evening. The paracentesis may be repeated from twice to four times a week. Its favourable effect in malignant myopia is due, as in all chronic affections of the uveal tract, to the diminution of intra-ocular tension and the clearing up, of the dioptric media, which it brings about. Perhaps it exerts also a beneficial action upon the circulation, both of blood and lymph.

In the most advanced stages, the extreme degrees of myopia, the inflammatory phenomena yield to those of atrophy; treatment has less and less effect upon the ailment and becomes more and more symptomatic. This is easy to understand. What can medicine do for an eye that is distended in all its dimensions, elongated by a deep staphyloma? It is as impossible to restore their functions to the destroyed choroid, to the retina in ruins or to the half-atrophied optic fibres, as to resuscitate a corpse. Hence we are here reduced to the necessity of treating the patient from day to day. Sometimes he will get along passably well, and will make shift to live with his field of vision in shreds. He will even imprudently read, by moving his eyes along the lines by jerks, for hours at a time. When he feels well, generally he will laugh at the end of his nose, daubed with ink, which has been grazing along his freshly written lines, and will proudly show us the small objects that he can distinguish, if it were only the watchmaker's name engraved on his watch, which for years he has seen so often.

But even by the next day, under the influence of poor digestion or any slight indisposition, he runs to us frightened by *muscæ volitantes*, dazzled by a "white field," by sparks, by flashes of light, rendered uneasy by a feeling of pressure in the eye, by frontal headache, or by other alarming phenomena. All this will ordinarily be calmed when the general condition shall have resumed its equilibrium, under the beneficent influence of Fioravanti's balsam, Hoffman's balm of life, or some other marvellous ointment rubbed on the forehead and temples.

It is to be noticed, too, that, for eyes in so precarious a condition, the commonest occurrences assume a much greater importance than they possess for healthy eyes; the mucous secretion of a conjunctivitis, which spreads itself over the cornea, and but slightly affects the sight of the normal eye, may easily reduce by a half the visual acuteness of an eye which before possessed only a tenth of the normal amount. The least swelling of the mucous membrane, by the pressure that it exerts on the hypertrophied eyeball, and the hindrance to its movements which may result from this, will make the patient believe that his eye has grown larger, that it is swollen and hard. It is for this reason that one will do well to pay the most careful attention to such eyes, however impotent one may feel to cope with their essential malady.

At other times, our patient's vision will be still more profoundly changed. He will no longer see at all, or only indirectly. The object at which he looks disappears; he distinguishes only its surroundings, and those but imperfectly. The ophthalmoscope reveals, as the cause of this *central scotoma*, a hæmorrhage in the macula. What is to be done? Will energetic rubbing of the forehead with iodine ointment (iodide of potassium 3 grammes, pure iodine 0·30 grammes, axunge, or, better still, lanoline, 15 grammes), to iodoform (one part for ten), be powerful enough to cause the resorption of the blood ; will iodine taken internally have any other effect than to bring about a certain irritation of the mucous membranes and spoil digestion ? Here, again, resting the eye will be the best remedy. Little by little the blood will disappear. Its place will be marked by a whitish patch and its colouring matter will have nourished the hyperplasia of pigment which generally occupies this region of the eye. The sight will certainly have suffered from it, on account of the irreparable destruction of the retinal elements.

If the impairment of vision is due to a *detachment of the retina*, the condition of affairs is still more serious. In very recent cases we may try puncturing the sclerotic, underneath the detached retina, by means of a Graefe's knife, which we turn a little in the wound, in order that the subretinal liquid may escape. This little operation, if carefully done, is not attended with any danger, and offers some chances of success when it is followed by a compressive bandage, and, above all, prolonged rest in bed. But this perfect quiet in darkness is just what many patients cannot endure, because either their constitution or their mental disposition is opposed to it. And it must be admitted that, considering the uncertainty of the result, one really does not dare insist too strenuously, in such cases, that the patient shall undergo this martyrdom, especially when he still has one useful eye remaining.

This other eye, in fact, now demands the greatest solicitude. It often requires even the suppression of its fellow, when the latter, years after the detachment of the retina and in consequence of the profound changes which this produces in the entire eyeball, has become the seat of a *cyclitis.* This inflammation of the ciliary body, the most dangerous of all those affecting the uveal tract, may threaten the other eye with sympathetic ophthalmia. The latter is all the more to be feared, because neither the inflammation of the worthless eye nor that of its mate, which has remained well, declares itself suddenly, nor is it always accompanied by acute symptoms or significant redness, as in cases of wounds of the eye by foreign bodies. These ophthalmias are slow and insidious, because they may manifest themselves by symptoms to which will not, at once, be attributed the character of a sympathetic ophthalmia. Frequently there is nothing but a gradual diminution of the acuteness of vision, without any other appreciable phenomenon, or slight attacks of serous iritis, &c. One must not permit one's self to be deceived in such cases, nor hesitate to save the only remaining good eye by the *enucleation* of the other.

As to the treatment of the partial, or even total, atrophy of the optic nerve, which, as we have said, sometimes ends the drama of which the extreme myopia was only one scene, we would not coerce the reader. We prefer to let him choose, from among the many remedies (strychnia injections, albuminate of silver, electricity, &c.), that with which he has obtained the best results, or in which some abler teacher than ourselves has succeeded in inspiring in him the most confidence.

## THE TREATMENT OF MYOPIA BY MEANS OF CORRECTING-GLASSES.

What we have just pointed out concerning the treatment of myopia has to do, on the whole, only with *malignant* myopia; these measures are directed only against the morbid accidents which accompany it, and which, as we have said, make up an *ensemble* of such gravity that the anomaly of refraction plays the *rôle* of only a secondary symptom. This symptom is not, however, of minor importance; the proof of this is that it has given its name to the affection. Moreover, it is almost the only thing concerned in typical myopia. Hence it is time that we should take up, in detail, the consideration of its remedy, the treatment of myopia as ametropia, properly so called.

This treatment is, in reality, as we know, only palliative, and consists in correction of the optical defect by optical means.

The refractive change peculiar to myopia consists in the fact that the eye is permanently adapted to a finite distance; that is to say, it can focus, on its retina, only such rays as emanate from a somewhat near point in front of it; in other words, divergent rays (p. 125). Now, since *concave glasses* possess the power of giving to luminous rays, whatever their origin, any desired divergence, according to the strength of the glass employed, such glasses may evidently serve to adapt the myopic eye to any point in space. They give, to rays emanating from this point, the divergence which is necessary in order that the eye may focus them on its retina.

Thus, in order to enable an eye which is myopic by four dioptries, *i.e.*, adapted to $\frac{1^{\text{m.}}}{4}$, to see at a distance, which means adapt it for parallel rays, we need a concave lens number 4, because this gives to parallel rays a divergence that makes them seem to come from $\frac{1^{\text{m}}}{4}$. In this special case, that of parallel rays, the point whence, after their passage through the glass, they seem to come, is, as will be remembered, the principal focus of this glass.

The problem is not much more difficult when we are dealing with rays which were divergent before reaching the glass. This is the case when one wishes to adapt a myopic eye, not to infinity, but to some certain distance which is greater than that at which it sees distinctly. Suppose that our myope of four dioptries wants to see clearly at a distance of one metre. To change the divergence of rays emanating from a point one metre distant, to divergence from a point $\frac{1^{\text{m.}}}{4}$ distant, we evidently need a weaker glass than in the former example. In fact, in order to see at a distance of one metre, one dioptry of positive refraction is necessary. Hence the myope of four dioptries has $4-1=3$ D too much. With the concave 3, which diminishes his refraction by as many dioptries, he is, therefore, adapted to a distance of one metre.

The same problem may be solved in still another way. An eye affected with four dioptries of myopia is adapted to $\frac{1^{\text{m.}}}{4}$, or 0·25 metres, or 9″. Its refraction is $\frac{1}{{}^1/_4}$, $= 4$ D, or $\frac{1}{25^{\text{cm.}}}$, or $\frac{1}{9''}$.

In order to see at a distance of one metre, one needs a refraction of $\frac{1}{1^{\text{m.}}} = 1$ D, or $\frac{1}{100^{\text{cm.}}}$ or $\frac{1}{7''}$, and, to change the first adaptation into the second, we evidently need a glass which is equal to the difference between the two:

$$4\text{ D} - 1\text{ D} = 3\text{ D};$$

$$\text{or } \frac{1}{25^{\text{cm.}}} - \frac{1}{100^{\text{cm.}}} = \frac{1}{33{\cdot}3^{\text{cm.}}},$$

that is to say, a glass whose focal distance is $0{\cdot}333 = 3$ D,

$$\text{or } \frac{1}{9''{\cdot}2} - \frac{1}{37''} = \frac{1}{12''{\cdot}2} = \frac{1}{33{\cdot}3^{\text{cm.}}} = 3\text{ D}.$$

Either method of finding the correcting-glass is simple enough, but the first, which is based upon the system of dioptries, and which we have found far preferable to the other in practice, is especially so.

But although it is easy to vary at will the adaptation of the eye, the choice of spectacle-glasses is not in reality as simple as it appears, and that for several reasons.

In the first place, the basis from which we work, in determining the correcting-glass, is often variable. This basis must be the static refraction of the eye, the distance to which it is adapted, the refraction which it presents, when in a state of repose. This refraction is often increased by a greater or less amount of dynamic refraction which is involuntarily added to it. We have already seen, when discussing hyperopia, what complications result, in choosing spectacles, from this intervention of the accommodation. Its influence is no less important to consider in the correction and adaptation of myopic eyes. The *rôle* which the contraction of the ciliary muscle may play in the etiology of malignant myopia makes this easy to foresee. Just as hyperopia may be more or less hidden by a spasm of accommodation, so myopia may be exaggerated from the same cause.[1]

Now, by taking as a starting-point, in the choice of a correcting-glass, the *apparent myopia*, the myopia represented by the static refraction *plus* a certain amount of the dynamic, one evidently incurs the risk of giving too strong glasses, and of keeping up, by this very means, the injurious spasm of the ciliary muscle. In taking, on the contrary, the *real* myopia as the basis, we remain below the number that the patient wants and adapt him only to a much shorter distance than that at which he wishes to see.

Nevertheless, we advise and lay down, as the first general rule for the choice OF SPECTACLE-GLASSES FOR MYOPES, THAT THE REAL MYOPIA BE TAKEN AS THE BASIS OF CALCULATION. We have shown (p. 227 and p. 252 *et seq.*) how the amount of this is most surely ascertained. It now becomes a question, not only of adapting a glass to the eye, but also of adapting the eye to its correcting-glass.

If the latter, determined according to the rule, does not at once suit, we shall often succeed, by gradual training, in diminishing the spasm of accommodation and accustoming the eye to its true correction. At other times we shall be obliged to have recourse to mydriatics in order to overcome the too rebellious spasm. We shall find this especially the case with young children. If the contraction of the ciliary muscle is so energetic that, after the correction of the real myopia, the refraction is still too high, that the eye is still unable to see sufficiently well at a distance, and that the myope holds objects much too near him, then we shall consider this spasm pathological. Far from trying to neutralise the effect of it with concave glasses, which would only increase it and fatigue the organ, we have recourse to the hygienic means pointed out above—abstinence from ocular work, &c., or a regular course of atropine-treatment.

After having continued this treatment a certain length of time, we shall give the patient correcting-glasses and let him accustom himself to use them for the proper distance, while he is still under the influence of the mydriatic. The use of the latter will be only gradually discontinued, and we should always be careful to watch the distance at which the myope holds the object of fixation.

It is only very exceptionally, and for very short periods, that we have permitted a myope to use glasses somewhat stronger than the myopia found, in his case, by the ophthalmoscopic examination. This is when there is no symptom of asthenopia, no change at the *fundus* and no suspicion of hereditary myopia; and again, when quitting work was out of the question, and the adaptation of the eye to a distance was absolutely necessary. Then, too, we have sanctioned concave glasses, in such cases, only in the form of hand-lorgnettes or eye-glasses, so that they might not be worn more than just the necessary length of time, and we have never failed to warn the patient, and especially the patient's parents (for we are generally dealing with children in these cases), of the dangers entailed by the abuse of such glasses.

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[1] See pp. 227 and 255.

When no accommodative spasm exists, or when it has been overcome, the question of choosing spectacle-glasses for a myope is still far from being a simple problem in refraction.

Thus, let us take the case of correcting-glasses properly so called, those which adapt the eye for distant vision and with which no myope can dispense, if he wishes to see at a distance. It might be supposed, *a priori*, that it would be quite proper to prescribe such glasses and let the patient keep them, even for near vision. They render him emmetropic, and we have repeatedly declared that the play of the intrinsic and extrinsic muscles, and above all, their functional solidarity, are developed with reference to this refractive condition of the eye.

However, one would make a great mistake to reason thus. In the first place, as we have also frequently explained, Nature has permitted a certain latitude in the relations between accommodation and convergence, apparently just for ametropes whose defect is not of very high degree. For these, therefore, correcting-glasses would be, at least, useless. But they even have serious disadvantages for the myope: they force him to make an effort of accommodation from which his ametropia grants him dispensation, and they deprive him of another advantage, *i.e.*, of the larger size of the retinal images obtained by the naked eye, and which the glasses make smaller.

Even for seeing at a distance, concave glasses are not always well received; many myopes would rather see indistinctly without glasses than see distinctly with them; others like to have clear retinal images for certain cases, but do not care to have them at all times; finally, a few myopes, as we have already had occasion to say, cannot even bear distinct images.

The first categories include, as must be self-evident, especially myopes whose refractive errors are of *low* and *medium degrees.* In this connection it is sometimes surprising to note how well these myopes get along in ordinary life, and how great acuteness of vision they obtain with imperfect correction or with none at all.

But what might seem still more astonishing is the difference in the visual power of non-corrected myopes of the same degree of ametropia and of the same acuteness of vision, when they wear their correcting-glasses. Thus it may happen that, of two myopes of 3 D, with $V = 1$, one has, with the naked eye, only $V = 0 \cdot 1$, while the other has $V = 0 \cdot 3$. It may happen, too, that, in the one case, the acuteness of vision increases gradually in direct ratio with the strength of the concave glass placed before his eye, and proportionately as this is nearer the number of the correcting-glass. With the other patient, on the contrary, the acuteness of vision will scarcely vary, in spite of stronger and stronger concave glasses, but finally become, all at once, nearly normal when the ametropia is entirely corrected.

This fact may find an easy explanation in the difference of the diameter of the pupil, and its influence on the circles of diffusion, which is less for a small than for a dilated pupil. But it is especially explicable by the more or less developed ability to analyse diffuse retinal images. We have already spoken of this when discussing high degrees of hyperopia. Here, again, we are forced to admire the skill, in interpreting diffusion-circles, which is acquired by certain ametropes. This ability is, in both cases, the result of training and of experience.

Thus, the same myope who, examined in our consulting-room with the usual test-types, obtains only a very slight acuteness of vision without a glass, will recognise perfectly a rabbit some distance away, will perhaps even distinguish between it and his dog, and will shoot it; another, on the contrary, will lose himself in the open country, while he will resolutely read the test-types on our card. This is evidently because the former, from long experience, has learned what significance is to be attached to the imperfect figure depicted on his retina, when it is a question of game, and the other patient when dealing with letters. To succeed in this, both have profited by a host of details which are lost upon those of us who enjoy clear images, which scarcely require any effort of interpretation—details of which such myopes are not even, in any precise sense, conscious. Thus the first will utilise, besides the image itself, the movements it makes, the colour it presents, &c. &c.

Judgments by exclusion also play, in both cases, a *rôle* of considerable importance. If the acuteness of vision be examined, the pupils being of equal size, with points or figures of shapes unknown to the persons under examination, the difference between various myopes, of the same degree of ametropia and of *intelligence*, will not be so great. We must not forget, indeed, that the latter represents the most powerful aid in the analysis of any question or in the solution of any problem.

As to the influence of practice, let us cite one case in a thousand, which presents itself opportunely at the very moment when we are writing these pages:—A boy, fourteen years old, is emmetropic in the left eye, whose acuteness of vision equals 1·5. The other eye is myopic by 1 D, and has the same acuteness. But, without the correcting-glass, the vision of this eye is only 0·3, or one-fifth of the real amount. The ametropia is, however, but very slight. We have met with myopes of the same degree, whose visual acuteness was much superior to this, amounting to 0·5 and even more, of the total. Whence does this difference arise? Evidently from the fact that this patient, having one emmetropic eye which is gifted with superior sight, has accustomed himself to distinct retinal images, and has never learned to analyse even so slightly diffuse images as those of his myopic eye. If his left eye had been no better than his right, no doubt the examination of our patient's visual acuteness would have yielded a much higher figure for this without the correction of the ametropia.

For the same reason, persons whose myopia has developed and increased rapidly must be placed in a much more troublesome position than those who have had time to accustom themselves to their ametropia.

If one stops to consider what we have just pointed out, one reaches a conclusion which is so natural that it might have been arrived at in advance, without waiting for its practical confirmation, viz., that those who have tried to determine the degree of ametropia from the fraction of visual acuteness presented by the non-corrected eye, must have utterly failed in the attempt.

It will also be understood, on the other hand, why we abstain from imposing the use of their correcting-glasses upon myopes who do not complain of the insufficiency of their sight. We even generally recommend young persons, and those who are but slightly myopic, to wear glasses only exceptionally. It is for this reason that we have them mounted in hand-lorgnettes or eye-glasses, as has been said above.

Although slightly myopic persons do not always feel the need of correcting their ametropia, those whose defect is of *medium* or *high degree* cannot dispense with the use of concave glasses, at least for distant vision. It might be supposed, then, that so long as the glasses do not entail an effort of accommodation,—that is to say, so long as those that are used for a distance, for instance, do not exceed the degree of the real myopia, they ought to be borne without either danger or inconvenience.

Such is not, howe ays th It is rare that a myope,
whose defect is of n age is not advanced, is
pleased with the ful If we give a myope

of 5 D the concave 5, we incur a great risk of having him return a few days later to tell us that the glasses tire his eyes, and of being obliged to give him weaker ones. Just as we recommend the slightly myopic person to use glasses only intermittingly, so it is prudent to prescribe glasses somewhat below the degree of their myopia for those who wear spectacles all the time. If they prove insufficient, there is always time enough to exchange them for stronger ones, or, as may happen especially in cases of strong myopia, to add auxiliary glasses to them.

We have already had occasion to say that the highest degrees of myopia are rarely exempt from changes at the fundus of the eye. This morbid condition of the organ seems to oppose the entire correction of the ametropia, and it is noticeable that the malignant form generally does not bear correcting-glasses so well as typical myopia. In such a case it is well to give the patient, in the form of spectacles, the strongest concave glasses that he can bear, without fatigue, even at a medium distance (30 to 50 centimetres). This may be number 12 for a myope of 16 D. At the same time, he is to be furnished with a hand-lorgnette containing glasses which complete the correction (−4 in our example). He will hold these in front of his spectacles when he wants to see distinctly for the moment.

*Extreme* myopes very often refuse any sort of permanent correction. If it is only partial, they find no advantage in it, and, as soon as it begins, by approaching complete neutralisation, to procure for them retinal images of some distinctness, it tires them. Thus, a myope of 20 D would, as a general thing, hardly thank us for giving him a concave 15 or 16. The number 18 will prove annoying to him after a short time. He will prefer the minus 20 in the shape of a hand-lorgnette. He will use it only a moment at a time, and in very exceptional cases, on account of the fatigue which he experiences from its use.

The inconveniences attaching to the use of very strong concave glasses are not to be explained solely by the bad condition of the eyes that need them. They are also due to certain optical effects, peculiar to such glasses, of which we shall speak hereafter.

After the correction of the ametropia for infinity, let us consider the visual functions of myopes for *finite distances*. The domain of "near vision" commences, practically, at about 50 centimetres, the length of the arm. The latter marks the limit of manual labour, which is what we are principally concerned with. The shortest radius of this domain is hard to determine. For binocular vision it could scarcely be less than 20 centimetres. But, if one eye be excluded, we know of cases in which this limit would be found even within the length of a long nose.

Now, myopic eyes being, from their refractive condition, adapted to more or less short finite distances, it goes without saying that, if the distance of their work is the same as that of their *punctum remotum* (R), they will need, for the latter, neither correction nor an accommodative effort. If the distance at which they wish to see is less than that of their *punctum remotum*, they must bring their accommodation into play or, in default of this, use a convex glass. If the object of fixation is farther away than their *punctum remotum*, they will, on the contrary, need a concave glass.

When it is a question of adapting a myopic eye to a near point, we shall still follow the same principles that have guided us in correction for a distance. We take the real degree of myopia as a basis. We lay down, as the second general rule, resulting in part from the first, that A MYOPE MUST BE PROHIBITED FROM WEARING A CONCAVE GLASS FOR ANY DISTANCE AT WHICH HE CAN SEE CLEARLY WITHOUT ACCOMMODATION.

We will, therefore, guard against giving their correcting-glasses to slightly myopic persons and, generally, to all who work at a distance less than, or equal to, that of their *punctum remotum*. A myope of 2 D ($R = \frac{1^m}{2}$) ought to be able to play the piano, write in large account-books,—in short, work at a distance of 50 centimetres, without using spectacles and without leaning forward. If it is impossible for him to do this, it is because he is affected with a spasm of accommodation, and he is to be treated exactly like a patient who has the same amount of myopia, and who, with the concave 2, does not see at a distance, and requires, for this purpose, the minus 3 or a higher number.

If the objects of fixation are nearer—at $\frac{1^m}{3}$, for instance—our myope will see them with an accommodation of $3-2=1$ D; or, if his accommodation is *nil*, as will be the case if he is old enough, we shall furnish him with a convex glass of 1 dioptry.

The same rules will necessarily guide us in what we do for myopes of *medium* degree. These will see still nearer without having recourse to their accommodation, and it will be much later before they require the help of convex glasses. A great many of them will even never need them, if their ametropia be of somewhat high degree and their avocations do not require a too exact application of vision.

It may happen, on the contrary, that, instead of being too short, and demanding an increase of refraction, the working distance is greater than that of the myope's *punctum remotum*. The domain of distinct vision is then to be extended by diminishing the degree of the myopia. This was the case in the example which we worked out at the beginning of this section. It proved to us that such correction is very simple. Hence we shall not return to it now, unless it be to reiterate that we must be on the look-out for spasm of accommodation in young persons, and take into account, when prescribing a glass, only the degree of the real myopia.

But difficulties of another kind may arise in the near vision of myopes of medium and high degrees. We have already discussed these quite at length: they are due to the mutual incongruity of accommodation and convergence. Thus a myope of 5 D might like nothing better than to read, engrave or sew, &c., at a distance of 20 centimetres, but will be unable to do so on account of the relative or absolute insufficiency of convergence (see p. 209).

It is not to be forgotten that the power of convergence has not so durable an energy as the apparatus of accommodation; in other words, that there must be in reserve a greater quota of convergence than of accommodation. Indeed, although one-third or one-fourth of the amplitude of accommodation may suffice for long-continued work, we have shown (p. 342) that an equivalent reserve of about two-thirds of the convergence is necessary,—that one's work ought not to require the exercise of more than one-third of this function at one time.

Knowing the great amount of convergence required for work, it will not astonish us to find this function so often in default in myopes, whose short range of sight exacts, of itself, a considerable degree of convergence, when vision is to be accomplished binocularly.

In such cases the first thing to be done is to ascertain the amplitude of convergence, especially the positive portion of it, and compare it with the amount required by the distance and duration of the work. This experiment may be made with the author's dynamometer, the method of using which has been explained on pages 282 *et seq.*

To correct low degrees of insufficiency, we sometimes have a choice between concave glasses and simple prisms. The first, by causing the limits of optical adaptation to recede, lessen the amount of convergence required; the prisms, without changing the working distance, diminish the necessary effort of convergence, owing to the deviation, of the luminous rays, which they cause.

Suppose, for instance, that our myope of 5 D is affected with a slight degree of insufficiency of convergence. By giving him concave glasses of 2 D, we enable him to see at $\frac{1^{m.}}{3}$, and we shall have diminished, at the same time, by two metre-angles, the convergence necessary. Nevertheless, the patient may not be quite satisfied; the spectacle-glasses, although permitting him to hold objects farther away, sensibly diminish the size of the retinal images, and may, in this way, substitute one cause of asthenopia for another. It is for this reason that concave glasses, prescribed for this purpose, and which are at all strong, are not generally well borne.

It is, however, impossible to establish a rule concerning the strength of the glasses that may be used; the acuteness of vision, the habits of the patient and other individual circumstances, are the first points to be consulted. But the minifying influence of concave lenses is such that they are rarely applicable, as working-glasses, in high degrees of myopia. In such cases, indeed, in order to gain even a few centimetres of distance, several dioptries are required. A myope of 16 D is obliged to pay 6 dioptries for the 4 centimetres by which we increase his working distance, in carrying it from 6 to 10 centimetres.

Hence one will often be tempted to resort to prisms. But here, too, a heavy piece of glass is often necessary, with which to realise a mediocre advantage. The table on page 189 shows that the metre-angle, for a base-line of 62 millimetres, amounts to 1°·77. The angle of the deviation produced by a prism being equal to half its angle of opening, a prism of 3°·54 is therefore necessary, simply to lighten the task of convergence by one metre-angle. Naturally 2 metre-angles would require double this amount, or a prism of 7° before each eye. We say, advisedly, *would* require, for it is not possible, practically, to use prisms of such strength; one of even 5° is a thick prism, and this would give in our example (the base-line measuring 62 millimetres) only 1·4 metre-angles.

But it will often be advantageous to combine both means. By giving our myope of 5 D prisms of 3°·5 whose concave surfaces are cut in such a way as to possess a divergent power of 2 D, we save him 3 metre-angles of convergence; instead of seeing with 5 metre-angles at $\frac{1^{\text{m.}}}{5}$, he will see at $\frac{1^{\text{m.}}}{3}$ with 2 metre-angles.

Concave glasses of a certain strength may acquire a sufficiently powerful prismatic effect simply by means of their decentration. We shall speak of this hereafter. At all events, the optical correction of insufficiency of convergence, which is so often a cause of asthenopia with myopes, opens a very vast field of research to the educated, intelligent and attentive practitioner.

But this infirmity often rapidly attains, especially in malignant myopia, such degrees that it resists all pacific attempts to remedy it, and necessitates surgical intervention. To know how to determine when the latter is opportune is to render the patient great service. He will not only be spared a long period of asthenopia and incapacity for work, a prospect which is doubly painful on account of the inevitable failure of the treatment, but the use of binocular vision will also be preserved for him.

Indeed a remedy which is much more powerful than any optical means, for lightening the task of the internal recti muscles, is the weakening of their antagonists. In diminishing the power of the external recti, the abductors, by setting back their insertion by means of a tenotomy, the positive portion of the amplitude of convergence will necessarily be increased. But we shall speak of the surgical treatment of high degrees of insufficiency of convergence when we come to discuss the divergent strabismus of myopes, which very often represents the final stage of it. It remains for us to say a few words more concerning the use of spectacles in the adaptation of myopes.

We bear in mind that their use for distant vision, although indispensable to myopes of high degrees, reserves nevertheless, for these very cases, sad disillusions, on account of the minifying of retinal images on the one hand and of the bad condition of the ocular membranes on the other. The services that they may render in adaptation for a short distance are still more restricted. It must be remembered, indeed, that near work requires of the organ infinitely more than the vast majority of men demand of it for seeing at a distance. Strongly myopic persons, especially, have long since given up all hopes of becoming coast-guards or pilots, if they ever cherished any such hope. They are content with being able to distinguish between a man and woman on the other side of the street, or to recognise their friends in a drawing-room, which satisfaction one is generally successful in procuring for them by means of correcting-glasses.

But for near vision the exigencies are very different. "Doctor, I can't see even to thread a needle any more:"—this is an exclamation of despair which we frequently hear, as if the threading of a needle, under the exact control of the vision, did not require the visual organ of an altogether superior race. To no longer be able to read a newspaper stands with many people for the same thing as being blind. And yet the analysis of such small retinal images as those of printed characters, under so unfavourable circumstances as those in which they are presented—such an analysis presupposes a retina in pretty good condition. Those who rule in these matters have not been content even to make the letters in many of our books, and especially in newspapers, with reference to healthy eyes; the interests of economy so far surpass those of hygiene, that frequently the smallness of the characters and the imperfection of their execution are pushed beyond the limits that a well constituted organ will tolerate. Let us add to this the real advantage that the myope has always had over other eyes in near vision, and we shall understand his despair when the ectasis of his eyeball has caused his myopia to increase to an excessive degree, and when the precarious condition of his nervous apparatus no longer permits it to fulfil the requirements made upon it.

To intervene with concave spectacles, in such cases, is evidently a waste of effort. These glasses can but notably augment, by their minifying effect, the difficulties of seeing, unless we are dealing with occupations which peremptorily require a certain range of sight and with eyes that are still relatively good. It is in such cases that one may render a teacher of piano playing, who is myopic by 16 D, some service with number 14 glasses. But for nearer work, the distance of which is a matter of choice with him who has it to perform, concave glasses have no *raison d'être*. Persons who are excessively myopic, as we have before remarked, have long since renounced binocular vision, and this cannot generally be re-established. Hence there is no reason why distinct vision should be carried beyond the distance at which it is accomplished with the naked eye. And, aside from this useless lengthening of the range of sight, the accessory effects of the concave glass, in such a case, stand for nothing but a series of inconveniences.

The sole advantage, of this monocular vision of strongly myopic persons, being the large size of the retinal images, one might, on the contrary, try to increase their size still more by means of a convex lens. It is unusual, however, to see myopes derive great profit from a magnifying glass. This is easily comprehensible: the management of this glass becomes very difficult, because of the short distance between the eye and its *punctum remotum*—for the least displacement of the glass, either forward or backward, suffices to destroy the clearness of the image.

### Accessory Effects of Concave Spectacle-Glasses.

Just as in the case of convex lenses, concave ones have, for vision, still other results than simply to vary the distance of its adaptation. We have already frequently mentioned the *diminishing influence* which they exert upon the size of retinal images. This influence is much more important to consider than is the magnifying effect of convex glasses, inasmuch as it constitutes an inconvenience which is often annoying, while the other is always an advantage.

The minifying effect of concave glasses is more pronounced in proportion as they are further removed from the eye, for distant vision. We have pointed out, on pages 130 *et seq.*, that the greater the distance between such glasses and the cornea, the stronger they should be, because under such circumstances they are nearer the *punctum remotum*[1] of the eye.

Hence there is always an advantage in placing a concave glass as near as possible to a myopic eye. In this way its necessary refractive power will, at the same time with its minifying effect, be reduced to a minimum.

If convex glasses sometimes diminish the *region of accommodation*, concaves always increase it. This fact needs no explanation. The myope of 5 D, endowed with an amplitude of accommodation amounting to 4 D, has, with the naked eye, his *punctum remotum* at 20 centimetres, and his *punctum proximum* at 9 centimetres, therefore a region of accommodation of 11 centimetres. If he puts on a minus 3, his *punctum remotum* recedes to a distance of 50 centimetres, and his *punctum proximum* to only 16 centimetres; the region then amounts to 34 centimetres. With the correcting-glass minus 5, it extends even from 25 centimetres to infinity.

At the same time the *amplitude of accommodation* is somewhat increased. Nagel has calculated that a concave glass number 1, placed 2 centimetres from an eye that is 2 D myopic and which has 8 D of amplitude of accommodation, causes the latter to increase by 0·29 D.

We have seen how convex glasses are able to *change the perception of relief*. The explanation, of this fact, given by Donders and Nagel, rests upon this peculiarity, that the retinal image undergoes an increase in its absolute dimensions, without being subject to the changes in its proportions, which would be necessary in order to provoke the same stereoscopic sensation. It is clear that we ought to expect an analogous effect from concave glasses, only in the opposite sense. Uniform diminution of the size of retinal images will cause us to think the posterior parts of a solid farther away than its anterior portions; hence the relief of an object will seem to us diminished on the most distant surfaces, and exaggerated in its most advanced details. A plane surface will appear hollowed out before the feet of the wearer of concave spectacles, and that in proportion as he is looking farther away, and as he is less accustomed to his glasses.

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[1] The myopic eye furnished with a concave glass is comparable with an opera-glass *reversed*. If we consider the convex glass in the latter as belonging to the eye, it renders it myopic, and this myopia is corrected by a concave glass (the ocular of the opera-glass), which must be very strong, since it is far from the eye. (Landolt, in de Wecker and Landolt, *Traité complet d'ophthalmologie*, i., p. 502.)

The lessening effect which concave glasses exert upon the size of retinal images, is explained by saying that, in the dioptric system which they, together with the eye, constitute, the second nodal point is farther from the retina in proportion as the concave glass is stronger and more distant from the eye.

Just as in the case of convex lenses, concave ones act as *prisms* if one does not look through their optical centre. Only, since they are thinner at the centre than at their edges, it is evidently the former which will represent the apex of the prism, while this was the edge in the case of the convex lenses. It follows from this, that if we place concave glasses before the eyes in such a way that the lines of fixation pass externally to their axes, they will increase the effort of convergence necessary for the fixation of the same point, while they diminish it when their optical centres are external to the lines of sight.

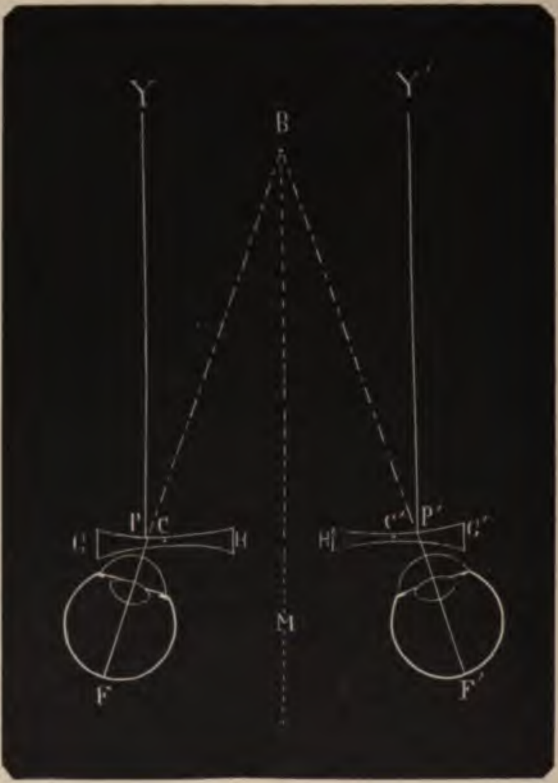

FIG. 136.

If, for instance, as is shown in Fig. 136, the pencils of parallel rays, Y P and Y′ P′, coming from a very distant object, pass through the *external* halves of the concave glasses G H and H′ G′, they are deflected outwards, that is to say, toward the bases of the prisms formed by these portions of the lenses. Hence the eyes, instead of being directed parallelly, are obliged to converge in order to receive the images on their *foveæ centrales* F and F′,—that is, give their lines of fixation the directions F B and F′ B, as if the object were at B.

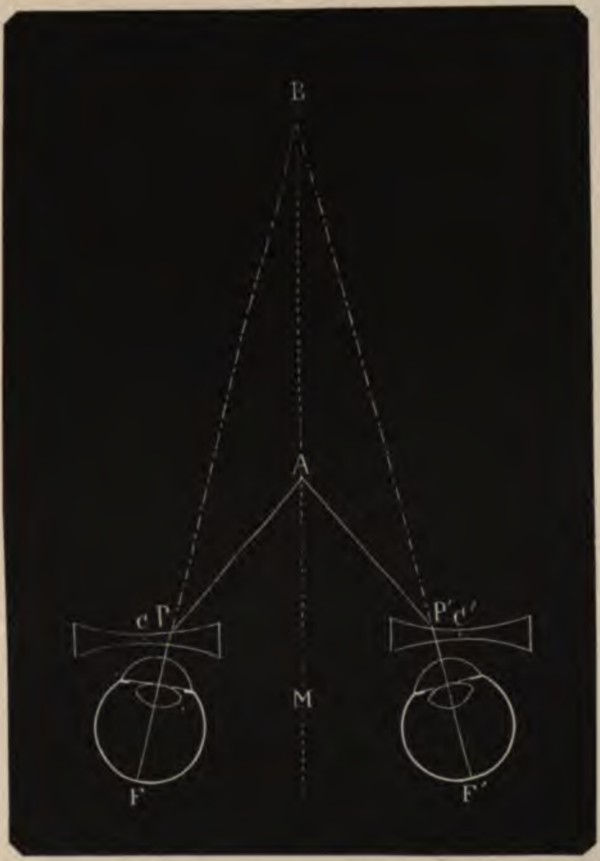

FIG. 137.

If, on the contrary, an object A be looked at through the *inner* halves of concave glasses (Fig, 137), the latter will act like prisms having their apices turned toward the temples. The rays coming from A will undergo, at P and P′, a deviation toward the bases of the prisms, and the lines of fixation will converge less, as if the object had changed its position from A to B.

axis of the lens. If the refractive power of the latter amounts to 4 D, it increases, by 4 dioptries, the refraction necessary for the distinct vision of the object, and the convergence, required by binocular vision by 4 metre-angles. Hence the object will seem to be at B, $\frac{1^{m.}}{10}$ distant from each eye, and the lines of fixation will be obliged to assume the directions F B and F′ B.

A lens of 5 dioptries would require, on the part of the emmetrope, in order to see at a distance, an accommodation-effort of 5 D and a convergence of 5 metre-angles. A myope of 5 D, who should use it as his correcting-glass, would not need to accommodate. It might initiate him into the art, so useful to myopes, of converging without accommodating.

But, for this purpose, we possess a much surer and simpler means —that is to say, atropine. Moreover, so powerful is the solidarity between the two functions that, in forcing convergence, we should probably only succeed in increasing the accommodation, which is already too much brought into play by young myopes. Indeed, since all attempts to increase, by exercise, the faculty of convergence—which is much less powerful than that of accommodation—offer scarcely the slightest chance of success, it is best to address ourselves to the means that we have already mentioned: a fortifying regimen in general, and ocular hygiene in particular, in the most benign cases. If the insufficiency of the internal recti is very pronounced, an operation will be indicated.

In the intermediate cases, we have recourse to a palliative treatment; instead of trying to overcome the evil by means of exercise with convergent prisms, we shall give the patient divergent ones, which enable him to see at the required distance with less convergence.

Thus it is that the prismatic effect of concave glasses may render real service. They are to be decentered, so that the patient shall look through the *inner* portion of the concave lens, and thus the effort of convergence necessary for the fixation of any object, seen through spectacles of this kind, will be diminished.

In virtue of the apparent displacement of objects which they may produce, it is very important to place concave glasses, as well as convex ones, at the same height in front of the eyes. Strong ones, especially, might, if this precaution were not taken, cause a very troublesome diplopia.

Just as in the case of biconvex glasses, the prismatic effect of biconcave glasses is manifested in all meridians. Only the lens may be regarded as composed of an infinite number of prismatic sections, whose apices all converge towards its centre. It follows from this that, if one looks through the edge of the glass, so that it cuts the pupil in halves, objects are seen *double*, in part directly and, on the other hand, through the prism which displaces them toward the centre of the lens. If, then, the eye, wearing a convex glass, has a gap in its field of vision, the concave glass, on the contrary, favours it with a zone in which it sees the objects twice.

Among the most important secondary effects of concave lenses, we have finally to bear in mind the *cylindric action* which they exert when a pencil of luminous rays falls upon them otherwise than perpendicularly to their plane. We have already explained that, when a spherical lens is turned about one of its diameters as an axis, its action in the plane perpendicular to this axis is thereby sensibly increased. Thus it is that, by inclining the concave glass in the vertical, we obtain the same effect as if we had added a concave cylinder with its axis horizontal, which increases the refractive power of the vertical meridian of the lens.

The same thing is evidently accomplished when the eye looks obliquely through the glass. If, for instance, through concave spherical glasses, one direct the gaze strongly to the right or left, one is thereby placed in the same conditions as if one were wearing concave cylinders with their axes vertical: whether the line of fixation be deviated horizontally, or the glass be turned around a vertical axis, the effect is necessarily the same.

This cylindric action of concave glasses may be of service to myopes who are obliged to wear them. By inclining these glasses in front of the eyes, which is very easily done, especially if they are mounted as eye-glasses, they may correct an astigmatism of which the vertical meridian is generally the most refractive. It appears, however, according to Mauthner, that myopes often have recourse to this stratagem, not to correct an existing astigmatism, but simply to increase, at least in one meridian, the insufficient strength of their glasses. In such cases they voluntarily make themselves astigmatic; they prefer retinal images furnished by *lines* of diffusion to those composed of diffusion-*circles* (see p. 298).

Myopes whose error is of high degree, to whom their glasses permit very extended variations of action, sometimes acquire remarkable skill in the management of these glasses. The acuteness of vision which they obtain when they are allowed to hold the glasses themselves, is notably superior to that obtained when the same glasses are placed straight in a trial-frame. This is easy to understand after what we have just said on the subject.

When dealing with a strong myopia, complicated by a slight astigmatism, it is better to let the myope correct the latter by a trifling inclination of his glasses than to combine a spherical concave 18, for instance, with a number 1 cylinder. Moreover, in order to apportion to a single surface the entire action of so strong a concave lens, it must be given such an excessive curvature that the glass becomes very heavy, unless its edges be clipped away, whereby the field of its action is limited.

Under any other circumstances, it need hardly be said, we shall profit by the correcting-glass of the myopia to neutralise, at the same time, the astigmatism, according to the rules given on p. 308 and the pages following. The correcting cylinder will be added to the distance glass, as well as to that for near vision. It will be worn even when the eye does not demand the aid of a spherical glass, provided the advantage to be derived from the correction of astigmatism is greater than the inconvenience of wearing spectacles.

## SURGICAL TREATMENT OF THE INSUFFICIENCY OF CONVERGENCE AND THE DIVERGENT STRABISMUS OF MYOPES.

When the insufficiency of convergence has resisted hygienic measures, general strengthening treatment, repose of the eyes, and when it is too great to be corrected by optical means, we may think of remedying it in a surgical way.

The operations which we have at our disposal for this purpose are—the tenotomy of one or both of the external recti, the advancement of the interni, or even the combination of both these operations.

But surgical intervention never requires more prudence and delicacy than just in the case of muscular insufficiency. However unpleasant it may be to commit a therapeutic error in the prescription of glasses, it is but seldom followed by serious consequences, especially when it is recognised in time and corrected. But it becomes quite another matter when it is a question of surgically affecting the muscular system of the eye, and, indeed, especially in the case under consideration, where there is a simple insufficiency of the muscles. The less considerable the motor disturbance, the more difficult and the more delicate is its surgical treatment. In high degrees of strabismus, one can boldly recede and advance.

It would require a great amount of awkwardness not to be able to give the eyes a better position by an operation than they had before it; and should the result not be quite satisfactory, it is always easy to assist it.

But it becomes a more serious matter where the functional disturbances are slight, as in the case of diminution of the power of convergence. Here the operation, undertaken at the wrong time, or falsely dosed, may be followed by most disastrous consequences. It would not only expose the patient to unnecessary pain, to the interruption of his work, and to the indulgence in vain hopes; but it would, above all, be apt to call forth a strabismus with diplopia, which is incomparably more troublesome, and much more difficult to remedy, than the original affection. It is, therefore, of the greatest importance to take accurately into account the indications for, and the dosing of, the operation to be undertaken against this trouble.

We must confess that our knowledge as regards muscular asthenopia and insufficiency of convergence is still in its infancy. The nature of this affection is still obscure, and trustworthy observations of the results of operations, with a view to cure it, have been neither numerous nor in accord with each other. For some years past I have tried to introduce into practice the notion of the amplitude of convergence, as it is explained in this book, and especially to study, with its aid, not only the influence of the different operations on the muscles, but also the nature of the insufficiency of convergence in general.

I shall now give, with all possible precaution, the results I have thus far obtained, with the hope that they will soon become more numerous and conclusive, and more firmly established.

Insufficiency of the power of convergence is quite a widespread affection, and a frequent cause of asthenopia. It is not by any means peculiar to myopes only, even if they do, for the reasons above given (necessary approach of the object to be fixed), suffer more from it than others, and are, on account of the conformation of their eyes, more inclined to it.

Taking all in all, it is possible to distinguish two forms of motor asthenopia, which are distinctly separated from each other.

The first we may designate *muscular asthenopia in the true sense of the word*. It depends upon the absolute or relative weakness of the adductors or upon their insertion. The second has its origin in the *central organ*, and depends upon a disturbance of innervation or upon a weakness of the power of fusion. The excursions of the eyes, the monocular fields of fixation, and the associated movements may with all this be quite normal, whilst the amplitude of convergence is much reduced, or even, at times, equal to zero.

The following schema[1] (Fig, 139) will render easy of perception the different ways in which the power of convergence may become altered. The vertical lines represent the amplitude of convergence, whose zero is indicated by the horizontal line O O. Below it (+) is the positive, above it (−) the negative domain of the amplitude of convergence.

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[1] Landolt, Congress of Ophthalm. at Heidelberg, 1885.

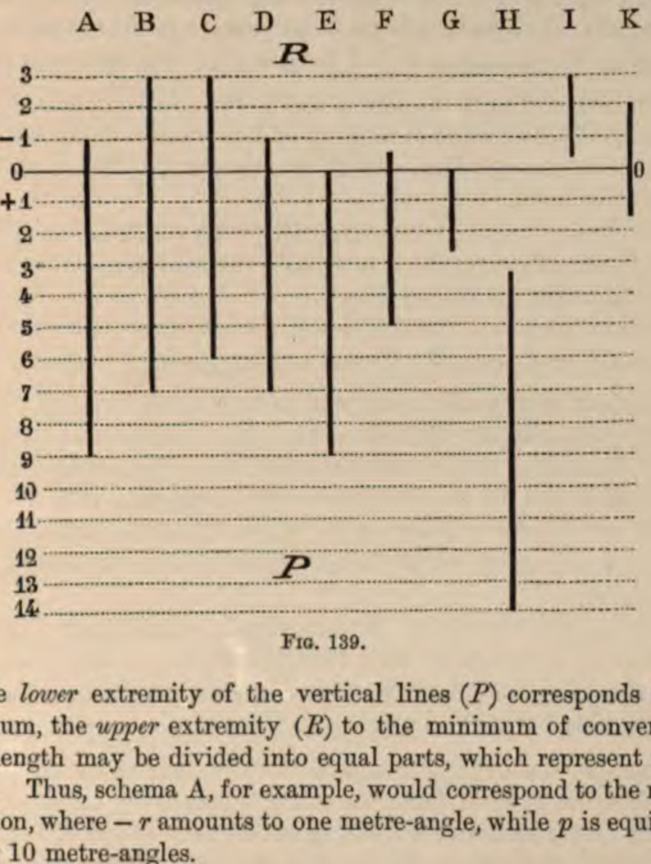

Fig. 139.

The *lower* extremity of the vertical lines (*P*) corresponds to the maximum, the *upper* extremity (*R*) to the minimum of convergence. Their length may be divided into equal parts, which represent metre-angles. Thus, schema A, for example, would correspond to the normal condition, where $-r$ amounts to one metre-angle, while $p$ is equivalent to 9 or 10 metre-angles.

$$\left.\begin{array}{r} p = \phantom{-}9 \\ r = -1 \end{array}\right\} a = 10\ m\,a.$$

It may happen that the total value of this function is normal, but that the entire amplitude of convergence has passed over to the negative side (B). This is the case where there is simple preponderance of the abductors, without alterations of the muscles, and without disturbances of innervation. The amplitude of convergence may, for instance, be expressed in the following manner: $\left.\begin{array}{r} p = \phantom{-}7 \\ r = -3 \end{array}\right\} a = 10\ m\,a$ ; $a$ has its normal value, but $-r$ is too great, $p$ is proportionately too small. In fact, according to what has been said of the quota of convergence, it is evident that a maximum of convergence of 7 $m\,a$ is not sufficient for near vision. Such persons, be they emmetropes, hyperopes or myopes, are, in spite of good vision for distance, exposed to asthenopic troubles, as soon as their work requires to be brought nearer.

Asthenopia is much more marked and makes its appearance much more rapidly when, as in schema C, its positive portion is shown to be reduced, as, for instance: $\left.\begin{array}{l} p = \phantom{-}6 \\ r = -3 \end{array}\right\} a = 9\, m\, a$. The excessive value of the negative $r$, that is to say of the power of divergence, has, it is true, as we shall soon see, a very great importance for the operative treatment of the affection, but it can, as such, interfere only with work near at hand.

This also holds good for the case D, where the negative portion of the amplitude of convergence is normal ($r = -1\, m\, a$), but where the positive part is reduced ($p = 7\, m\, a$).

In fact, a person whose amplitude of convergence amounts to only 9 *m a* but is all positive $\left.\begin{array}{l} p = 9 \\ r = 0 \end{array}\right\} a = 9\, m\, a$ (schema E), will be able to work, without asthenopic symptoms at the usual distance, a thing it would be impossible for B to do, notwithstanding his $a$ of 10 *m a.*

But the state of affairs becomes much more complicated when, as in schema F and G, the amplitude of convergence is considerably reduced at *both* extremities, when $r$, for instance, represents only $-$ 0·5 (as in F), or even 0· as in G, while $p$ does not exceed a few metre-angles.

This happens especially in *neurasthenic insufficiency.* There the amplitude of convergence may become reduced to less than 3 *m a* (G), or it may even go down to zero, that is to say that, at the most, binocular vision may exist only for the distance.

Let us cite, as a striking example of the reduction of the amplitude of convergence, two cases of locomotor ataxia, observed by our assistant Dr. Hübscher. They had, both of them, passed the state of irritation, and had entered upon that of troubles of co-ordination. They still possessed a visual acuteness = 1, in spite of a marked limitation of the fields of vision, especially for red, and of the pronounced discoloration of the papilla. The pupils were immoveable, but accommodation was still normal.

The examination of the fields of fixation did not show any muscular paresis, but the maximum of convergence was in one case only $p = 4{\cdot}5$ *m a*, and in the other only $p = 3{\cdot}5$ *m a.* Both showed spontaneously the most characteristic symptoms of muscular asthenopia.

Schema H shows an amplitude of convergence which is altogether on the positive side: $\left.\begin{array}{l} p = 14 \\ r = 3{\cdot}25 \end{array}\right\} a = 10{\cdot}75\, m\, a$. In this case the individual will have good binocular vision, but he cannot relax his convergence above 3·25 *m a.* In other words, beyond the distance of a third of a metre, convergent strabismus with homonymous diplopia will be found to exist.

Paresis of one external rectus may give rise to such an amplitude of convergence, but it is especially produced by careless tenotomising of the abductors.

Schema H corresponds to the first case, a paresis of the left external rectus. In such cases, the *punctum proximum* of convergence approaches sometimes much nearer than is illustrated by our example. It may reach even 20 *m a* and perhaps more.

The reverse is found in schema I. The entire amplitude of convergence is negative: $\left.\begin{matrix} p=-0{\cdot}5 \\ r=-3 \end{matrix}\right\} a = 2{\cdot}5\ m\,a.$ This corresponds evidently to a divergent strabismus, where spontaneous binocular vision does not exist. It can only be accomplished with the aid of abducting prisms. Without them there will be crossed diplopia. Did the strabismus arise suddenly, in consequence of paresis, then the diplopia will be most annoying, and the negative $r$ may reach a high degree. These cases in reality do not belong to the subject now under consideration.

Schema K is, on the contrary, very characteristic of high degrees of myopia. Here it happens that, in consequence of the hyperplasia of the globes, especially in their antero-posterior axes, the lines of fixation can no longer be brought into a convergent direction, and this so much the less in proportion to the loss of contractile power of the over-stretched muscles. This is the reason why, in such cases, the total amplitude of convergence does not amount to more than a few metre-angles, whether the *punctum proximum* lies on this or the other side of infinity, as in K, where, at the expense of all power, it is possible to converge only 1·5 metre-angles.

It is true that, in the great majority of cases of extreme myopia, binocular vision no longer exists. It may be that one eye has succumbed to one of the many dangers to which such eyes are exposed; or that, in consequence of the divergent position of the globes and the difficulty of converging, binocular vision has never been fully developed or else has been gradually lost. Nevertheless, it is sometimes possible, in such cases, to bring binocular vision into play, by making use of a luminous object of fixation and by directing the patient's attention to the image of each of his eyes, in the manner above indicated, and, when necessary, to project them, by means of diverging prisms, simultaneously upon the *maculæ* of both eyes. In such a way I obtained schema K, which corresponds to a myopia of 18 and 20 D. The amplitude of convergence is, as one may readily see:

$$\left.\begin{matrix} p= & 1{\cdot}5 \\ r= & -2 \end{matrix}\right\} a=3{\cdot}5\ m\,a.$$

Now, in all cases where the positive convergence does not reach 9 metre-angles, asthenopic troubles may become developed when the eyes are used for near vision. This is, of course, the more apt to happen, the more the maximum of this function falls below the given measure.

To these belong our schemata B, C, D, F, G, I and K. In all these examples the deficiency of the convergence is too considerable to be equalised by optical means (prisms). On that account, one might think it advisable to come to its aid operatively—that is to say, by means of a tenotomy of one or both abductors, the advancement of one or both adductors, or of a combination of the setting back and the advancement of the muscles.

What can be the effect of these operations—for example, of a tenotomy of one externus—upon the amplitude of convergence? The setting back of one or both abductors must evidently diminish their strength—that is to say, the power of divergence. At the same time, the resistance which the adductors meet in contracting is also diminished, and it is precisely upon this fact that we base our hopes of a cure. In other words, we reduce the amplitude of convergence at the end of the *punctum remotum*, and bring nearer that of the *punctum proximum*.

Now, the *punctum remotum* of convergence must never lie this side of infinity,—$r$ must never become positive, otherwise there will be convergence for the distance, with homonymous diplopia. In all cases, therefore, where $r$ is equal to zero as in schema G, I would, *eo ipso*, abstain from a tenotomy.

If, on the other hand, $r$ is negative (as in B, C, D, F, I, K), we may think of sacrificing the power of divergence, which is of no use whatever, in order to benefit that of convergence.

Hence this important question arises: Is the quantum of convergence taken off at one end of the amplitude simply added, as such, to the other end? For instance, if, before the operation, the amplitude of convergence was

$$p - r = a$$

$$7 - (-3) = 10\ m\,a, \text{ will it be}$$

$$10 - 0 = 10\ m\,a,$$

if we have succeeded, by a well-dosed tenotomy, in exactly sacrificing the 3 metre-angles of divergence?

In this case the amplitude of convergence would remain 10 metre-angles, just as it was before, only that it is brought nearer in its totality (Fig. 140).

This is, in fact, possible, and we have had the opportunity to observe the very example given in the adjoining figure, and to relieve, in this manner, muscular asthenopia.

Most frequently, however, the matter does not thus run its course. The amount of positive convergence gained by means of a tenotomy may be greater, but may also be less, than that which has been taken from the negative. In other words, a tenotomy may increase the amplitude of convergence more at one end than it will reduce it at the other, and also *vice versâ*. In the first case, this function, as a whole, will be increased and in the latter it will be diminished. If the former result is very gratifying, the latter may be quite as unpleasant. Whether the one or the other happens, will depend upon the nature of the asthenopia and upon the condition of the muscular system.

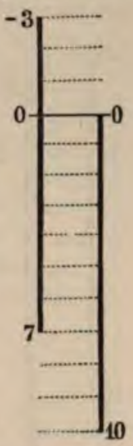

FIG. 140.

If, for instance, a but slight reduction of an amplitude of convergence, as a whole, shows that the muscles properly perform their functions, and that the impulse to binocular vision is well preserved, it is possible, by operative means, to gain, notwithstanding a small amount of the negative $r$, a quite considerable increase of $p$.

The following example may serve as an illustration:

Left eye — H·0·75 ; V = 0·6.
Right eye—H·0·75 ; V = 7.
$a^a = 2{\cdot}5$ D.

Muscular asthenopia—

$$\left.\begin{array}{r} p = 7 \\ r = -1 \end{array}\right\} a = 8\ m\,a.$$

After tenotomy of one external rectus—

$$\left.\begin{array}{r} p = 11 \\ r = -0{\cdot}7 \end{array}\right\} a = 11{\cdot}7\ m\,a.$$

Things will naturally take a more favourable turn when $-r$ is even more considerable, as in the following case:

Robust man, aged thirty. Left eye, myopia 6 D; right eye, myopia 7 D. Astigmatism of 1·25 in each eye; after correction, normal acuteness of vision. Insufficiency of convergence, which interferes with his work—

$$\left.\begin{array}{r} p = 7 \\ r = -3 \end{array}\right\} a = 10\ m\,a.$$

After a simple tenotomy of one externus—

$$\left.\begin{array}{r} p = 12 \\ r = -2 \end{array}\right\} a = 14\ m\,a.$$

Asthenopia disappeared. This result, with the same amplitude of convergence, has remained unchanged for more than a year. Here we have sacrificed 1 metre-angle of divergence, and have gained 5 of positive convergence. The total amplitude of these functions has been increased 4 metre-angles (Fig. 141).

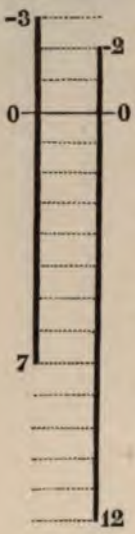

Fig. 141.

Such favourable results are not to be expected of a tenotomy, when, as in schema F, $r$ is very low and the amplitude of convergence, as a whole, much reduced. The latter fact appears to me to be particularly important. The diminution of the amplitude of convergence usually indicates weakness of the muscles of the eye. We must therefore not expect much support from them, and if we do not, by operative suppression of the power of divergence, gain much more $p$ than the amount $r$, that is to say, only 0·5 of metre-angle as in our example, we will have rendered no service to the patient. His maximum of convergence, even if somewhat increased, will still not be sufficient to permit of continuous work. Indeed, as we shall soon see, these cases lie upon the boundary between muscular and neurasthenic insufficiency, and in the latter case we run the risk of gaining even less in convergence than we have lost in divergence.

Such cases appear to be better adapted to *advancement of one or both interni* than to tenotomy of the externi. Advancement for the purpose of doing away with muscular asthenopia has but seldom been undertaken until of late. It is an operation which is more aggressive than tenotomy. It is more difficult to dose, and may expose the patient to the danger of a very annoying diplopia, produced by a difference in the height of the eyeballs, or by the rotation of the operated eye on its antero-posterior axis. At all events such is the report of very trustworthy and able surgeons. I must observe, however, that, although I have performed the operation of advancement, with and without tenotomy of the antagonist, for many years, and that probably more often than has been done elsewhere, I have never seen these fears realised. On the contrary, the favourable results which this operation has given in cases of strabismus, encouraged me to try it also against simple insufficiency of convergence. Advancement of the interni is, in reality, also more logical than tenotomy of the externi. If it is a question of the insufficiency of the adductors, the strengthening of these muscles ought to be taken into consideration, before an attempt is made to weaken the abductors. In fact, advancement of the internus aims to give this muscle a more favourable insertion, and thus a greater leverage upon, and more power over the eyeball. It goes without saying that, with the increase of the power of the adductors, we also raise the resistance which is encountered by the abductors. It was, however, to be hoped that the gain of power of convergence would be more considerable than that which was lost by divergence, that the amplitude of convergence would gain more by an advancement, that is to say, the direct invigoration of certain muscles, than by a recession, *i.e.*, a weakening, of others.

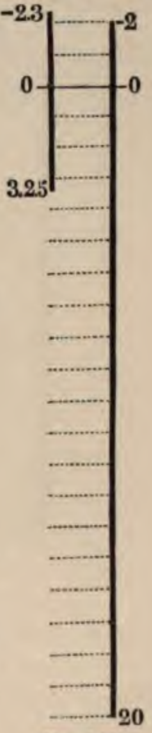

Fig. 142.

This prevision is confirmed in certain cases, as is shown by the following examples:

Boy aged thirteen; both eyes myopic 1·25, L.E.V. = 0·7; R.E.V. = 1. High degree of asthenopia, which has resisted all general and local treatment for months:

$$\left.\begin{array}{l} p = \phantom{-}3{\cdot}25 \\ r = -2{\cdot}3 \end{array}\right\} a = 5{\cdot}5\ m\,a.$$

Advancement of one internal rectus. After recovery:

$$\left.\begin{array}{l} p = \text{ over } 20\ m\,a. \\ r = -2 \end{array}\right\} a \text{ at least} = 22\ m\,a \text{ (Fig. 142).}$$

This result has remained unchanged for more than a year, and asthenopia has not returned.

Another example (Fig. 143):

Woman aged thirty-five—Left eye hyperopic, 0·75; V = 0·7
Right „ 0·5; V = 0·8 to 0·9

Motor asthenopia:

$$\left.\begin{array}{l} p = \phantom{-}7 \\ r = -1 \end{array}\right\} a = 8\ m\,a.$$

Advancement of one internus. After six days:

$$\left.\begin{array}{l} p = \phantom{-}11 \\ r = -1 \end{array}\right\} a = 12\ m\,a.$$

After several weeks:

$$\left.\begin{array}{l} p = \phantom{-}14 \\ r = -1{\cdot}5 \end{array}\right\} a = 15{\cdot}5\ m\,a.$$

This operation was performed according to the method indicated (p. 403), and indeed we did not fear to boldly approach the muscle in one of the resistance of its intact antagonist. It goes without saying that we bestow the greatest care not only upon the operation, but also upon the after-treatment. We will spare the reader a summary of the necessary details, which ought to be familiar to all surgeons. Should there become apparent, soon after the operation, a considerable amount of over-correction, that is, convergent strabismus, we will simply remove the sutures a little earlier, or, in case of a difference of height of the eye-balls, give them a better position. A moderate surplus of convergence, far from being a thing to be dreaded the first few days after the operation, is, on the contrary, to be desired if the latter is to fulfil its purpose.

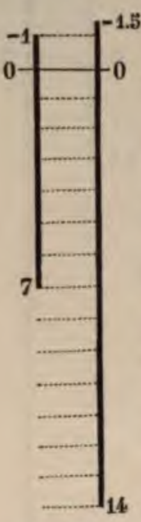

FIG. 143.

In our attempts to treat asthenopia operatively, we have also met with cases in which a setting back as well as an advancement, or even a combination of the two, has given most unsatisfactory results. These are, first, the cases of *neurasthenic asthenopia*, where the amplitude of convergence is considerably reduced, as, for instance, in schema G. Here the muscular force is so slight, that a not very energetic operation produces hardly any result, while a bold tenotomy calls forth a permanent convergent strabismus; the effect of an advancement is, on the other hand, owing to the relaxation of the muscles, in great measure soon lost. Indeed, even the combination of both operations, in such cases, is disappointing.

Let us cite two characteristic examples:—

A young tutor, a good observer, who willingly submitted to all examinations and experiments. General muscular debility.

Left eye—myopia, 1·5; V = 1
Right eye—myopia, 2·; V = 0·8

Motor asthenopia. Fields of fixation generally limited.

$$\left.\begin{array}{l} p = \quad 4{\cdot}5 \\ r = -\,0{\cdot}5 \end{array}\right\} a = 5\ m\,a.$$

Advancement of one internal rectus. Four days after the operation:

$$\begin{array}{l} p \ \text{ over } 20 \ \ m\,a. \\ r = +\ 1{\cdot}5\ m\,a. \end{array}$$

Consequently, noticeable over-correction. During the following days, convergence for distance even increased, but for near vision it, on the contrary, diminished. After a while, it disappeared entirely for distance; patient no longer saw double and was able to do near work without difficulty. But after three months the asthenopic troubles reappeared. He had recourse to spectacles, with adducting prisms, which brought him relief for a short time. But finally these no longer sufficed, and then we found, upon examination:

$$\left.\begin{array}{l} p = \quad 4 \\ r = -\,0{\cdot}5 \end{array}\right\} a = 4{\cdot}5\ m\,a.$$

The state of affairs is, therefore, no better than it was before the operation.

Even more striking is the case of a young nun, with myopia of 2·25 in one eye, of 2 in the other and normal acuteness of vision in both. She suffered from a high degree of asthenopia and divergent strabismus of 5°, without diplopia. With employment of all her power of convergence, she was hardly able to see binocularly at the distance of one metre ; $p$ was, therefore, scarcely 1 *m a.* In spite of *successive* tenotomy of both externi and advancement of both interni, the convergence gained was only just enough to allow of work for a short time. Asthenopia soon returned, and after five months we found :

$$\left.\begin{array}{l} p = \phantom{-}4 \\ r = -2 \end{array}\right\} a = 6 \ m\,a.$$

Therefore still a tendency to diverge, insufficient convergence, and especially poor amplitude of convergence.

From these cases we must not hastily draw the conclusion that neurasthenic asthenopia is not to be treated operatively, but rather that, without being excluded, surgical treatment ought to be assisted by general invigorating treatment, and, moreover, that we ought to be guarded in operating, as well as in making our prognosis.

Finally, a special category of insufficiency of convergence is formed by those cases where divergence is the result of myopia of high degree. This, however, but exceptionally becomes the cause of muscular asthenopia, for binocular vision, in such cases, has long been lost, and near vision is accomplished, as we have seen, in an altogether different manner from what it is in emmetropia and moderate ametropia. Every attempt to re-establish binocular vision for near objects, in extreme myopia, must fail on account of the difficulty which the hypertrophied eyeball encounters in its movements, and especially on account of the inertia of the muscles which, over-stretched and for want of use, have lost their elasticity. I operated only once, *experimendi causa*, upon such a case, and cite it here as a discouraging example.

Young man, left eye, myopia 18 D ; right eye, myopia 20 D. Eyeballs greatly enlarged, limited in their movements, and divergent ;

$$\left.\begin{array}{l} p = 1{\cdot}5 \\ r = -2 \end{array}\right\} a = 3{\cdot}5 \ m\,a.$$

Tenotomy of one external rectus brought—

$$\left.\begin{array}{l} p \text{ to } 7 \\ r \text{ to } +4 \end{array}\right\} \textit{ergo, } a \text{ to } 3 \ m\,a.$$

Patient's convergence still remained insufficient for near objects, but there was considerable surplus,—that is to say, convergent strabismus, with homonymous diplopia—for distance. Besides which, the amplitude of convergence had become reduced. Convergence diminished more and more, and a slight tenotomy of one internus did away with the diplopia entirely. The amplitude of convergence then became and remained—

$$\left.\begin{array}{l} p=2{\cdot}5 \\ r=0 \end{array}\right\} a=2{\cdot}5\ m\,a.$$

This amplitude of convergence was, therefore, again reduced, as appears to be the rule in cases of muscular inertia. The operation accomplished nothing but the cosmetic effect which followed the removal of the divergent strabismus for distance. The impulse to binocular vision was so slight that the patient was not annoyed by his relative divergence for near objects.

The *technique* of the operations, tenotomy and muscular advancement, undertaken to cure insufficiency of convergence, is evidently the same as that already explained. If it is question of a setting back of the externi, the operation as performed with our forceps is particularly to be recommended. It is, indeed, often necessary, in such cases, to loosen the conjunctiva but very little from the muscle, while the introduction of the muscle-hook requires so extended a separation that the tenotomy may but too easily overstep the desired effect. The operation has to be performed with great caution, especially when the power of divergence is only about 1 *m a.* If, after several weeks, it appears that the result is insufficient and that there still remains some divergence, it is then easy to increase the effect by means of a tenotomy on the other eye. Although, in cases of genuine monocular strabismus, we always operate upon the deviating eye only, in insufficiency of convergence we prefer to divide the operation between the two eyes.

The first tenotomy has also the advantage of giving us a very valuable indication as to the manner in which the eyes react to surgical interference. If, for instance, the effect of the first tenotomy has been but slight, then the second may be undertaken more boldly and more extensively, and *vice versa.* Besides, the immediate effect of a tenotomy may be increased or diminished by means of the sutures above mentioned.

Immediately after the operation, we generally find homonymous diplopia to exist when the patient looks straight ahead. There is nothing surprising about this, since, at this moment, the tendon of the external rectus, detached from the eyeball, has no direct hold upon it. Even if this excess of convergence persists for several days, there is no reason for being frightened, provided it does not exceed certain limits, that the strength of the abductors was shown to be great enough before the operation, and that the patient carefully carries out the treatment which we shall prescribe for him after the operation.

If after the third day the diplopia has disappeared, and if the effect of the tenotomy should prove to be moderate, then the patient may set the eye that was operated upon at liberty, and begin to exercise his convergence. In order to do so, it will only be necessary for him to return to his ordinary occupation. Forced training of convergence by means of an object of fixation, gradually brought nearer, will act still more powerfully. The stereoscope, also, may here render good service. But all these exercises must not be so long continued as to produce fatigue. That we must bear in mind the position of the *punctum remotum* of convergence, goes without saying. It is just as evident that the result of our operation will be sooner obtained if we combine with it an invigorating general treatment, especially in the case of anæmic persons.

If, on the contrary, the effect of the tenotomy appears to be somewhat too great, we keep both eyes bandaged, instil atropine, and, above all, recommend the patient to avoid all movements of convergence. Louchettes may perhaps be of use in such cases. But still more effective are the exercises with our stereoscope. These evidently are meant to increase the strength of the abductors. The two objects to be fixed are first given the position in which their images are most easily fused, and then they are separated more and more from each other.

On the other hand, an intelligent patient readily finds the position of the head which enables him to see single at a distance. He is then told to turn his head slowly toward the side where the diplopia manifests itself, taking care meantime not to go so far as to bring about an apparent doubling of the object. The best object to use for this purpose is a candle flame at a distance of at least 5 metres. The patient will in this way succeed in extending the domain of single binocular vision.

The latter can be perfectly re-established after some weeks or months, if the immediate excess of correction was not too great. When only a few degrees of it remain, and the patient is obliged to resume his work, we may afford him assistance with prismatic glasses. Since this strabismus manifests itself only for distant vision, we need generally, only decenter the concave correcting-glasses as is shown in Figure 137. Little by little the muscular equilibrium will be re-established, so that the patient will see single even without this artifice. It is, however, absolutely indicated to watch such cases, until entire and permanent cure has been effected. It is especially necessary not to regard the simple disappearance of diplopia as such a cure. This disappearance may be due to the suppression of the retinal image in one eye, principally in cases where there is a marked difference between the two eyes as regards refraction and acuteness of vision. Here it inaugurates, on the contrary, a new and very troublesome condition—permanent convergent strabismus.

But no careful surgeon will ever allow matters to come to this pass. He will have remedied any considerable surplus of convergence, immediately after the operation, by means of a conjunctival suture. Indeed, even after a few days, the effect of a tenotomy may be modified at will. The new insertion of the muscle is, at first, not so firm but that, with the hook, it can easily be detached from the eyeball and advanced by means of a suture.

If, instead of tenotomy of the externi, we are concerned with *advancement of the interni*, we also follow the method above indicated.

The operation demands, indeed, very great precaution, so that there may not result a difference in the height of the eyeballs. In return, however, an over-effect is not so much to be feared as in tenotomy, for in such cases of advancement of the interni, the externi are generally intact. In order to effect a perfect and rapid healing of the advanced muscle, we strongly recommend the practice of keeping both eyes bandaged for several days, of removing the sutures as late as possible and with the greatest care, and of not allowing resumption of work until after complete cicatrisation.

As in all medical and surgical undertakings, Nature here also offers us the most valuable aid. He who understands how to make her his ally will rarely have to complain of failure. Nourishing diet, judicious mode of life, well regulated orthoptic exercises, &c., will complete the cure which an operation alone would not have been able to effect.

Except in cases where the faculty of divergence is lacking, tenotomy of the external recti is contra-indicated whenever the muscles are generally too weak. This occurs especially in high degrees of myopia. Here, the muscles, distended by the excess of development of the eyeball, have lost a great part of their contractility. The field of fixation is restricted, as well as the amplitude of convergence. The excursions of the eyes may be reduced to 18° on the inner side and 26° on the outer.[1] There may be a maximum of convergence ($p$) of only 1·5 *m a*, and a minimum ($-r$) of 2 *m a*, which gives an amplitude ($a$) of 3·5 *m a*.

In such a case, tenotomy of the external recti could never give a good result. It diminishes, it is true, the range of convergence at the end of the *punctum remotum*, and increases it at the nearer extremity, but the decrease is greater than the increase; hence the amplitude is in reality restricted, and, if it was already too weak to begin with, it is materially impossible to bring the *punctum proximum* to working-distance without producing an enormous convergent strabismus which will manifest itself even at a very short distance. In other words, the minimum, as well as the maximum of convergence, will be of a very high positive value. In the example cited, $p$ may therefore become $7\,m\,a$, while $r$ becomes $+\,4\,m\,a$. Homonymous diplopia would commence, at most, at 25 centimetres, if not still nearer, and the amplitude of convergence would not be greater than $7-4=3\,m\,a$. Oftentimes it will lose still more by tenotomy.

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[1] Landolt, "Etude sur les mouvements des yeux" (*Arch. d'opht.*, p. 603, 1881).

Moreover, it would be, in any case, so weak that, even though the *punctum proximum* were at working-distance, work would not be possible, because there would not be sufficient disposable muscular power to maintain it for any length of time.

One should bear in mind here what we have said, concerning the extreme degrees of myopia, at the beginning of this clinical chapter, and in the section relating to that form of ametropia: such eyes are diseased, profoundly altered in all their functions. Just as we are impotent to restore to them a normal acuteness of vision, so our attempts to re-establish the power of a binocular vision would be stricken in advance with sterility.

When insufficiency of the adductors has given way to a true *divergent strabismus*, but when the latter is still periodic, it may sometimes be remedied by prismatic and concave glasses, but such cases are very rare. The divergent strabismus, which we meet with in practice, has generally long since far exceeded the degrees that are corrigible by optical means.

Even *orthoptic treatment* has very little hold upon it. And this is susceptible of ready explanation. Every person, gifted with two eyes which see, is, so to say, constantly under the influence of the stimulation to converge. Unless looking at an infinite distance, he is always obliged to converge more or less in order to see single the objects about him. He incessantly practises exercises which ought to accustom him to converge, and prevent his falling into divergent strabismus. If this, notwithstanding, becomes established, there is very little to hope that exercises of this kind will suffice to cure the strabismus which they have been incompetent to prevent. The internal recti muscles are too weak to maintain, or even produce, convergence. In the case of debilitated young people, with overworked eyes, forced manœuvres of convergence could only contribute to increase the fatigue of the ocular muscles.

The surgical treatment is, indeed, the only efficacious one in high degrees of insufficiency of convergence, and, with all the more reason, in divergent strabismus.

If the divergent strabismus is of low degree, we shall correct it by means of a single or double tenotomy, being careful to take fully into account the absolute and relative strength of the conflicting muscles, the total amplitude of convergence and the relative value of its component elements, as well as all the means that surgical and pacific therapeusis places at our disposal.

When divergent strabismus has lasted a long time, and has reached a high degree, binocular vision is almost always irrevocably lost. The operation is not then performed, unless for a cosmetic purpose. It is sufficient to suppress the divergence and re-establish the parallel direction of the eyes for distant vision. The contractility of the internal recti, favoured in their action by the setting back of the external recti, will restore to the eyes their normal aspect when looking at a distance, even though it should not bring the convergence up to the myope's *punctum remotum.*

We have rarely to fear, in such cases, an excessive effect from the operation, on account of the inertia of the adductor muscles and of the tendency to divergence, which is natural to many myopic eyes. When the acuteness of vision is good and the myopia not too strong in either eye, we may still, however, try to re-establish binocular vision after the operation, with the aid of the stereoscope. These attempts are, too, but rarely successful, and success, when achieved, does not always present any great advantage for the patient. Binocular vision is indeed useful, especially for work near at hand, which is just the distance for which, in such cases, convergence is insufficient.

When divergent strabismus exceeds 15 degrees, the most extensive tenotomy does not suffice for its correction. We may then add a tenotomy performed on the other eye. If the deviated eye is much inferior to its fellow, we prefer the setting back of the external rectus, combined with advancement of the internal rectus, according to the method devised by the author (p. 406).

It sometimes happens that the maximum effect of this energetic procedure is still unable to correct the extreme divergent strabismus frequently met with in the highest degrees of myopia. In such a case we shall add a tenotomy of the external rectus of the non-strabotic eye, or even the advancement of its internal rectus.

## ATYPIC MYOPIA FROM UNUSUAL CAUSES.

The elongation of the axis of the eye, although being the most frequent origin of myopia, and being able to carry this refractive anomaly to very high degrees, is not, however, the only cause capable of making the focus of the dioptric system pass in front of the retina.

It is often in consequence of an *increase in the refractive power* of the system itself that its focus is displaced.

Thus we sometimes see an eye, whose length does not exceed even the minimum compatible with emmetropia, which is myopic on account of an *increased curvature of its cornea.* Such cases are exceptional. We remember, indeed, that Donders' measurements, and all that have been made since his, prove that the cornea, far from being more convex in myopic eyes, often shows itself more flattened with them than with emmetropes.

The most striking and most characteristic example of this *myopia of curvature* ($M^c$) is *conical cornea.* The first refractive surface of the eye affects in such cases, as its name indicates, an acuminate form, easily recognisable in profile, and giving rise to catoptric and dioptric phenomena which have already been mentioned on page 327. The apex of the cone presents a curvature of so short a radius that the myopia of an eye affected with kerato-conus may readily attain 30 D or more, while axial myopia rarely exceeds 20 or 22 D. We have yet to meet with a figure greater than 25 D, resulting from excessive length of the eyeball.

Unfortunately, these conical corneæ are not only excessively convex, a difficulty which could easily be remedied by glasses of equivalent concavity, but their curvature is generally very irregular. In the first place it is much further removed from sphericity, and approaches much nearer to that of a hyperboloid than the curvature of the normal cornea. However, the attempts of Raehlmann and others to neutralise the effect of the corneal deformity by means of hyperbolic glasses, have not, as yet, given very satisfactory results. This is because, in most such cases, the surface of the cornea is not even a surface of revolution; its curvature differs from one meridian to another. In short, it is strongly astigmatic. The regular astigmatism of these kerato-cones may vary from 5 to 15 dioptries,[1] and its correction, combined with that of the general myopia, sensibly increases the visual acuteness of such eyes. This, however, never reaches the normal, inasmuch as only one part of the corneal irregularity is corrigible, and since there always remains some irregular astigmatism which resists any correction.

Curvature myopia may be produced by an *excess in the curvature of the crystalline.* This occurs notably when the crystalline is withdrawn from the action of its suspensory ligament, which tends to flatten it more or less. Thus, a rupture of the zone of Zinn, which may be brought about by a traumatism, almost always increases the refraction of the eye, sometimes to such an extent as to render it strongly myopic. In cases of subluxation of the crystalline, we even find myopia co-existing side by side with hyperopia in the same eye. The portion of the pupil occupied by the crystalline is myopic, while the aphakic portion is hyperopic.

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[1] Laqueur, "Ueber die Hornhautkrümmung, &c." (*Klin. Monatsbl.*, p. 99, 1881).

Luxation of the crystalline may produce myopia in still another way, that is, by the forward displacement of this lens. The *advancement of the crystalline*, from any other cause, likewise produces an increase in the refraction of the eye.

Finally, the same organ can also occasion a third form of myopia, due to the *increase of the index of refraction* of its nucleus. We have often enough pointed out that the refractive power of the crystalline is more considerable in proportion as the difference in refraction between its periphery and its centre is greater. The effect of an increase in the density of the nucleus, sometimes recognisable in the reflex obtained from it in oblique illumination, is therefore readily explainable. Thus we often meet with persons who, far from becoming hyperopic on account of the senility of the crystalline, notice, on the contrary, that their refraction increases, that they can read without the aid of convex glasses, and enjoy a sort of "second sight," as the Americans call it. This improvement in sight is, however, only very relative. Although near vision is possible without convex glasses, it is indistinct for distance. Such persons have become true myopes. And, unfortunately, this *myopia from alteration of the index of refraction* ($M^i$) is almost always succeeded by blindness, since this increase in the density of the crystalline is generally the prodrome of a nuclear opacity, which is soon followed by total cataract.

Increase of the index of refraction of the aqueous humor, as well as diminution of that of the vitreous body, might likewise bring about myopia; but cases of this kind have not yet been made out with certainty.

The correction of atypic myopia is necessarily the same as that of the typical variety. But the importance of the refractive anomaly generally vanishes, in comparison with that of the alteration of the eye which has given rise to it.

## ANISOMETROPIA.

We designate by the term *anisometropia* (privative ἀ and ἴσος equal) the state in which the refraction of the two eyes is unequal.

Slight differences between the eyes, in the same person, are frequently found, and are certainly commoner than absolute equality of the two. But it is not of such trifling inequalities, which deserve no special name, that we are about to speak. For our purposes, *anisometropia* begins when the difference of refraction between the two eyes exceeds the value of the minimum interval presented by our notations for the different degrees of refraction. In other words, anisometropia exists whenever the two eyes demand, in order that each shall possess its maximum of visual acuteness, or in order to present to the observer the same clearness in their ophthalmoscopic images, two different numbers of spectacle glasses.

All imaginable varieties of anisometropia, all combinations of refraction, are possible between the two eyes of the same individual. One eye being *emmetropic*, the other may be *myopic* or *hyperopic*. Both may be *hyperopic* or *myopic* in *different degrees*. Finally, one may be *hyperopic* while the other is *myopic*.

The difference in refraction may attain very high degrees. Six dioptries is not a very rare figure for congenital anisometropia, and, by unilateral extraction of cataract, we create even much greater inequalities. Let us add that astigmatism is of very frequent occurrence with anisometropes, and it will readily be understood how dissimilar the refraction of an eye may be from that of its fellow. It is true that astigmatism is the very thing that at times re-establishes some little analogy in this inequality, by the symmetrical disposition of its principal meridians. But this symmetry is itself not more constant than the degree of astigmatism, and even less so than its nature in each eye.

Anisometropia may be *congenital* or *acquired*. After what we have said concerning the development of the eyeball, it is evident that, when one eye becomes myopic in the course of its evolution, while the other remains hyperopic, or becomes emmetropic, this difference in refraction, although coming on with age, can, nevertheless, be considered as a congenital anisometropia. The disposition to the state of refraction is, in fact, congenital, at least in the majority of cases.

An incontestably acquired anisometropia, on the contrary, is that which has already been mentioned as being created by the suppression of the crystalline in one of the eyes. But it is more than doubtful if one eye can become myopic in consequence of its exclusive employment for near vision, while the other remains hyperopic, for instance. The organ which does not participate in vision partakes of the fatigue of its mate, as is readily verified. Thus we often see one eye which is primitively weaker, and consequently excluded from vision and deviated, become more staphylomatous and more myopic than its fellow, which is in effective activity. Hence work is but the determining cause of the development of myopia in eyes which are congenitally or accidentally predisposed to it. It is this predisposition which is lacking in the active eye which has remained hyperopic, while the other has become myopic.

Except when it is the consequence of an operation, an accident, as the unilateral luxation of the crystalline or of detachment of the retina or of a corneal facet, &c., anisometropia may therefore be regarded as being congenital and attributable to the unequal development of the eyes. Hence it is perfectly logical to try to ascertain if this difference in the evolution of the two eyes is not associated with a similar inequality between the corresponding orbits, and even the two halves of the head. This supposition is all the more justified from the fact that asymmetry of the face and cranium are frequently met with, and in forms that are as various as its degrees are diverse. We have already had occasion to speak of the relations which seem to exist between the development of the cranium and that of the eyes. It often happens that the smaller eye corresponds to the less developed half of the cranium. But if we watch the form of the face and that of the head generally, in all cases, and not alone when we have to do with anisometropia, we find that very marked degrees of facial asymmetry may exist without there being a noticeable difference in the refraction of the eyes. It even happens, at times, that the less refracting eye corresponds to the more prominent side of the forehead, the more projecting cheek-bone and the larger side of the chin.

It would, however, be a mistake to deny, on that account, all relation between the development of the eyes and that of the cranium. Although Donders[1] modestly declares that he has not succeeded in discovering the laws which should govern these anomalies, it is not to be denied that they exist, nor should we despair of finding them at some future time. Here is already one point to be considered: the investigation ought not to bear solely upon the apparent development of the two halves of the face, but upon that of the entire head. It is not rare, indeed, that the relief of one-half of the face—the left, for instance—appears more marked than that of the other, which seems retiring; but when the head is looked at from above, or the outline of its circumference is taken with a *conformateur*, such as hatters use, it is found that the occiput presents the converse asymmetry, that here the right half is the more prominent; so that one has an impression as if the two halves of the head had simply been displaced relatively to each other, antero-posteriorly.

In the *second place*, the development in length, and that in width, of the two halves of the cranium, may have a distinct influence. Thus Donders says (*loc. cit.*) that, "at the side where the strongest refraction, or rather the longest visual axis occurs, the orbit (and with it the eye) is situated closer to the median line, while its surrounding edges are placed more forward."

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[1] Donders, *loc. cit.*, p. 558.

Finally, it must not be forgotten that the shape of the head is not the sole determining cause of the shape and refractive condition of the eye, but that many other factors influence these in ways different from, and even the opposite of, that of the development of the cranium.

Although it is prudent to be very reserved in one's conclusions drawn from the face of a person, as to the absolute or even relative refraction of his eyes, still an attentive observer succeeds in pretty readily distinguishing, by outward inspection of the eyeballs, which of the two possesses the stronger and which the weaker refraction. That is to say, that since we have to do almost exclusively with axial ametropia, he will easily see which is the longer and which the shorter eye. One has only to separate and push back the edges of the lids, having the patient meantime look as much as possible in the opposite direction, as we have previously described, and complete this examination by palpation.

## VISION OF ANISOMETROPES.

The vision of anisometropes may be accomplished in three ways:—

1. *Both eyes fix simultaneously, and binocular vision exists.*
2. *The anisometrope uses each eye alternately.*
3. *Vision is accomplished with one eye only, this being always the same, and the other being permanently excluded.*

1. In order to ascertain whether or not a person, who sees with both eyes, *fixes binocularly*, one has only to designate some object of fixation, and suddenly cover one and then the other of his eyes. If, at the instant when one eye is excluded from vision, the other makes a movement in order to direct itself toward the object, it is certain that it had not taken part in the fixation, and that the other alone had been fixing. It is well to make the experiment for a distant as well as for a near point.

The same end will be attained, with still more certainty, by quickly interposing a prism, with its apex toward either the temporal or nasal side, between the object of fixation and one of the eyes. If the latter immediately makes a movement of rotation toward the prism's apex, this proves that binocular fixation exists.[1]

But it was long supposed that, notwithstanding the simultaneous and precise fixation of both eyes, binocular vision,—that is to say, the stereoscopic impression, was wanting in anisometropes. Its existence is, however, easy to demonstrate. The simplest proof of it consists in Hering's experiment. The experimenter takes a tube about 30 centimetres long, covered with a diaphragm in which there is a horizontal slit 15 millimetres broad, but long enough to allow both eyes to look through it at the same time. Having the subject of the experiment fix a small, pointed object, placed vertically at some distance from the tube, this object being on the median line, small balls are dropped from a certain height, in front of and behind it. If binocular vision exists, an attentive observer will always be able to tell whether the balls fall between him and the object or on the other side of it. If, on the contrary, his answers are not in accord with the facts, one may safely conclude that he does not enjoy binocular vision.[1]

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[1] Donders, *loc. cit.*, p. 560.

By means of experiments of this sort, it has been established beyond doubt that anisometropes, who fix correctly, have stereoscopic vision. It might be supposed from this that it would be allowable for them to vary their refractive condition by means of the accommodation, using this faculty unequally in the two eyes. One would think that the weaker eye might thus raise its refraction to the level of that of the stronger, and do away with the difference between the two. Such is not the case, however, and this fact is an extremely important one to note: *an anisometrope almost always makes the same effort of accommodation on both sides.* It follows from this that he combines, in one stereoscopic impression, the distinct retinal image of the adapted eye with the diffuse image of the other.

This does not prevent the eye that is more favoured by its refraction from preserving a preponderant value in vision. If we give this anisometrope an object to *aim at,* if we ask him, for instance, to place his finger some distance in front of him, so that it shall just hide a distant point, these two points will always be found to be on the line of sight of the better of his eyes. He has only to alternately close his eyes. If he closes the eye which is imperfectly adapted, the finger will keep exactly the same position, relatively to the distant object, that it had when both eyes were open; if, on the contrary, he shuts the better eye, the finger will appear sensibly displaced toward the opposite side. So when he looks at a distance, or when he is reading,—that is to say, when binocular vision is of no great use to him,—he will scarcely notice the elimination of the poorer eye by an object which conceals the point of fixation from it, while the same obstacle would bother him considerably, at least for an instant, if placed in front of the other eye. And, if the exclusion of the former is very sensible to him when he needs binocular vision, that of the other is, however, much more so.

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[1] Donders has improved upon Hering's experiment, employing an apparatus which he calls a *Ptotostereoscope* (Van der Meulen, Thesis, Utrecht., 1873, and Landolt, *Traité complet d'ophth.*, vol. i., p. 933).

It goes without saying that, in work common to both, the value of the less favoured eye depends upon its visual acuteness, and upon the clearness and size of its retinal images, as compared with those of its congener. The two last factors are evidently in direct proportion to the difference of refraction existing between the two eyes,—that is to say, with *the degree of the anisometropia.*

In this connection, it is very interesting to observe how great the difference between the retinal images may be, without their ceasing to be united in one stereoscopic impression. A person who is emmetropic in the right eye and hyperopic by 3 D in the left, who has been willing to serve as the subject of a great many experiments on anisometropia, has perfect binocular vision, although he always puts forth a like amount of accommodation on each side. Under such circumstances, the image for his hyperopic eye is so diffuse that its acuteness of vision amounts to but 0·1, while that of the other is 1·25. We have convinced ourselves of this by suddenly covering our patient's good eye. It was necessary to wait a moment—the time required for him to bring into play his 3 D of accommodation—before the visual acuteness of his hyperopic eye was equal to 1.

But the degree of anisometropia may be still higher—6 D and more—without hindering the production of the stereoscopic impression. Von Graefe even found that an eye deprived of its crystalline could co-operate with the other in binocular vision. The difference between the two would be about 11 D. It is true that the case on which von Graefe founds this assertion was that of a child who had been operated upon for congenital cataract, and for whom it was easier to accustom itself to the fusion of such dissimilar images, than it would be for a person who had become aphakic at an advanced age. The old people, upon whom we have operated for cataract, have not seemed to us, at least, to enjoy any very certain binocular vision, even when their anisometropia was corrected.

As to the difference in size of the retinal images which may be united in a single impression, our hyperopic anisometrope has claimed to be able perfectly to fuse images whose diameter was to each other as 1 to 1·25, or as 4 to 5. This result was attained by correcting the hyperopic eye by means of weak convex glasses, which, placed at a great enough distance from the eye, markedly amplify the retinal images. This experiment is not, however, much more conclusive than if it had been made upon a person who, by optical means, had procured different images for the two eyes. Indeed, the anisometrope is in the habit of uniting a diffuse image with a clear one, and not two distinct ones of different sizes. Only, the experiment is easier for him, because it can be made with a single spherical glass, which is impossible in the case of an isometrope.

2. *The anisometrope uses either eye alternately*, especially when one of them is emmetropic, or slightly ametropic, and the other moderately myopic, and when each possesses a good visual acuteness. The former is used in distant vision, and the other for seeing near at hand. Binocular fixation sometimes exists, in such cases, but not always.

It very often happens that the anisometrope is far from being aware of the way in which his vision is accomplished, and, supposing that he has at disposal, in both eyes, an extraordinary amplitude of accommodation, he flatters himself that he escapes from the best established laws of the physiology of the eye. Donders gives an example of this kind: one of his friends, whose right eye was emmetropic and the other myopic by 7 D, used either eye alternately, according to the requirements of his visual work. At the age of twenty years, when a person generally has 10 D of amplitude of accommodation, he seemed to have 17; the *punctum remotum* of his emmetropic eye being at infinity, that of the myopic one was at $\frac{1}{7+10}=\frac{1}{17}$ of a metre, or six centimetres, instead of 10 centimetres. At the age of sixty-five years, when any emmetrope sees only at infinity, he still saw perfectly at one-seventh of a metre, the *punctum remotum* of his myopic eye. The mystery was explained in a very simple way, when one eye and then the other was successively covered, and it was thus proved to him that each eye furnished only a portion of the region of accommodation, the part assigned to it by physiology. So, if at the age of twenty years he could see distinctly anywhere between infinity and a distance of 6 centimetres, because the *punctum proximum* of the emmetropic eye was still nearer than the *punctum remotum* of the myopic one, at sixty-five years of age he ought not to have clearly distinguished anything between infinity and a distance of 14 centimetres $\left(\frac{1\text{ m.}}{7}\right)$, because this space was not included in his region of accommodation.

We have met with several similar examples, notably that of one of the most esteemed *savans* of the Paris Faculty of Medicine. With his frank and open glance, which has nothing about it of the uncertain expression of the myope, he can recognise you at a great distance; his hair is growing grey, without his becoming presbyopic, and spectacles will never disfigure his sympathetic countenance. This is because one of his eyes is emmetropic, while the other is myopic, and he has accustomed himself to use them alternately.

At another time there came to us a colleague, fifty-seven years of age, who said he had constantly worn the same convex glasses, number 2·5, for both eyes, for near and for distant vision. According to that, he should have been hyperopic. But what was strange about it, in the first place, was that he really had normal acuteness of vision for a distance, and still read at 40 centimetres $\left(\frac{1\text{ m.}}{2\cdot5}\right)$ with his glasses, which would lead us to suppose that his amplitude of accommodation amounted to 2·5 D, instead of the 1 dioptry which corresponds to his age. On examining, as usual, each eye separately, I found that there was a hyperopia of 2·5 in the left eye and a hyperopia of 1 D in the right. Hence one of his positive lenses, 2·5, corrected the hyperopia of the left eye for distant vision, while the other adapted the right for working distance.

3. Anisometropes of a third catagory *use only a single eye and always the same one,* as we have said, the other being absolutely set aside. This is especially the case when not alone the refraction but also the visual acuteness of the two eyes is very different. The poorer eye then takes the direction given to it by the relation of its form with that of the orbit, by the equilibrium of its muscles or by the synergic action of the latter with those of the better eye. This direction in the great majority of cases is abnormal; there is strabismus, either divergent or convergent. The former is the rule when the deviated eye is strongly myopic. Convergent strabismus is most often met with in hyperopes, especially in their youth. We have given the reasons for this above. Sometimes the convergent strabismus of hyperopes diminishes or disappears with age, or even changes into divergence; sometimes it asserts itself more and more by the contraction and shortening of the internal rectus.

In any case, the movements of the deviated eye only very imperfectly accompany those of its normal congener. When the former diverges, it sometimes gradually carries itself inward during the energetic convergence of the other, without, however, directing itself toward the object fixed. Having reached a certain degree, it stops, and as the convergence of the better eye goes on increasing, it yields and returns to its outward direction.

Although not serving directly in vision, and contributing in no way to binocular perception, a deviated eye may nevertheless offer some advantages in going about, by extending the field of vision on its side. It often happens, too, that the abandoned and disdained eye contains a treasure of vision, whose existence its possessor does not suspect. Although many an anisometrope is unaware that he sometimes sees with one eye and sometimes with the other, many another supposes one eye to be blind and good for nothing, simply because he does not know how to use it. This is especially the case when the excluded eye is strongly myopic. The patient is then very much astonished when, by bringing the book up very near to him, he is made to read with the eye "with which he has never seen."

At other times, on the contrary, it is the correction of a strong hyperopia, or of a formidable astigmatism, which restores its usefulness to an eye thus neglected. It is generally too late to re-establish at the same time binocular or even alternate vision. This is not regenerated even when the direction of the eye is corrected by surgical means. But the service thus rendered to the individual is none the less great; if an accident should ever happen to the good eye, the neglected one will then appear to him like a veritable saviour.

It is unnecessary to cite examples. Each of us has had, and will again have, occasion to observe them, especially in myopes, who are so subject to all kinds of accidents; hæmorrhage into the macula, retinal detachments, &c. It is useless, too, consequently, to insist upon the importance of a careful examination of each eye by itself, even though the oculist should thereby prove himself "more of a royalist than the king," by trying to make an eye see in spite of the indifference of the patient, who has long since ceased to mourn its loss.

## TREATMENT OF ANISOMETROPIA.

Although it is our duty to conscientiously examine the refraction of each eye, and try every means of procuring for each one its maximum of visual acuteness, our advice, not to allow one's self to be led into optical polytherapy, is, however, nowhere more applicable than when we have to do with anisometropia.

Knowing, on the one hand, that the stereoscopic impression is vivid, and binocular vision precise, in proportion as the retinal images of each eye are distinct, and, on the other hand, that it is impossible for the anisometrope to correct his inequality of refraction by a different effort of accommodation for each eye, one might be tempted to re-establish, by means of correcting-glasses, the identity of the adaptation of the eyes. If we are dealing with a person, for instance, who has an emmetropic left eye and a myopia of 3 D in the right eye, it seems perfectly logical to correct the myopia of this eye by a concave 3, in order to equalise the extent of the view for both eyes when looking at a distance; and to give the emmetropic eye the convex 3 for near work, in order to adapt it for the same point with its myopic congener. By acting on this plan, one would, however, run absolutely counter to success in the vast majority of cases. *An anisometrope endowed with binocular vision*—and in our example it can be a question of no other category—will readily admit that he sees better with his eye corrected than when it is not, but he will still more energetically declare that he much prefers binocular vision without the glass to the same with the correction; "that the latter occasions him considerable and insurmountable annoyance, that he is dazzled, that his head swims, that objects dance before his eyes, that the spectacles tire him to the utmost and that he could never accustom himself to them."

There exist cases, indeed, in which the anisometrope is unable to bear the least modification of the difference between his eyes. Hence we are obliged to let it subsist. If one of the eyes is emmetropic and the other ametropic, we shall abstain from prescribing glasses for distant vision, unless the ametropic eye is the favoured one, which will, however, rarely be the case where binocular vision exists.

For near vision we are to be governed by the eye which the patient prefers. We will have him read, for instance, with both eyes open, and then suddenly cover one after the other. The eye that the patient uses more readily is, evidently, the one whose exclusion causes him appreciable annoyance. He stops reading, holds the book nearer or farther off, or at least waits a moment until he has supplied, by an effort of accommodation, what the refractive power of his eye lacks. This is what occurs when the patient habitually uses his more refractive eye—the emmetropic one, for example, when the other is hyperopic. In such a case we should give him, for each side, the glass required by the emmetropic eye.

If one of the eyes is myopic and specially charged with near vision, we shall rarely be obliged to prescribe spectacle-glasses for work. It will be done only in inferior degrees of myopia, when the accommodation is weak, or in exceptional cases of very strong myopia.

The choice of glasses, for anisometropes who do not bear any correction of their difference in refraction, is very simple as soon as we know which eye is the more used. This is the only one we take into account, and the rules to be applied to it are those which have been formulated above. The same number is given for the other eye, so that the anisometropia itself is not modified.

At other times the patient tolerates *a certain degree*, but not the whole, of the correction. If the difference in refraction is one of 5 D, it may be reduced, for instance, to 2 D, by means of a concave 3 placed before the more refractive eye, or the convex 3 placed before the other. The rest of the interval is not filled up by the accommodation. The patient still receives rather indistinct images in his imperfectly corrected eye, but yet clearer ones than with the naked eye. Is it, then, always proper to correct the corrigible portion of the anisometropia?

It might be thought that the answer to this question could not be doubtful, and that, inasmuch as the patient bears the glasses and declares that he sees better with their help, there ought to be no hesitation about prescribing them for him. There are cases, however, in which we hesitate. The reader is aware, by this time, that we are not fond of burdening patients with spectacles when they are able to do without them. Now, a person who is emmetropic, and endowed with good visual acuteness in one eye, and ametropic in the other, seldom needs the little help that the partial correction of this eye can procure for him. His vision will be a little clearer, but not much, and the sensation of relief, of very distant objects, does not depend upon binocular vision. Hence he would have purchased a slight advantage at the price of the inconveniences attending the wearing of glasses.

It will be objected, however, that this is at times indicated for other reasons—for instance, to protect the eyes against dust, excessive light, &c.—and that, under other circumstances, it entails only the slightest inconvenience, as in near vision. It would, therefore, seem preferable to reserve the correction of anisometropia for special cases, and to procure for the patient the benefit of glasses only under circumstances in which they are in nowise disadvantageous to him, without imposing upon him their continuous use.

Thus, for an anisometrope who is emmetropic in the right and hyperopic by 3 D in the left eye, and who sees better with the convex 2, in front of the latter eye, than he does without any glass, we would give this number to be worn in his protective glasses, and in those for working. He would then use it only incidentally.

But his correction accustoms him to retinal images of some distinctness in his hyperopic eye. He will be less and less satisfied with such diffuse images as he had before, by which, however, he knew how to profit. He will become more and more dependent upon his spectacles, and if he leaves them off, will incur a sort of asthenopia, due to the fruitless efforts of the hyperopic eye to procure, by accommodation, the relatively distinct images for which it has acquired a taste. Anisometropes of this kind are perfectly aware that, at the moment when they take off their correcting spectacles, something disagreeable happens in the less refractive eye; they feel a sensation of cramp, or contraction, which constrains them to close the eye or to rub it; they are obliged to stop looking at anything for a little while, on pain of being annoyed, dazzled and fatigued. This is a struggle for clear images.

We are not to be surprised if, impelled by this desire, a young eye, wishing to see again, with the usual distinctness, and at any cost, some particular object, discovers the forbidden artifice of strabismus. This distinctness requires an effort of accommodation equal to the degree of anisometropia corrected by the spectacles. It can be furnished only at the expense of an equivalent convergence, *i.e.*, convergence exaggerated relatively to the distance of the object. This excess of convergence will be imposed upon the more refractive eye, so long as the other remains master of fixation; but having once become conscious of the sensation of convergent strabismus, the individual will produce it with greater and greater facility, and, if the more refractive eye were inferior to the other, in point of visual acuteness, the strabismus might become permanent, at least during fixation.

The same precautions are necessarily indicated when the patient bears the correction of not only a portion, but of the whole of his anisometropia. We are always to ask ourselves if the advantage of this correction may not be compensated by inconveniences greater than the imperfection of retinal images in an eye which takes a useful part in binocular vision.

Let us hasten to add, however, that, if a person is obliged, for any other reason than that of being anisometropic, to wear spectacles, there is almost always an advantage in equalising the refraction of his eyes as much as he finds agreeable.

Sometimes this relative or absolute equalisation is imperatively demanded. Anisometropia which, in the majority of cases, is borne without any disagreeable subjective phenomenon, at times becomes the cause of an extremely troublesome asthenopia. This case, although being the exception, presents itself, nevertheless, not very rarely. Thus, without looking through our case-records, we cite a few examples of it, which have come under our notice during the preparation of these pages.

The first concerns a young man who has been obliged, for several years, to give up his studies, because, as soon as he reads or writes, his right eye becomes tired and gives him "frightful pains in the head." I find, in the left, M 3, V = 1; right, H 1, V = 1. The refraction, as determined with the ophthalmoscope, does not differ from that found subjectively. Binocular vision exists, otherwise his anisometropia would not trouble him.

The patient does not bear the correction of his anisometropia, but is entirely relieved, for distant vision, by the concave 2 before his myopic eye. He derives still more advantage from the convex 1·5, with which we furnish his hyperopic eye for work. He is very much pleased, and is immediately able to resume his occupations.

This case is interesting, as showing the capriciousness of anisometropia toward correction. We tried all sorts of combinations, gradually diminishing the number from full correction until the patient said that vision was agreeable to him and, as is seen, we were obliged to leave an anisometropia of 2 D for distant, and of 2·5 for near vision.

The same day, I saw a young lady, an artist, who had been sent to me, the preceding year, for advice concerning an extreme asthenopia, which had been unsuccessfully treated in France and abroad, and which prevented her engaging in her work as a sculptress. She had, in the left, a myopia of 7 D; the acuteness of vision, equal to 0·4, was, by the correction of an astigmatism of 1 D, raised to 0·7. In the right there was an apparent myopia of 3 D, V = 0·6.

The myopia of the right turned out to be due to a spasm of accommodation. It diminished to 1 D, under the influence of a course of atropinisation, which we instituted in both eyes. The patient then accepted with pleasure the concave spherical 4, with the cylinder number 1, for the left eye, and the concave 1 for the right,—that is to say, the correction of 3 of her 6 dioptries of anisometropia. This year we have even tried to do away with the concave 1 for the right eye, *i.e.*, to neutralise one more dioptry of the difference in refraction between the two eyes, at the same time facilitating the near vision of the patient, whose accommodation might soon demand some aid. She was very well satisfied with her new spectacles.

It is true that, in this case, the anisometropia was not the sole cause of asthenopia; there was some astigmatism, which had previously been left uncorrected, and the accommodative spasm of the left eye. But the latter was, perhaps, attributable to the difference of refraction in the eyes, though unequal contraction of the ciliary muscles, in anisometropia, is quite exceptional.

Three months ago I was consulted by a young merchant, who complained of having been, for some time, considerably annoyed in his business by fatigue of his left eye, which made it necessary for him to close that eye during near work. I found, in the left, a hyperopia of 1 D, V = 1; the right was emmetropic, V = 1: with the ophthalmoscope—left H 1·5, right H 0·5. The patient very gladly accepted the full correction of his anisometropia—that is to say, the convex 1 for his left eye. Since that time he has had no asthenopia.

Let us cite, on the other hand, an absolutely opposite observation which we made a week ago. A boy, twelve years old, complains of asthenopia. His father is anisometropic, and presents a marked difference in the development of the two halves of his face. The son himself exhibits a still more noticeable asymmetry of the face. The arrested development of the left side is marked. There is, however, but a slight anisometropia: left M 0·5, V = 1; right M 1·5, V = 1; no astigmatism. With the ophthalmoscope, the left eye shows itself to be hyperopic by 1 D, and the right by 0·5 D. Suspecting that the asthenopia might arise from the anisometropia, we tried to remedy it by placing the convex number 0·5 before the left eye. Weak as this glass was, the patient could not bear it a single moment, and we shall be obliged to have recourse to treatment with atropine, which will doubtless give us a satisfactory result.

These few observations, taken at hazard, demonstrate how much there is of the unforeseen in the correction of anisometropia: one case calls for total correction, another rejects it utterly; a third patient demands a partial correction, in order that he may be able to use both eyes, and a fourth patient gets used to his anisometropia, without, however, disdaining optical help, which diminishes it somewhat.

All these difficulties are scarcely met with in the cases of *anisometropes who do not enjoy binocular vision.* The eyes are independent of each other; each may be treated by itself, due regard being had for the needs growing out of their special uses. The attempt to equalise them, and make them work together, would almost surely be fruitless and contrary to the interests of the patient.

Thus anisometropes who use one eye for seeing at a distance and the other for near vision, would hardly thank us for adapting both their eyes to any one distance, long or short. In the first place, not having learned to fuse clear images of both retinæ, they would experience a feeling of extreme perplexity and would reject the spectacles from the outset. Or, if we succeeded, by persistent training, in getting the patients used to them, they would think themselves worse off than before, just because they were obliged to wear glasses, while they could formerly dispense with them, or to use an extra pair of glasses.

In fact, if the individual has learned to spontaneously dissociate his eyes—to use one eye for seeing at a distance and the other for his work—this is just because the difference in refraction between the two organs is equivalent to that required by the isometrope of his accommodation, or of spectacle-glasses, when he wishes to vary the extent of his sight from infinity to the distance of his occupations. An anisometrope has, therefore, the great advantage of being able to dispense with this muscular effort, or this optical aid, and of possessing an extraordinarily extensive amplitude of adaptation, owing to the fact that each eye dominates a different space, and that the length of their regions are added to each other, instead of being parallel. We necessarily reduce this amplitude by bringing, artificially, the refraction of both eyes to the same level.

Therefore, when the difference in refraction of such an anisometrope's eyes is just 3 to 4 D, it is better to let them alone; if one of the eyes is emmetropic and the other myopic by 4 D, the patient will always see at a distance and will read without spectacles.

If both eyes are hyperopic, and the more hyperopic one is used for distant vision, we give each the same convex glass,—the correcting-glass of the strongest hyperopia, according to the example furnished by the anisometrope cited above. If the hyperopia of one of the eyes equals 4 and that of the other 1 D, we shall prescribe the convex 4 for each. The first eye will see at a distance and the other near at hand. The converse case will not present itself; the less refractive eye will never be found to serve for near vision, if both have the same visual acuteness.

If both eyes are myopic, we may try giving each the correcting-glass of the weaker of them. Suppose, for instance, the myopia of one to be 2 D and that of the other 5 D; then the former will see at a distance, and the latter at 33 centimetres, with the concave 2. In such a case, however, the patient will probably prefer not to use his glasses at all for work. He will therefore reserve his concave 2 for distant vision.

It may, however, happen that one is obliged to give different glasses to the two eyes, in this class of anisometropes. Let us suppose a person who is emmetropic in the left, myopic by 4 D in the right eye, and sixty years old. He wants to see clearly at 50 centimetres $\left(\frac{1^{m.}}{2}\right)$, *i.e.*, give his eyes a positive refraction of 2 dioptries. Neither the left eye nor the right can furnish this amount; the former because it no longer has more than 1 D of accommodation, the second because it possesses no means of diminishing its refraction. We are, therefore, limited to these two alternatives: to give a convex glass (a little stronger than 1 D) to the emmetropic, or the concave 2 to the myopic eye. We shall try one of these combinations after the other, leaving the choice to the patient. It cannot be foreseen to which of these glasses he will accord the preference, but, in any case, he would not be able to accommodate himself to the simultaneous correction of both eyes, nor will he want the same convex or concave glass for each. We should have artificially diminished the anisometropia by a certain degree.

The same necessity may present itself when the difference in refraction between the two eyes is a little less than is necessary in order to carry the adaptation from infinity to the working-distance; for example, if the hyperopia of one eye equals 1, and that of the other 2 D. As long as he has a perfect amplitude of accommodation at disposal, the anisometrope will find the convex 2 sufficient to make work, with the less hyperopic eye, easy; but if, with age, this eye calls for a convex 3, we could not impose the same number on the other, because the static refraction—the degree of hyperopia—will not have changed. Hence we shall be obliged to leave the convex 2, and substitute, for the glass of the other eye, the convex 3, which will be replaced, later on, by a still stronger one. This difference between the correcting-glasses will, moreover, be easily borne by our anisometrope, simply because the eyes are independent of each other.

This is also the reason why we need not multiply the number of instances. One is to be guided, in the adaptation of each eye, by the rules before given, including the advice not to neglect the sensations of the patient, and one will thus safely accomplish the end in view. The only risk of being deceived will be in wishing to improve upon Nature, and forcing the disunited couple to resume the common labour.

When *the anisometrope uses one eye exclusively*, this is the one to be corrected, in case of need, according to the well-known principles. The other generally being far inferior in every respect, not accustomed to fix and even deviated, the question of bringing it back into the path of binocular vision, by an operation for strabismus, by the partial or total correction of its ametropia, or isolated visual or by stereoscopic exercises, could only very exceptionally arise.

We do not mean, by that, that an eye thus abandoned deserves no attention. Although it can rarely be utilised in binocular vision, it may sometimes render estimable service, as, for instance, when its fellow is the victim of an accident. Only, in order that it may always be ready to enter upon the performance of its function, it is well to train it and accustom it to seeing. One may succeed in this, as we have already said, by means of exercise, which will be especially facilitated by the correction of astigmatism, by giving concave glasses to myopes, convexes to hyperopes and even to emmetropes for near vision, and one will seldom have reason to regret the patient and attentive care, and the persevering solicitude in behalf of the disgraced eye.

# ANOMALIES OF ACCOMMODATION.

We have shown, on pages 143-166, that accommodation is, at once, a function of *the ciliary muscle and of the elasticity of the crystalline.* The anomalies of accommodation must, therefore, find their origin in alterations of one or the other of these two factors, or of both together.

The diminution of elasticity, to which the crystalline is subject from early childhood on, occupied our attention through one entire chapter (page 174 *et seq.*). To this is due the gradual decrease in the amplitude of accommodation, and the complete extinction of this function toward the age of seventy-five years.

If the crystalline becomes rigid from any other cause than senility, the effect upon the accommodation must be the same. Cases of this kind seem, however, to be rare. It is probable that the lens becomes inelastic when it becomes opaque. But the change in structure which produces cataract is accompanied by so intense visual disturbances, that the diminution in the amplitude of accommodation disappears in comparison with them.

It oftener happens that the crystalline is withdrawn, by subluxation, from the action of the ciliary muscle. It still occupies the pupillary space, but its attachment with the suspensory ligament, the zone of Zinn, being ruptured, at least in part, it assumes its most convex form, brings the refraction of the eye to the maximum, and, at the same time, renders any modification impossible. Accommodation is as thoroughly abolished as if the crystalline no longer existed, as in *aphakia.*

Changes in the dynamic refraction of the eye, which depend upon the crystalline, are, however, much less frequent and less important to be acquainted with than those resulting from irregularities in the working of the ciliary muscle. The latter are as multiple as complex, and often even only imperfectly explained.

As in the pathological working of every muscular organ, we have here to consider *spasm, weakening, paresis and paralysis of the ciliary muscle.*

But, before entering upon this discussion, we ask permission to briefly recall a few anatomical and physiological facts relating to the accommodative apparatus of the eye.

The innervation of Bruecke's or of the ciliary muscle is furnished, as Hensen and Voelckers[1] have demonstrated, by the branches of the

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[1] Hensen and Voelckers, *Experimentaluntersuchungen über den Mechanismus der Accommodation:* Kiel, 1868.

*ciliary ganglion.* As they also innervate the iris, and since the movements of this diaphragm are intimately associated with those of accommodation, and since, moreover, the physiology of the ciliary ganglion has scarcely been studied except with reference to the pupillary movements, we will recapitulate, in a few words, the action of this ganglion upon the iris.

The *ciliary ganglion* sends to the muscle of accommodation and to the iridian diaphragm some fifteen nerve branches, the *short ciliary nerves.* They perforate the sclerotic in the vicinity of the optic nerve, pass along between this membrane and the choroid, and, having reached the anterior portion of the eye, spread out into a plexus of fine fibrillæ, which lose themselves in the ciliary muscle, and the sphincter and radiating muscle of the iris.

The afferent branches of the ciliary ganglion arise from three sources:—1. From the *common motor oculi* (motor root); 2. From the *trigeminus,* through the intermediation of the *naso-ciliaris* (sensory root); 3. From the *great sympathetic* (vegetative root).

The *first* of these roots appears to hold under its sole dependence the working of the *accommodative muscle* (Trautvetter),[1] and of the sphincter of the iris. H. Mayo's[2] experiments have, indeed, already indubitably proved that the irritation of this nerve produces a contraction of the pupil. Pathology demonstrates, too, that paralysis of accommodation, as well as the mydriasis which often accompanies it, is due to lesions of the third pair or of the motor root which it gives to the ciliary ganglion.

The *vegetative* root, emanating from the great intracranial sympathetic, controls the *dilator of the pupillary orifice* (Petit,[3] Budge, and Waller[4]), and the vessels of this region. It was long ago established that the experimental or pathological irritation of the great sympathetic, in the neck, provokes mydriasis, and that the section or paralysis of the same nerve entails the preponderance of the pupillary sphincter.

As to the root furnished by the *trigeminus* it very probably contains only *sensory fibres,* although Magendie,[5] Claude Bernard[6] and others have seen produced, in consequence of the isolated irritation of this nerve, a constant contraction of the pupil, of which nobody has yet been able to give a satisfactory explanation.[7]

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[1] Trautvetter, *Arch. f. Ophth.*, xii., 1, p. 65, 1866.

[2] H. Mayo, *Anatom. and Physiolog. Commentaries*, No. 2: London, 1823.

[3] J. L. Petit, *Mémoirs de l'Acad. des Sciences*, p. 1: Paris, 1827.

[4] Budge and Waller, *Comptes rendus*, t. xxxiv. and xxxv., *passim*, 1852.

[5] Magendie, *Journal de Physiol.*, t. iv., p. 176, 1824.

[6] Cl. Bernard, *Leçons sur la Physiol. et la Pathol. du système nerveux.*

[7] See Donders, *loc. cit.*, pp. 580-583.

The contraction of the ciliary muscle and that of the sphincter of the pupil are almost always simultaneously exerted, while relaxation of the accommodation is usually accompanied by dilatation of the pupil. These two acts are not, however, indissolubly associated with each other. We have only to recall the fact that the pupil contracts and dilates under the influence of variations in illumination, without the accommodation undergoing any change. The reflex movement of the iris remains intact, not only when the accommodation is abolished entirely by the rigidity of the crystalline, but also in many cases of paralysis of the ciliary muscle.

Inversely, immobility of the pupil, under the influence of light as well as under that of accommodation, has been observed when the ciliary muscle worked normally.

Toxic substances which act upon the pupillary sphincter generally have a similar action on the accommodation: *mydriatics*, which dilate the pupil, paralyse the accommodation, while *myotics*, which contract the former, make the ciliary muscle contract likewise. It is, however, to be noticed that the effects of these medicaments upon the two functions is neither synchronous nor equal in intensity. Thus we shall see that, under the influence of certain mydriatics, the accommodation commences to diminish somewhat later than the beginning of the pupillary dilatation, and that the latter attains its maximum long before the paralysis of the ciliary muscle. Some have a much more pronounced action upon one than upon the other of the two functions, as, for instance, muscarine, which promptly provokes an accommodative spasm, while it has less power over the sphincter of the pupil.

Changes in the accommodation are, therefore, generally, though not always, accompanied by modifications in the diameter and mobility of the pupil; *spasm of accommodation* coincides with *myosis*, and *paralysis* with *mydriasis*. In these two conditions the pupil presents a diminution of contractility, or even *complete immobility under the accommodative impulsion*. Nevertheless, the iris reacts, in a certain degree, to variations of illumination, unless there be complete paralysis of the ciliary fibres of the third pair. In ataxia, the contrary is observable: the pupils, contracted under the influence of the preponderant tonicity of the sphincter, which is no longer compensated by the action of the dilator fibres, cease to respond to luminous excitation but still contract under the influence of the accommodation.

The capital symptom of an anomaly of accommodation is the *change in position of the punctum proximum* or of the *punctum remotum*, as well as the visual disturbances resulting from it.

Non-artificial spasm of accommodation, *i.e.*, that which is not provoked by myotics, brings the *punctum remotum* nearer, without altering the position of the *punctum proximum*. The latter never comes closer to the eye than the position of the absolute *punctum proximum*.

Paresis of the accommodation causes recession of the *punctum proximum*, without influencing the *punctum remotum*.

Hence both affections reduce the amplitude of accommodation, the spasm at its farther extremity, and paresis at its nearer.

Now accommodation is in inverse ratio to age and, although this relation is not absolute and mathematical, it nevertheless admits of less latitude than most physiological laws. Donders' table (p. 174) corresponds, with almost surprising accuracy, to reality, so that, whenever the amplitude of accommodation is less than that required by Donders' law, there is reason to seek some pathological cause of its restriction.

The differential diagnosis between spasm and paresis of accommodation will generally be facilitated by the other symptoms accompanying them.

## SPASM OF ACCOMMODATION.

A spasm of accommodation may be either *clonic* or *tonic*. In the former case, it is produced only under the influence of fixation, convergence, a desire for distinct vision, or of certain causes which excite the sensibility of the eye, while it ceases as soon as the organ is in repose. In the second case, the ciliary cramp is permanent and yields only to mydriatics.

We have already become acquainted with a very common form of spasm of accommodation. It is the one that increases the refraction of nearly all young persons and makes it appear stronger than it really is. This spasmodic contraction of the ciliary muscle is, in most cases, inoffensive. It ceases at the same time with fixation, and disappears with age. At times, however, it causes serious asthenopia, and some authors even ascribe to it, as will be remembered, an active part in the production of myopia.

We have already discussed this ciliary spasm, as well as its therapeusis, in the place mentioned. Hence we shall not return to a consideration of it at present. We shall add only that this affection, which is nearly always binocular, may, however, manifest itself in a single eye, or affect one eye more than the other. The visual disturbances resulting from it are then more troublesome than when it exists on both sides.

The treatment is necessarily identical with that of binocular spasm.

We even advise the simultaneous use of the mydriatic in both eyes, and that they be allowed to rest for a few weeks.

Three months ago we saw a girl, ten years old, who belonged to a family which is myopic on both the father's and mother's side. Her father, now dead, is said to have been strongly myopic in the left eye. The child complained of rather vague symptoms of asthenopia. She showed, in the left, a myopia of 3 D; V = 1; in the right, emmetropia; V = 1. There was no asymmetry of the face nor any difference in the aspect of the optic papillæ. With the ophthalmoscope, each eye proved to be hyperopic by 1·5 D. We instituted a course of atropine. At the end of a month, there was, on either side, a hyperopia of 1·5; V = 1. After the use of the mydriatic was discontinued, refraction and accommodation showed themselves to be equal on both sides; a portion of the hyperopia remained manifest and facultative; the asthenopia had absolutely disappeared, as well as the troublesome tendency to bring objects too near the eyes.

We have cited, when on the topic of anisometropia, another case of unilateral spasm of accommodation, likewise producing strong asthenopia, cured by a course of atropine and the partial neutralisation of the difference in refraction between the two eyes.

Let us bear in mind, too, the abnormal contraction of the ciliary muscle, which may produce an apparent regular astigmatism, or conceal a real one (p. 301 *et seq.*) The former yields to mydriatics, and is radically cured, whereas the latter becomes manifest, especially when a correcting-glass supplements the treatment with the collyrium.

In the forms of accommodative spasms already cited, the pupil generally presents nothing worthy of special notice. It seems to react normally in all respects. Its diameter does not appear to be influenced by the contraction of the ciliary muscle; even in cases of monocular spasm, we have found no difference between the pupils. Nevertheless, the diagnosis is very easy, because the clonic spasm ceases during the examination with the ophthalmoscope. Thus we ascertain directly that the static refraction has been increased, during the subjective examination, by a given quantity of dynamic refraction.

When it is a question of *tonic spasm* of accommodation, this diagnosis is not so simple, because the real condition of the refraction is brought out only under the influence of a mydriatic, which paralyses the ciliary muscle. The diminution in the amplitude of accommodation, which accompanies the tonic spasm, could not, however, be mistaken for that produced by paresis, since, in the former case, the pupil is rather contracted, while it is dilated in the other; and the anomalies in its size hardly escape the observer, however variable the size may be in a physiological condition.

Patients, moreover, generally complain of a very disagreeable sensation of constriction in the affected organ—a sensation which is often accompanied by a feeling of discomfort in the entire region of the eyeball, or even by a cephalalgia. When the spasm is due to an irritation of the sensory nerves of the eye, a hypersecretion of tears is often observable. The acuteness of vision is nearly always diminished and, above all, very variable. In some cases we find a narrowing of the field of vision. Sometimes there exists a tendency to convergence, which may amount to genuine strabismus. This phenomenon does not astonish us, when we take into account the intimate relations existing between the contraction of the ciliary muscle and that of the internal recti.

Idiopathic tonic spasm of accommodation is rather rare. Donders cites, under the head of "painful accommodation,"[1] two observations in which any effort of accommodation was accompanied by such pain in the eye that all work became impossible. Both patients were hyperopes. The cure was brought about by a course of atropine continued several months.

In a case published by von Graefe,[2] the spasm of accommodation was accompanied by a painful cramp of the orbicularis; in another, it was due to a superficial lesion (a scratch with the finger-nail) of the cornea.

It has been observed, moreover, as a consequence of *contusion of the eyeball;*[3] sometimes it accompanies *inflammation of the ciliary body*, notably that due to sympathetic ophthalmia.[4]

It is difficult to give an exact idea of the mechanism according to which local causes may determine contraction of the ciliary muscle. This is either the result of a lesion of the muscular tissue, or it is produced by direct or indirect irritation of the motor nerves.

*Reflex irritation of the motor branches of the ciliary ganglion* is the only admissible hypothesis in cases of spasm co-existing with affections which have no other immediate relation with the ciliary muscle: of this kind is the accommodative spasm which Samelsohn is disposed to place under the influence of *external inflammation of the eye*, as conjunctivitis, keratitis, episcleritis and even blepharitis (Nagel).[5] This spasm, which may increase the refraction of the eye by as much as 4·5 D, seems to be, for the greater number of cases, clonic, *i.e.*, to cease during the ophthalmoscopic examination, while a tonic cramp of the ciliary muscle, concurrent with *blepharospasm*, has been observed by the same author, as also by Hagedorn.[1]

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[1] Donders, *loc. cit.*, p. 622.

[2] Von Graefe, *Arch. f. Ophth.*, ii., 2, p. 308, 1856.

[3] Voelckers, Berlin, Steffen, Seligmüller, *Klin. Monatsbl.*, p. 124, 1875. Just, *Klin, Monatsbl.*, p. 226, 1872. Leber, Dehenne, *Ann. d'ocul.*, t. lxxxi., p. 180, 1870.

[4] Lüders, Würzburg Thesis, 1871. Cuignet, *Recueil d'ophth.*, p. 198, 1878. Reich, *Ann d'ocul.*, t. lxxv., p. 13, 1876.

[5] Samelsohn, *Soc. ophth. de Heidelberg*, 1878. Berthold, *Deutsche med. Wochenschr.*, p. 430, 1880.

We may also range under this heading the curious cases described by A. Graefe and Leber,[2] in which the cramp of the ciliary muscle was only one of the phenomena of a more generalised *spasmodic affection*, extending, in the first case, to the muscles of the lids and the internal recti, in the second to the entire face. Compression of the supra-orbital and infra-orbital nerves caused cessation of the tonic spasm.

We might call attention here to the spasm met with in what is called *hyperæsthesia of the retina*, and in the cases of persons who have *overworked their eyes*, who are probably, besides this, affected with some refractive anomaly.[3]

The synergy which exists between accommodation and convergence, and which necessitates the admission of a common centre of innervation, readily accounts for the spasm of accommodation observed in certain cases of *insufficiency of the internal recti.*[4] The obstacles to their working, which these muscles encounter, or the inferiority of their structure, provokes an exaggerated nervous impulsion, which manifests itself in the ciliary muscle by a pathological increase in its tonicity.

Like all muscular organs, the ciliary muscle is subject to the *contractions which are symptomatic of affections of the central nervous system.* The serious functional disorders generally entailed by the latter, and the slight attention paid by neuro-pathologists to accommodative disturbances in cerebro-spinal diseases, are to blame for the fact that our knowledge of this subject is still very limited. The same is true of the principal causes of symptomatic contractions, viz., *meningitis* and *meningo-encephalitis.* These affections are generally accompanied, in their first stage, by a marked narrowing of the pupil and spasmodic strabismus, phenomena which are due to irritation of the trunk of the common *motor oculi.* It is probable that the ciliary muscle participates in this pathological excitation, although this has not yet been indubitably proved.

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[1] Hagedorn, Halle Thesis, 1872.

[2] A. Graefe, *Arch. f. Ophth.*, xvi., p. 90, 1870. Leber, *Arch. f. Ophth.*, xxvi., 2 p. 249, 1880.

[3] Webster, "Observations in Dr. Agnew's Clinic," *Arch. of Ophth. and Otol.*, vol. iv., Nos. 3 and 4, 1875. Van Santen, *Weekbl. van het Nederl. tijdschr. voor Geneesk.*, No. 15, p. 209, 1875. Cuignet, *Rec. d'ophth.*, p. 24, 1873. Weiss, *Klin. Monatsbl.*, p. 124, 1875. Manz, *Berliner Klin. Wochenschr.*, pp. 2-3, 1880. Pagenstecher, *Klin. Monatsbl.*, p. 41, 1871. Block, *Klin. Monatsbl.*, p. 155, 1875.

[4] A. Graefe and Hagedorn, *loc. cit.*

Knies[1] has observed a clonic spasm of the ciliary muscle in the case of an *epileptic*. Gradle[2] has seen it in connection with hemiplegia. Heidenhain and Gruetzner[3] have found that the first act of *hypnotisation* is opened by an accommodative spasm. Rumpf[4] and Cohn[5] have also observed it in such cases.

A certain number of conclusive observations, taken from the works of competent observers, show us that the *cervical sympathetic*, although excluded from the direct innervation of the ciliary muscle, may nevertheless, in a pathological condition, be the cause of a prolonged increase in refraction, due to the more considerable bulging of the surfaces of the crystalline. Is this change of form simply passive and a consequence of the diminution of intra-ocular tension which generally accompanies paralysis of the sympathetic? Or is it, on the contrary, ascribable to an exaggeration of the tonicity of the muscle of accommodation, from some unexplained cause? These questions have not yet been satisfactorily answered, notwithstanding the careful study of cases of this kind by Jany,[6] Fraenkel[7] and Schliephake.[8] It was almost always a question of paresis of the sympathetics, with its characteristic oculo-pupillary phenomena.

The pathogeny of the accommodative cramp is still more obscure in the case of Stilling's patient,[9] wherein it constituted the sole symptom of an *intermittent fever*. The miasmatic poison had doubtless, by a caprice of localisation which is customary with it, fixed itself upon some region of the nervous system (medulla, ciliary ganglion), whence it determined, in a direct or reflex manner, the contraction of the ciliary muscle.

Let us cite, finally, a very interesting observation which Nagel communicated, in 1871, to the Heidelberg Ophthalmological Society.

A boy twelve years old suddenly acquired, as the result of a very severe cold in the head, a myopia of 5·2 D. At the same time there was a violent cephalalgia, a continual tendency to convergence and a very pronounced convergent strabismus with each effort at fixation. A collyrium of atropine caused the apparent myopia to entirely disappear, it being succeeded by a slight hyperopia. The other symptoms likewise ceased; but they all manifested themselves anew when, at the end of a few weeks, the use of atropine was discontinued. They yielded to the application of an artificial leech to the temple. A recurrence of the same trouble, which occurred six months afterward, was radically cured by injections of strychnine.

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[1] Knies, *Ophthalmologenversammlung in Heidelberg*, 1877.

[2] Gradle, *Journal of Nervous and Mental Diseases*, New York, p. 164, 1881.

[3] Heidenhain, *Breslauer aerzt. Zeitschr.*, March 13, 1880.

[4] Rumpf, *Deutsche med. Wochenschr.*, p. 279, 1880.

[5] Cohn, *Breslauer aerzt. Zeitschr.*, No. 6, 1880.

[6] Jany, *Berl. klin. Wochenschr.* (gôitre compressing one of the sympathetic nerves and producing cephalalgia, myosis and spasm of accommodation on the same side).

[7] Fraenkel (Breslau Thesis, 1874). Migraine ascribable to paralysis of the great sympathetic, blepharospasm, myosis and cramp of accommodation—all unilateral.

[8] Schliephake, *Arch. f. Augen- u. Ohrenheilk.*, v., p. 286, 1876. Paralysis of the cilio-spinal centre, which determines the myosis of *ataxic* patients, is only exceptionally accompanied by normal tension of the accommodation (Hempel, Coppez).

[9] Stilling, *Klin. Monatsbl.*, xiii., pp. 5-30, 1875.

## TREATMENT OF SPASM OF ACCOMMODATION.

We know that mydriatic collyria and resting the eyes are trustworthy remedies with which to overcome idiopathic spasm of accommodation.

They will evidently be insufficient when the cramp of the ciliary muscle is directly or indirectly associated with a local or general cause. In such cases their use must be accompanied by treatment of the fundamental affection. Inflammations of the external organs and membranes of the eyes are to be treated with the greatest care. An overtaxed retina will call for absolute repose, protection, by means of smoked glasses, against all sorts of bright light, and even demand that the patient remain, for a time, in total darkness. When there is present, at the same time, hyperæmia of the retina, and the patient's general condition does not render them impracticable, local depletions from the temples may be very useful,—as we have seen, for instance, in the observation cited by Nagel. According to the same author, hypodermic injections of strychnine are sometimes efficacious.

Insufficiency of the internal recti is to be treated according to the rules which we have given on page 502.

An eye which is likely to give rise to spasm of accommodation, from sympathetic ophthalmia, is to be immediately enucleated.

The general diseases and the affections of the nervous system which may be concerned, will furnish the principal therapeutic indication. The latter will naturally vary according to the nature of the case, according to the seat of the lesion to be combated and according to the constitution of the patient affected with it. So it is impossible for us to enter into the details of the treatment applicable to each particular category. Pagenstecher (*loc. cit.*) tells of radically curing a case of spasm of accommodation, due to a condition of spinal irritation, by means of the constant current, applied along the vertebral column. Leber also obtained, in his patient, a speedy recovery by using morphine in subcutaneous injections.

## MYOTICS.

Spasm of the ciliary muscle may be *artificially* produced by means of certain substances, called *myotics*, because they have, at the same time, the further effect of narrowing the pupil or producing *myosis.*

The most powerful myotics, and those most used in practice, come from the *Calabar bean* (*Faba calabarica,* or *Semen physostigmatis*), the fruit of the *Physostigma venenosum* (Balfour). This plant, a species of Phaseolus, belongs, therefore, to the Leguminosæ.

The *alcoholic extract of Calabar* (Sharpey) was formerly used as a myotic collyrium. Th. Fraser[1] discovered its action on the pupil, and Argyll Robertson[2] that which it has on the accommodation. The merit of having introduced the new medicament to scientific practice belongs specially to Robertson. It afterwards became the object of scholarly investigations,[3] among which the most noteworthy are those of Donders and Hamer.

The alcoholic extract of Calabar, employed in the form of a solution which was never quite limpid, or incorporated in gelatinous tablets, or placed on the conjunctiva by means of paper soaked in this substance, was always more or less irritating to the mucous covering of the eye. It was not long tolerated, and its action was rather uncertain, whether from the difficulty of its dosage or from the changeableness of the preparation.

Hence the discovery of *alkaloids* of this substance was a valuable acquisition for ophthalmology. In 1863 Jobst and Hesse derived from it *physostigmine,* and Vee and Leven *eserine.* These two alkaloids have since been recognised to be identical. Harnack and Witkowsky discovered, in 1866, a second alkaloid of the Calabar bean, which they called *calabarine.*

Under this name was formerly understood, not this substance, but physostigmine itself. It is well to call attention to this point, since most pharmacists still consider these two alkaloids identical. But calabarine is, in reality, a product of decomposition of physostigmine. It differs from it by its insolubility in ether and by some differences in its physiological action.[4] Calabarine, moreover, keeps much better than physostigmine.

Solutions of any of these substances introduced into the conjunctival sac, act at once on the diameter of the pupil and the state of the accommodation. Their influence upon the sphincter iridis is, however, much more prompt and energetic than that upon the ciliary muscle. Doses that are too weak to change the contractility of the latter, still suffice to cause myosis; and this myosis, when it is produced by too strong doses, still persists long after the action of the medicament upon the accommodation has entirely ceased.

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[1] Th. Fraser, Edinburgh Thesis, July 1862.

[2] Argyll Robertson, *Edinburgh Medico-Chirurg. Soc.*, Feb. 1883.

[3] Harley, Hulke, Bowman, Soelberg, Wells, von Graefe, Rosenthal, Schelska, and others.

[4] Cloetta, *Lehrb. der Arzneimittellehre*, p. 43, 1881.

The narrowing of the pupil, due to a strong solution of one of the myotics mentioned, is greater than the contraction provoked by the most energetic effort of accommodation, or by the most intense light. Nevertheless, the reaction of the pupil to light is not wholly abolished; it manifests itself, even as a consensual movement, in a calabarised eye, when its fellow, not under the effect of the medicament, is exposed to variations of illumination. The pupil, in a state of maximum contraction, is seldom perfectly round, but its shape is generally irregular, and even angular.

The effect of myotics, on the *ciliary muscle*, is very curious. Weak doses stimulate its contractility and increase its energy without producing any spasm. They therefore act on the *punctum proximum*, permitting it to come nearer to the eye than it is under purely physiological circumstances, without altering the position of the *punctum remotum*. *Hence the amplitude of accommodation is thereby increased.* This influence also manifests itself at times at the beginning of the action of strong doses, but especially during the period of decrease of the myotic effect.

The stimulating property of these medicaments makes itself felt, moreover, through the entire duration of their action. Thus, in one of Donders' experiments, the convergence which provokes, in an eye not subjected to the action of the medicament, a proportional effort of accommodation, for instance 3·7 D for 3·7 metre-angles, was accompanied, in the other eye, by an accommodative tension amounting to 8·3 D.[1]

When the action of the myotic has reached its *maximum*, the *punctum remotum* is relatively nearer the eye than the *punctum proximum*. Hence the range of accommodation has been diminished, having lost more at its far end than it has gained at the near one. But it is to be noticed that, at least in the case of young persons on whom experiments have been made, no myotic has ever been powerful enough to bring the *punctum remotum* sufficiently near to make it coincide with the *punctum proximum*. The will has, therefore, always retained a certain empire over the accommodation. The latter has never been entirely abolished.

The accompanying diagrams represent, better than any description could do, the influence of a myotic on the eye. They are due to the investigations made by Donders and Hamer with the extract of

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[1] Donders, *loc. cit.*, p. 614.

Calabar.[1] We have only adapted them to the metric system (Fig. 144).

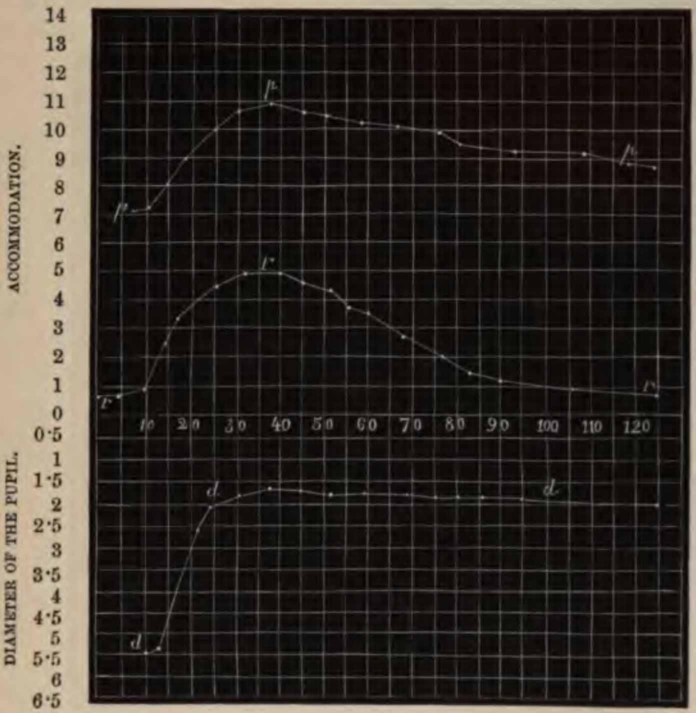

Fig. 144.

The lower portion is devoted to the pupil. The figures at the left, below 0, represent millimetres, and the figures on the horizontal, commencing with 0, represent minutes in Fig. 144, and hours or days in Fig. 145. The curved line *d d d* shows the variations in the diameter of the pupil.

The upper portion of the diagrams shows the influence exerted by the myotic on the refractive condition of the eye. The figures at the left indicate dioptries, those along the horizontal still stand for units of time. The curve *r r r* designates the course of the *punctum remotum*, *i.e.*, of the minimum of refraction, and the curve *p p p* that of the *punctum proximum*, *i.e.*, of the maximum refraction of the eye. The space included between the two is, consequently, nothing more nor less than the amplitude of accommodation, directly expressed in dioptries by the horizontal divisions passing through it.

It is to be noticed that the beginning of this scheme does not abso-

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[1] Donders, *loc. cit.*, p. 610.

during the maximum action of the myotic there was found to be . . . . . . $p' = 11$

$$\begin{array}{r} r' = \phantom{1}5 \\ \hline a' = \phantom{1}6\text{ D.} \end{array}$$

The descent of the three curves is accomplished very unequally; that of *d d d* is the slowest. It is only at the end of three days (Fig. 145) that the pupil has resumed its physiological diameter; *r r r* descends more rapidly. At the expiration of 120 minutes (Fig. 144), the *punctum remotum* has already returned to where it was before the experiment. The person examined could then entirely relax his accommodation. But the *punctum proximum* was still nearer than its physiological position. The maximum of accommodation was $p = 8{\cdot}6$
the minimum was only . . . . . $r = 0{\cdot}6$

hence the amplitude of accommodation, . . $a = 8$ D,
instead of 6·6, which would correspond to the age of the person.

The alkaloids of Calabar give, as has been said, entirely similar results. Let us cite an example taken from a monograph on myotics and mydriatics, written by Dr. Jaarsma, under the direction of our friend, Professor Doyer.[1]

A person subjected to the instillation of a one per cent. solution of the *sulphate of eserine* had, before the experiment, a maximum of refraction, . . . . . . $p = 8$
a minimum, . . . . . . $r = 0$

Hence an amplitude of accommodation, . . $a = 8$ D,

At the end of fifteen minutes, the *maximum* of the myotic action had been reached, and there was—

$$\begin{array}{rr} & p = 12{\cdot}5 \\ & r = \phantom{1}5{\cdot}\phantom{5} \\ \cline{2-2} \text{Hence} & a = \phantom{1}7{\cdot}5\text{ D.} \end{array}$$

When, at the end of an hour and a quarter, the *punctum remotum* had resumed its normal position, the maximum of accommodation was still—

$$\begin{array}{r} p = 12{\cdot}8 \\ r = \phantom{1}0\phantom{{\cdot}8} \\ \hline \end{array}$$

The amplitude of accommodation $a = 12{\cdot}8$ D, was, in this case also, noticeably increased during the period of diminution of the myotic action.

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[1] Jaarsma, *Leiden; Thesis*, p. 21, 1880.

The weakest solution of the sulphate of eserine, which can be ascertained to have an effect on the eye, is that of 1 to 12,800. It, however, influences only the sphincter of the iris. The *punctum remotum* commences to approach the eye under the influence of a solution of 1 to 800. But, before this effect makes itself felt, the excitability of the ciliary muscle, and, consequently, the amplitude of accommodation, increase.

The strongest cramp of the ciliary muscle which the author has been able to produce has never exceeded an hour and a half in duration, while the contraction of the pupil has been maintained for seventy-two hours.

The *sulphate of eserine* is very much used in practice, either to stimulate the action of the ciliary muscle, or on account of its myotic power, or, finally, to diminish intra-ocular tension. One-third or one-half per cent. collyria of eserine are generally very well borne. At times, however, the conjunctiva proves intolerant of the prolonged use of this medicament, and it seems to us that there would be an advantage in replacing it with the *salicylate of eserine* (Merk), which is less irritating and more stable.[1]

Indeed, the sulphate of eserine, like other chemical agents, changes pretty quickly under the influence of the atmosphere and light. The solution reddens and the eserine passes into the state of *rubreserine*, which is readily distinguishable by its absolute insolubility in ether. Rubreserine possesses no myotic action; therefore red solutions of the salts of eserine lose a part of their efficacy.

The action of *physostigmine* does not differ from that of eserine (Hamer, Krenchel). This alkaloid is, in fact, as we have already said, identical with eserine.

*Pilocarpine*, an alkaloid derived by Hardy, in 1875, from the leaves of Jaborandi (*Folia Pilocarpi*), is a myotic capable of occasionally replacing those derived from the Calabar bean. With hydrochloric and nitric acid, it forms crystallised salts which are readily soluble in water.

According to Jaarsma, the action of the hydrochlorate of pilocarpine, used as a collyrium, is similar to that of eserine. It affects at once the sphincter of the pupil and the ciliary muscle, which contract simultaneously, though independently of each other. It is to be noticed, however, that, although the myotic effect of eserine makes itself felt even when so small doses are used that the accommodation is not changed, and although myosis from eserine always somewhat precedes ciliary spasm, the weakest solution of pilocarpine that acts on the sphincter iriditis likewise acts on the accommodation. This minimum solution is, moreover, more concentrated (1 to 400) than that of eserine, which is 1 to 12,800 for the pupil and 1 to 800 for the accommodation.

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[1] It is advisable to add an antiseptic, as, for instance, a solution of sublimate, to all these collyria, especially in the case of a lesion of the membranes of the eye.

The promptness of the action of pilocarpine, according to the author quoted, is independent of the concentration of its solution. This action reaches its maximum in half an hour, and lasts only eighty hours for the pupil and two hours and a half for the accommodation. The pupil never entirely loses its reaction under the influence of light. That of the other eye does not dilate, even when the myosis of its fellow is at its highest degree.

As an example of the influence of this medicament on the ciliary muscle, we will cite the following observation of Dr. Jaarsma.[1]

| **Before the experiment.** | **A quarter of an hour after the instillation of a 2 per cent. solution.** | **Two hours and three quarters after the instillation.** |
|---|---|---|
| $p = 9{\cdot}5$ | $p = 10{\cdot}5$ | $p = 10{\cdot}2$ |
| $r = 0$ | $r = 1{\cdot}25$ | $r = 0$ |
| $a = 9{\cdot}5$ D. | $a = 9{\cdot}25$ D. | $a = 10{\cdot}2$ D. |

The action of pilocarpine on the accommodation is, therefore, quite analogous to that of the extract of calabar and its derivatives.

The former of these myotics, according to this author, would not alter vision, while vision is lessened during the maximum action of the last mentioned.

The *hydrochlorate of pilocarpine* is the form of the drug generally used; this crystallises readily, and a solution of it keeps very well for a long time; but, in this respect, the *salicylate* is still preferable, for a solution of it remains unchanged for more than three months.

*Muscarine* is an alkaloid discovered by Schmiedeberg and Koppe,[2] first in *Amanita muscaria*, and, afterwards, in the albumen of hens' eggs, by Schmiedeberg and Harnack.[3]

These authors were the first to ascertain the influence exerted by this drug on the eye; it consists in a narrowing of the pupil and a spasmodic contraction of the ciliary muscle. But the latter action greatly predominates over that to which the sphincter iriditis is subject. Six minutes after a hypodermic injection of 4 milligrammes of muscarine, its action began to be manifest, and sixteen minutes later it had attained its maximum on the refraction, which, from 6·5 D had increased to 13 D. The accommodation was, at this moment, abolished. It has not been possible, however, to demonstrate this fact in the later experiments of which we shall make mention. A 1·5 per cent. solution instilled in the eye left the pupil intact, although it produced an intense spasm of the accommodation.

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[1] Jaarsma, *loc. cit.*, p. 47.

[2] Schmiedeberg and Koppe, *Das Muscarin*, Leipzig, 1869.

[3] Schmiedeberg and Harnack, *Centralbl. f. d. med. Wissensch.*, p. 598, 1875.

Krenchel[1] has made more extensive experiments with this drug. He has ascertained that the action of muscarine on the pupil is very inconstant: at times the pupil remains passive, while, simultaneously, there is a very energetic spasm of the accommodation; at other times an intense myosis accompanies a very moderate ciliary spasm.

Quite to the contrary of what we have seen to be the case for the other myotics, muscarine acts first on the accommodation and afterwards on the pupil. The modifications, which the latter undergoes, persist from four to six times as long as those of the former.

The spasm of accommodation announces itself five or ten minutes after the instillation of a one to five per cent. solution, reaches its maximum in half an hour, and disappears at the expiration of from one hour to two hours and a half. The amplitude of accommodation is never entirely abolished, although the action of muscarine commences by bringing the *punctum remotum* nearer, and does not change the position of the *punctum proximum* till later. The ciliary muscle is at first put into a condition of tonic cramp, without, however, its contractile energy being increased. The latter effect does not manifest itself till afterward, and is never so marked as the initial tetanic spasm. The augmentation of the static refraction always greatly surpasses the increase in dynamic refraction; in other words, muscarine always restricts the amplitude of accommodation. This is the principal difference between this myotic and the preceding ones, especially physostigmine, a weak solution of which markedly increases the energy of the ciliary muscle without altering the static refraction of the eye. Muscarine, therefore, does not seem destined to render very great services in ophthalmic practice.

We have exhausted the list of the most important myotics known at present. Indeed, although certain authors have seen a strong myosis supervene under the influence of inhalations of nitrogen-protoxide,[2] or hypodermic injections of morphine,[3] the action of these agents upon the iris is as accessory as it is uncertain. As to the changes undergone by the accommodation, they are unknown and are, at all events, absolutely useless in practice.

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[1] Krenchel, *Arch. f. Ophth.*, xx., 1, p. 135, 1874.

[2] Bordier, *Journ. de thérapeutique*, t. xxiii., p. 885, 1876.

[3] Von Graefe, H. Lawson, *Med. Times and Gazette*, vol. liii., p. 671, 1876.

It still remains for us to say a few words concerning a phenomenon which generally accompanies the action of myotics. Objects appear enlarged while the eye is under the influence of a spasm of accommodation.[1] This *macropia* is very naturally explained by the fact that the effort of accommodation expended, in order to see distinctly, is less than when the eye is in its normal condition. Hence the individual is deceived as to the real distance of objects. He supposes them to be farther away than they are, and, since he refers his retinal image to an exaggerated distance—*i.e.*, projects this image too far—the object must appear to him larger than it really is.

It is to be noticed, before concluding, that the influence of all myotics, on the ciliary muscle, which we have always said to be independent of that which they exert upon the sphincter of the iris, persists even in cases of aniridia.

### WEAKNESS, PARESIS, AND PARALYSIS OF ACCOMMODATION.

As a symptom of paresis of the *common oculomotorius*, the diminution or abolition of the accommodative faculty is met with in the most diverse affections, capable of provoking alterations either of the ciliary ganglion and its branches, or of the trunk of the third pair, or of its nuclei of origin.

Thus Maingault,[2] Trousseau[3] and Donders[4] long ago pointed out the absolute or relative weakness of the ciliary muscle in persons convalescing from, or long since cured of *diphtheritic angina*. These learned observers admit that the virus peculiar to this disease, localising itself in certain points of the nervous system, produces paralytic accidents which figure among its frequent sequelæ. Recent investigations have confirmed their opinion, although it is still a matter of doubt as to whether the diphtheritic poison acts more especially on the nervous fibres (Buhl,[5] Oertel,[6] Voelckers),[7] or on the ganglion cells of their nucleus of origin (Déjerine).[8]

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[1] Donders, *loc. cit.*, p. 614.

[2] Maingault, "De la paralysie diphth.," *Arch. gén. de méd.*, *Paris*, ii., pp. 285 and 674, 1859 and 1860.

[3] Trousseau, *Clinique méd. de l'Hôtel-Dieu*, t. i., 1861.

[4] Donders, *Hollaendische Beitraege z. Natur- u. Heilkunde*, p. 10, 1861.

[5] Buhl, *Zeitschr. f. Biologie*, Bd. iii., p. 341.

[6] Oertel, *Deutsches Arch. f. klin. Medicin*, viii., p. 242.

[7] Voelckers. See Scheby-Buch, *Arch. f. Ophth.*, xvii., p. 265, 1871.

[8] Déjerine, *Arch. de Physiol.*, 1878.

It is noticeable, too, that the most diverse throat affections, without being of a diphtheritic nature, may be complicated by paralysis of the accommodation (Hutchinson, Mooren).[1]

Diphtheria is not the only infectious disease whose effects make themselves felt on the ciliary muscle. *Typhoid fever, recurrent typhus*[2] and *articular rheumatism* are cited in several observations as the alleged cause of accommodative paresis, whose genesis is doubtless the same as that just pointed out.

If we pass from acute diseases to chronic constitutional changes, we meet, in the first place, *syphilis,* that universal cause of all the pathological lesions imaginable. Its injurious action on the apparatus of accommodation may arise either from a lesion of the centres of the *common oculomotorius* or, oftener, from the compression or destruction of the fibres of this nerve by an infiltration of its sheath, by gummy productions or proliferations of the periosteum. Alexander[3] has even found unilateral paralysis of the ciliary muscle in cortical syphilitic affections of the brain, and attributes to this symptom a certain unfavourable prognostic value. Hutchinson,[4] on the contrary, has collected several examples of paresis limited to a single eye and attributable to a more peripheral lesion, that of the ciliary ganglion. Isolated paralysis of the muscles of accommodation and of the iris, called by that author *internal ophthalmoplegia,* is nearly always of specific origin; moreover, it is generally accompanied, later, by multiple manifestations in the other ocular nerves.

Although we are as yet unacquainted with the nature of the nervous changes produced by *diabetes,* there are numerous examples demonstrating the influence of this anomaly of nutrition on the workings of the cranial nerves, especially of the third pair and the filaments of it destined for Bruecke's muscle. Trousseau wrote, long ago, that accommodative disturbances might be the first symptom to reveal the existence of glycosuria. And, indeed, in many cases, it is the oculist who, led by a paralysis of one of the nerves of the eye to make an analysis of the urine, gives the first warning of this disease.

Certain parasitic affections likewise produce paralysis of the accommodation. Among their number is *trichinosis*.[1] The functional disorders produced in the muscles from the invasion of their tissue by trichinæ are not sufficient to account for the participation of the ciliary muscle, inasmuch as these parasites have not yet been found in the unstriated muscles. It is still more difficult to explain the abolition of the accommodative faculty observed in certain cases of *acute poisoning produced by the meat of raw sausages* (Scheby-Buch,[2] Ulrich[3] and Eichenberg) or *tainted meat* (Cohn).[4]

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[1] Hutchinson, *Ophth. Hosp. Rep.*, p. 436, 1873, and Mooren, *Fünf Lustren ophth. Wirksamkeit,* Wiesbaden, 1882.

[2] Scheby-Buch, *Bericht über* 38 *Faelle v. Accommodationslaehmung aus den Kieler Kliniken* (*Arch. f. Ophth.*, xvii., p. 265, 1871).

Alexander, *Berl. klin. Woch.*, No. 21, 1878, and *Deutsche med. Wochenschr.*, No. 41, 1881

[4] Hutchinson, *Medico-Chirurg. Transact.*, lxi., and *Med. Times and Gazette,* April 1878. It is doubtless in this class of cases that we must place that of Sichel (*Gaz des Hôp.*, p. 347, 1876), wherein the paralysis of accommodation was accompanied by paralysis of the external motor oculi.

Primitive lesions of the *central nervous system*, principally of *the brain*, are very frequently accompanied by paresis of the accommodation. They are then concerned with the nuclei of origin of the common oculomotorius or the trunk itself of this nerve, which is changed by pressure, by inflammation or by destruction. Simple vascular disturbances in this region are capable of producing a weakening of the accommodative faculty: anæmia or hyperæmia is sufficient, without the existence of any more serious cause.

It is to these anomalies of circulation that we are to refer the paralysis of the ciliary muscle observed in *digestive troubles*.[5] There is scarcely any other possible explanation for the strange case published by von Hasner,[6] in which a young girl was regularly attacked, at each menstrual epoch, by a complete paralysis of the common oculomotorius, which disappeared at the end of three days, to be renewed at the following menstrual period.

It need not be said that profound lesions in certain regions of the brain, in more or less direct relation with the third pair of its protuberance-nuclei, as well as increase of the intra-cranial pressure by a tumour, always manifest themselves by partial or total paralysis of the nerve of accommodation. It would be superfluous to enumerate all the spontaneous lesions and those of traumatic origin which enter into this category. It must be said, moreover, that, as in the case of spasm of the ciliary muscle, the general symptoms which they produce are of such gravity that accommodative disturbances often pass unperceived in their midst. Let us cite, however, *disseminated cerebro-spinal sclerosis* and, as an example of local lesions, the observation of

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[1] Foerster, in Graefe-Saemisch, *Handbuch der ges. Augenh.*, vol. vii., chap. xiii., p. 179.

[2] Scheby-Buch, *loc. cit.*

[3] Ulrich, *Klin. Monatsbl.*, p. 230, 1882.

[4] Cohn, *Arch. f. Augenh.*, ix., 2, p. 148, 1880. See also Leber, *Arch. f. Ophth.*, xxvi., 2, p. 236, 1880.

[5] Coccius, *Die Heilanstalt, &c.*, Leipzig, 1870 ; Steffan, *Nagels Jahresb.*, 1874, p. 25 ; Leber, *Erkr. d. Auges bei Diab. mell.*, pp. 318-322 (Acute diabetes following gastric catarrh) ; and Unterharnscheidt, *Klin. Monatsbl.*, p. 37, 1882 (Gastralgia).

[6] Von Hasner, *Arch. gén. de méd.*, p. 732, 1883.

Cohn,[1] in which a fracture of the base of the skull, by a fire-arm, had brought about a paralysis of the accommodation. H. Parinaud[2] has also observed this in a patient who had died of a cerebral tumour, which flattened, among others, the trunk of the common oculomotorius.

Among spinal affections, *locomotor ataxia* is the only one that appears to give rise to accommodative paresis, owing to the frequent manifestations of this disease on the nerves of the eye. When the third pair is affected, it is natural that the ciliary muscle should suffer in its contractility.

Only two observations exist in which compression of the *sympathetic* has been accompanied by a weakening of the power of accommodation. They are those of Eulenburg[3] and Warner.[4] Eulenburg's case was one of irritative lesion, and Warner's one of paresis of this nerve. These contradictory data furnish a presentiment of the obscurity which still reigns concerning this point. According to Eulenburg, the intra-ocular tension, increased under the influence of the excitation of the sympathetic, opposes itself to the bulging of the crystalline surfaces when the zone of Zinn is relaxed.

Hutchinson's[5] case, in which a sudden and transitory paralysis was seen to develop as the result of *great fear*, remains unexplained, unless we resort to the hypothesis of vaso-motor disturbance, or a peculiar action of arrest exerted upon the accommodative centre.

We have already had occasion, when speaking of accommodative asthenopia, to call attention to the unfavourable influence of a state of poor health upon the energy of the ciliary muscle. Any cause, capable of impeding nutrition and determining a general depression of strength, will therefore find an echo in the accommodative faculty. Thus it is not surprising that weakness of the accommodation should have been pointed out in *essential anæmia*, and in a multitude of other symptomatic anæmic conditions, such as that which succeeds *acute diseases*,[6] or which is provoked by *lactation*,[7] *alcoholism*,[8] *venereal excesses*, *masturbation*,[9] *uterine lesions*,[10] *abundant hæmorrhages*,[11] &c.

Let us point out, finally, among the peripheral nervous affections, neuritis of the ciliary terminations of the common oculomotorius, which, in a case of von Hasner's, accompanied an eruption of *herpes*.[1] The paralytic complications of this trophic lesion were here manifested in the ciliary muscle.

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[1] Cohn, *Schussverletzungen des Auges*, p. 6, 1872.

[2] Parinaud, *Arch. de Neurol.*, vol. v., No. 14, p. 145.

[3] Eulenburg, *Berl. Klin. Wochenschr.*, pp. 169-172, 1873.

[4] Warner, *Brit. Med. Journ.*, April 1877.

[5] Hutchinson, *Ophth. Hosp. Rep.*, vii., p. 38, 1871.

[6] Coccius, *Die Heilanstalt f. arme Augenkranke, &c.*, Leipzig, 1870.

[7] Hutchinson, *Ophth. Hosp. Rep.*, vii., p. 38, 1871.

[8] Romiée, *Rec. d'ophth.*, p. 33, 1881.

[9] Landesberg, *Med. Bulletin*, iii., No. 4, 1881.

[10] Mooren, *loc. cit.*

[11] Rampoldi, *Ann. di Ottalm.*, xi., p. 318. 1882.

It is doubtless to local changes of these nerve endings, or of the tissue itself of the ciliary muscle, that we are to refer the accommodative paresis occurring as a consequence of *traumatisms of the eye or of the orbital region*.[2] Perhaps it must also be brought in to explain the paresis which has been observed in *sympathetic ophthalmia*,[3] in *glaucoma* and in *neuralgia of the dental ramifications*[4] of the second branch of the trigeminus.

*Diagnosis.*—Whatever may be its provoking cause, partial or total paralysis of the ciliary muscle, regarded by itself and without respect to any other concomitant lesion, announces itself by one constant functional trouble, *viz.*, *the diminution or complete abolition of the amplitude of accommodation*, in consequence of the recession of the *punctum proximum* from the eye, or its absolute coincidence with the *punctum remotum*. The latter generally undergoes no displacement.[5] Static refraction remains the same, while the dynamic refraction is enfeebled or annulled.

This is followed by visual disturbances, which differ according as to whether the patient is emmetropic, myopic, hyperopic or astigmatic. They must evidently resemble those which also accompany the physiological diminution of the amplitude of accommodation (see p. 177). They are to be distinguished from the latter by their not corresponding to the age of the patient, and frequently also by their sudden appearance. The physician will, therefore, suspect a paresis or a paralysis of accommodation, as soon as it is shown that a person, who formerly had good vision for near objects, can now see well only in the distance, or when a young myope can no longer change his range of vision; when a supposed emmetropia or a facultative hyperopia is unexpectedly converted into an absolute hyperopia, or when astigmatism is suddenly developed.

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[1] Von Hasner, *Allg. Wiener med. Ztg.*, p. 120, 1873; and von Arlt, *Klin Monatsbl.*, p. 333, 1877.

[2] Harlan, *Amer. Journ. of the Med. Sc.*, vol. lxi., pp. 130-140, 1871; Hirschberg, *Klin. Beobachtungen*, pp. 33-38, 1874.

[3] Cuignet, *Rec. d'Ophth.*, p. 193, 1878.

[4] Schmidt-Rimpler, *Arch. f. Ophth.*, xiv., 1, p. 107, 1868.

[5] Jacobson and Weiss (*Arch. f. Ophth.*, xxiv. 2, p. 236, 1878) have observed the singular fact that, during diphtheritic paralysis of the accommodation, the refractive condition was notably inferior to that possessed by the same persons, before or after recovery, under the influence of atropine. The proof, that this diminution of refraction is not attributable to more complete inertia of the ciliary muscle in the former case than in the latter, lies in the fact that, while diphtheritic paralysis was at its height, the mydriatic mentioned was still capable of causing a very sensible recession of the *punctum remotum*.

The absolute and unmistakeable means of recognising this trouble is found in the direct measurement of the amplitude of accommodation. We have spoken of the different methods (pp. 275 *et seq.*). If the amplitude of accommodation is found not to agree with the age corresponding to it—that is to say, should it fall below it—then the diagnosis of paresis of accommodation may safely be made. But should there be absolutely no power of accommodation present—that is to say, should the eye be reduced to its minimum of refraction—the patient being able to see distintly only at a given distance, that of his *punctum remotum*, and that at an age when he ought still to possess a certain power over his lens to allow him to alter its state of refraction—then there exists a paralysis of accommodation. Donders' diagram (p. 174) corresponds so perfectly to the natural condition of things, that, in every case where the amplitude of accommodation is less than is indicated thereon, we may safely diagnose a weakness of accommodation, and, in case of any considerable difference, we may admit a paresis of this function.

If this has been firmly established, it will then be necessary to discover its cause. Anamnesis will generally disclose one or the other of the etiological circumstances mentioned above. In most cases we will have to do with young people recovering from a diphtheritic affection, or those who are anæmic, either from a weak constitution or on account of bad hygienic conditions. We attach no importance to certain objective phenomena, such as mydriasis, ptosis or a paralytic deviation, since they may be lacking in paralysis of the ciliary muscle,[1] and may exist in its absence.

But it is important, from a prognostic point of view, not only of local changes, but even of those of the entire organism—it is important, we repeat, to ascertain as accurately as possible the cause of this functional disturbance, and the seat of the lesion which provokes it. In such cases, the ophthalmologist ought to prove himself familiar with general pathology, and with all the methods of examination it possesses. It will generally be easy to find out whether or not the patient has had diphtheritic or simple throat affections; if there are syphilitic antecedents; if he is threatened with some serious affection of the central nervous system; or if he has only been exposed to influences injurious to health, whether in his alimentary regimen, in his habits or in other hygienic surroundings. Even the previous existence of an acute infectious disease could not be ignored.

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[1] Jacobson, Maingault, Scheby-Buch, and others besides, have not observed pupillary disturbances in diphtheritic paralysis of the accommodation. According to the same authors, the coincidence of paralysis of the extrinsic muscles is rare.

If this information is lacking, or is insufficient, the doctor will not hesitate to resort to an attentive and thorough examination of all the patient's organs and functions. He will scrutinise his general facies, his digestive organs, the disturbances which he may present in general motility, or sensibility, or in the special senses. He will examine his urine for albumen and sugar. He will devote himself especially to a circumstantial examination of the eye, its motility, the width of the palpebral slit, the diameter and reaction of the pupil, and the retinal functions—all these are notions of capital importance to acquire in order to get an idea of the point, in the course of the common oculomotorius, which may be compromised. It would not be consistent with ourplan to give, in this place, the symptomatology of all the nervous or general diseases which may abolish the accommodative faculty. We will content ourselves with calling the reader's attention to the so frequent coincidence of paralysis of the velum palati in convalescence from diphtheria. This paralysis may, moreover, extend to the members, and even to the muscles, of the trunk, in which case life is seriously threatened. It is remarkable that, of all the terminations of the common oculomotorius, it seldom affects any other than the ciliary ramifications.

The etiological diagnosis having once been made, the prognosis readily follows from it. It is well established that diphtheritic paralysis, when it does not extend to regions that are important for the preservation of life, is certainly cured in a few weeks. The same consoling assurance may be given to the patient who presents himself with symptomatic anæmia due to some dietetic vice, or consecutive upon an acute infectious disease. One must be more reserved in the presence of a chronic intoxication, or a disease of long duration, such as syphilis or diabetes. The reader doubtless knows numerous and brilliant successes obtained, in certain syphilitic nervous affections, by an energetic mixed treatment. But it is none the less true that there are too many cases in which this medication is powerless, and we have seen that syphilitic paralysis of the accommodation is often the prelude to a serious and incurable alteration in the nervous centres. Finally, every one knows the sad fate reserved for ataxic patients and the victims of disseminated sclerosis or cerebral tumours.

As in every other muscular paralysis, electric irritation might here furnish us with valuable hints as to the prognosis, if it were applicable to Bruecke's muscle. Fortunately, however, we possess, in myotics, a reactive which is almost as convenient and sure. Generally speaking, whenever the ciliary muscle responds, through a period of some length, to the excitation produced by this agent, the prognosis of its paralysis is favourable.

*Treatment.*—Although curable paralysis of the accommodation disappears in most cases spontaneously, it is none the less beyond doubt that local treatment may sometimes hasten the recovery.

If we do not speak specially of general medication, it is because the latter is a matter of course in every case where it is applicable to the cause of the accommodative weakness. It is self-evident, for instance, that one will prescribe strengthening hygiene and tonic remedies for every person in whom nutrition is defective. This rule is especially applicable to patients convalescing from diphtheritic diseases, who are generally very greatly debilitated; they are to be put upon a nitrogenous, reparative diet; one will procure for them, if possible, the benefits of the pure air of the country, of the mountains, or of the sea-side; they will be recommended to rest their eyes and to take an amount of bodily exercise commensurate with their strength. Ferruginous preparations will accelerate the regeneration of the blood corpuscles, and bitter tonics will restore the digestive powers.

Suppose one has to do with a syphilitic patient, who is the victim of tertiary lesions. Besides the above prescriptions, which cannot but be beneficial to any cachectic patient, the physician will institute an energetic treatment by means of mercurial inunctions and iodide of potassium in large doses. The diabetic patient's diet will be minutely regulated, and one will make use, in the case of unfortunates who are the prey of sclerosis of the central nervous system, of the uncertain resource of thermal medication, hydrotherapy, revulsives, electricity and subcutaneous injections of strychnine.

We have said that one may greatly hasten the favourable termination of accommodative paralysis by acting directly on the ciliary muscle itself or on the nervous twigs contained in it. Several valuable observations demonstrate,[1] indeed, that the instillation of a collyrium of eserine notably abridges the duration of the recovery. It is necessary, however, to guard against using this remedy in a concentrated solution, or requiring of it the maximum effect from the outset. One should not lose sight of the fact that eserine here takes the place of the faradisation applied to the other muscles of the body, and that, like the latter, when imprudently administered, it may destroy the vitality of the paralysed muscle, instead of restoring and sustaining it until the normal innervation returns. A drop of a solution of 1 to 500 is to be instilled once a day. If the injurious action of this drug on the conjunctiva is feared, one may resort to pilocarpine

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[1] See, among others, Manz, *Klin. Monatsbl.*, viii., pp. 245-246, 1870. Stammeshaus, *Ueber die Functionsstörungen d. Auges nach Ang. diphth.*, Inaug. Diss., Bonn, 1870. Woinow, *Med. Bote*, p. 301, 1875. Badal, *le Monit. méd.*, March 1877.

(one per cent.), which, according to Georges,[1] will act even in cases wherein the alkaloid of Calabar remains impotent.

In cases in which it is desired to still further accelerate the recovery of the patient, one may have recourse to the local application of electricity. Duchenne recommends faradisation, which he practises by placing two small metallic electrodes at the extremities of the horizontal diameter of the cornea, about three millimetres from the border of this membrane. Some electro-therapeutists fear the action of faradic currents on the nervous apparatus of the eye, and consequently advise that only continued currents be employed for this organ. The positive pole is then applied opposite the first cervical ganglion of the sympathetic (immediately below the mastoid apophysis) and the negative pole to the closed eyelids. The current, of from eight to twelve elements, is kept up for two or three minutes, and any interruption of it is to be avoided (Onimus).[2]

Because of the great authority of the *savant* who has extolled it, we ought to mention also Coccius'[3] treatment, which consists of instilling in the eye a few drops of a solution of camphor (1 to 300). We have not had sufficient personal experience as to the value of this medicament.

Finally, the specialist may be called upon to pronounce upon the advisability of a palliative optical treatment of paralysis of the accommodation. When this affection is recent and bilateral, it is best, in the beginning, to allow the eyes perfect rest, and only to protect, by means of smoke-glasses, the retina, which is exposed to the exaggerated illumination resulting from mydriasis. After a while, we may permit a return to the usual work and assist the reviving powers of the ciliary muscle by means of the glasses appropriate to the patient's refractive condition. When the paralysis is of long standing, and there is no hope of curing it, there is no reason why the subject should not be treated as in the case of the physiological diminution of the accommodation, and the suitable working-glass prescribed for him.

There is nothing easier than to adapt an eye, which is deprived of of accommodation, to any desired distance. We remember, indeed, that it is precisely the dynamic refraction which often complicates the determination of the state of refraction, as well as that of the working-glass. Here we have only to reckon with the static refraction and that required for the desired distance. The number of the necessary glass will be represented by the difference between these two. We have only to remember that the hyperope's refraction is negative while that of the myope is positive, like that demanded for any finite distance. Examples of this may be dispensed with here. What has been said in this connection on page 181, as well as in the clinical chapters on hyperopia and myopia, suffices for the guidance of the reader in selecting the glass necessary for an eye deprived of accommodation.

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[1] Georges, *Ein Beitrag. z. Wirk. d. Jaborandi auf. d. Sphincter pup. u. Accomm. Apparat.*, Inaug. Dissert., Greifswald, 1875.

[2] See Webster, *Phil. Med. Times*, Oct. 24, 1874. Camuset, *Gaz. des hôp.*, p. 209, 1874.

[3] Coccius, *Tagebl. der deutschen Naturf.-versamml. in Leipzig*, p. 173.

When the paralysis of the ciliary muscle is not complete, one will determine, first of all, the amplitude of accommodation, and prescribe the glass which will leave the patient the quota required for continued work. This quota will generally need to be pretty large (about one-third or more), as we have previously had occasion to show.

When the paresis or paralysis is unilateral, and the other eye works normally, it is often difficult to make both eyes work together. The correcting-glass of the affected eye changes the size of its retinal images, so that they are not always readily fused with those of its fellow. Moreover, they are distinct, for binocular vision, for only one single distance, as well when the paralysis is incomplete as when it is total, while the other eye enjoys a range of distinct vision which corresponds to the patient's age. The physician then finds himself in a position similar to that which he occupies when dealing with anisometropia. In default of absolute rules, which it is impossible to formulate, he must be guided, in selecting optical correctives, by his own judgment, and not disregard the patient's appreciation of the glasses tried.

## MYDRIATICS.

The accommodation can be artificially paralysed by certain substances which, at the same time, dilate the pupil. The latter effect, which is more apparent and was also earlier recognised than that on the ciliary muscle, has won for such substances the name of *mydriatics*. They are all strongly toxic, and are used in the form of collyria or ointments; or small bits of gelatine or paper are soaked in them and introduced into the conjunctival sac.

The four most active mydriatics are alkaloids of the *solanaceæ* which bear the greatest resemblance to each other in their chemical composition and physiological action. We use the soluble salts which they form with sulphuric, hydrochloric, salicylic or hydrobromic acid. These are:

1. *Atropine*, obtained in 1831 by Mein from *Atropa Belladonna*,[1] and prepared in a pure state by Geiger and Hesse

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[1] Van Swieten (*Commentaria in Boerhavii Aphorismos*, t. iii.) had already mentioned the influence of *belladonna* on the pupil. Himly (*Goetting. gelehrte Anzeig.*, 1800) dis-

in 1833; and *homatropine*, a derivative from atropine, discovered by Ladenburg;

2. *Duboisine*, extracted by Gerard and Petit from *Duboisia myoporoides;*
3. *Hyoscyamine*, an alkaloid of *Hyoscyamus niger*, discovered by Geiger and Hesse in 1833.

*Daturine*, which Geiger and Hesse have extracted from the leaves and seeds of *Daturia stramonium*, is identical with atropine.

Duboisine and hyoscyamine are also considered identical by von Planta and Ladenburg. Hence there exist, essentially, only two mydriatic alkaloids furnished by the solanaceæ, *atropine* and *hyoscyamine;* these two substances are also isomeric. It is scarcely worth while to make special mention of *hyoscine*, which differs from hyoscyamine only in its slight solubility, as a double chloride which it forms with gold. A mydriatic which does not belong to the family of the the solanaceæ is:

4. *Gelsemine*, a resinous substance extracted by Wormsly from *Gelsemium sempervirens* (Aiton, Apocynaceæ), from which the chemist has obtained the hydrochlorate of gelsemine. This salt, as well as the sulphate of gelsemine, is applicable in the form of a collyrium.

All of these mydriatics, when introduced into the conjunctival sac, filter through the cornea and have the effect, when employed in a certain concentration—(1) of dilating the pupil, and rendering it immobile; (2) of wholly paralysing the accommodation.

There is, therefore, an essential difference between the action of the mydriatics and that of their antagonists, the myotics. The latter, although they reduce the pupil to a diameter which is much less than the smallest that it presents in a physiological state, do not, however, deprive it of all mobility: in the same way they always allow a certain contractility of the ciliary muscle to persist. They act at once on the *punctum remotum* and *punctum proximum*, bringing both these points nearer the eye, without, however, making them coincide, *i.e.*, without abolishing the amplitude of accommodation. Mydriatics, on the contrary, have their influence solely on the *punctum proximum*, which they cause to gradually recede from the eye until its position is identical with that of the *punctum remotum* in the complete paralysis of the ciliary muscle, the nullification of the accommodation and the reduction of its amplitude to zero.

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covered the mydriatic action of hyoscyamus, and generalised the use of these substances in ophthalmology. Wells (*Philosoph. Transactions*, vol. i., p. 378, 1811) studied more particularly their influence on the accommodation (Donders).

In order to have a better perception of the action of a mydriatic let us consider the two diagrams (Figs. 146 and 147), which we have borrowed from Donders[1] work, and adapted, as in the former instance, to the metric system. They represent the influence of a drop of a solution (1 : 120) of the sulphate of atropine on the accommodation (upper portion) and on the diameter of the pupil (lower portion).

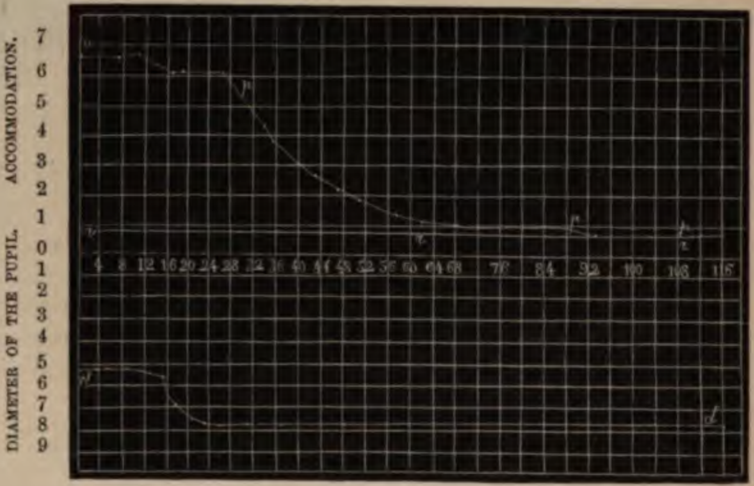

FIG. 146.

The figures at the left, above the zero, stand for dioptries; the curve *p p p* corresponds to the maximum of refraction (*punctum proximum*), the line *r r r* to the minimum (*punctum remotum*). Hence the divisions, included between the two, give the amplitude of accommodation.

The figures along the horizontal, opposite zero, indicate the time elapsed since the instillation—in minutes for Fig. 146, in days for Fig. 147.

The figures at the left, below the zero, indicate, in millimetres, the diameter of the pupil, whose variations are represented by the curve *d d d*.

It will be seen (Fig. 146) that the pupil commences to dilate at the end of twelve minutes, and that, in twenty-four minutes, the mydriasis has reached its maximum, which it maintains for more than a day. It returns but slowly and gradually to its normal diameter and physiological mobility—that is to say, at the expiration of about thirteen days (Fig. 147).

According to Donders' text, the diminution of accommodation begins to make itself felt somewhat later than the mydriatic influence.

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[1] Donders, *loc. cit.*, pp. 585 and 586.

But it is, above all, the maximum effect on the ciliary muscle—the complete paralysis of accommodation—which is attained noticeably

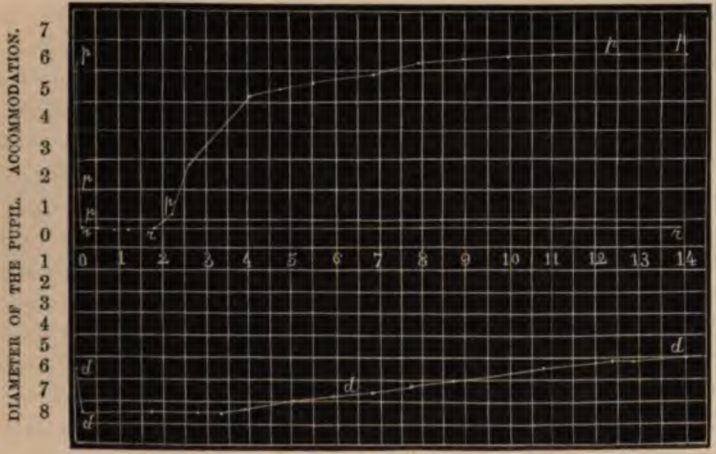

Fig. 147.

later than absolute mydriasis. At the end of three minutes, the curves *p* and *r* coincide, thus marking the loss of accommodative power. In the experiment cited this effect persisted more than thirty-six hours (Fig. 147) and then disappeared more, promptly than the mydriasis, as is indicated by the rapid ascent of the curve *p p p* (Fig. 147).

The line *r r* was not changed, but remained straight, since this drug has no influence on static refraction. The fact that it does not coincide with the zero abscissa, but is above it, shows that the person examined, Dr. Hamer, was slightly myopic.

The same experimenters have also made the following most interesting observations :—

When the eye is under the influence of the mydriatic, without its ciliary muscle being entirely paralysed, as happens, for instance, a few days after the instillation, its *relative amplitude of accommodation* behaves like that of a myope. The amount of accommodation brought into play is always less than the amount of convergence accompanying it. Donders cites the following instance: "Thus, Mr. Hamer, on the sixth day, found that, while with convergence to 9″ of the eye which had not been subjected to instillation, about the half of the total accommodation came into play, the eye which had undergone the operation attained only a fifth of what it was now again capable of at the maximum of convergence."[1]

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[1] Donders, *loc. cit.*, p. 586.

Now, in order to see at 9″—that is to say, at approximately ¼ of a metre, one requires 4 metre-angles of convergence and 4 D of positive refraction. The amplitude of accommodation of the eye not subjected to the mydriatic was doubtless the same as that of the other before the experiment, *i.e.*, about 6 D (Fig. 146). With a myopia of nearly one dioptry, it would need, in fact, half of its amplitude, or 3 D, to adapt itself to the desired distance. With this convergence, the atropinised eye brought into play only a fifth of its amplitude of accommodation, or 1 D. Indeed the diagram, Figure 147, shows that, on the sixth day, the amplitude of accommodation amounted to 5 D.

It will be remembered that the reverse is true of myotics, which increase the excitability of the ciliary muscle to such an extent that accommodation always greatly exceeds convergence.

Another phenomenon, likewise the opposite of that provoked by myotics, is the *micropia* which manifests itself under the influence of mydriatics: objects appear smaller to an atropinised eye, because the abnormally great effort of accommodation, required for distinct vision, begets the supposition that they are nearer than is really the case. In projecting its retinal image to so short a distance, the eye necessarily judges objects to be smaller than they are.[1]

The excessive dilatation of the pupil of one eye causes, on account of the dazzling which results, the contraction of the pupil of the other eye. For the same reason, objects appear much brighter to the first eye than to the second.

Myosis in one eye does not produce mydriasis in the other. However, the brightening of objects is, in this case also, in relation with the diameters of the two pupils. The weakest solution of the sulphate of atropine, a drop of which will still produce mydriasis, is, according to Jaarsma,[2] that of 1 : 80,000. A drop contains only 0·000,000,8 of a gramme of the substance; the effect commences to show itself at the end of an hour and lasts twenty-four hours. The weakest solution capable of paralysing the accommodation, is, according to the same author, in the strength of 1 : 1200. The duration of its action is likewise twenty-four hours, while it manifests itself for ninety-six hours in the pupil.

It is well to call attention here, however, to the fact that these authors experimented on normal eyes, which responded to the tests to which they were submitted with a promptness which left nothing to be desired. The case is quite different in pathological conditions. We occasionally meet with more recalcitrant ciliary muscles, and with spasms of accommodation which a much stronger solution of atropine requires days, and even weeks, to overcome. Leaving out of the question synechiae and extensive adhesions between the iris and the crystalline, which oppose the dilatation of the pupil, the latter is obtained with greater or less facility according to the contractility of this membrane, the energy of its muscles and of its innervation.

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[1] Donders, *Nederlandsch Lancet*, t. vi., p. 607, 1851.

[2] Jaarsma, *loc. cit.*, p. 68.

The salt of atropine most commonly used in practice is the *sulphate*. The *salicylate* is, however, preferable, for, although it is hygroscopic, a solution of it keeps unchanged for a very long time. Two parts of the sulphate of atropine are equivalent in their action to three parts of the salicylate, inasmuch as the same proportion of the base is contained in each of these quantities.

Pettorelli[1] has lately recommended the use of a substance which he calls *nitro-atropine*. It is obtained by submitting atropine to fuming nitric acid, and adding an alcoholic solution of caustic potash. A similar product, *nitro-daturine*, is also prepared from daturine. These two substances have the same effects as the original bases.

The ordinary solution of the sulphate of atropine contains 5 centigrammes of this salt to 10 grammes of distilled water. This concentration is amply sufficient for most cases. It is even well to weaken it a little for children who are subjected to prolonged atropinisation, in treating a spasm of the accommodation or hyperopic convergent strabismus. A solution, diluted by one-half, will be perfectly suitable for such cases. It is only in exceptional conditions that we shall find ourselves obliged to resort to stronger doses, as when it is a question, for instance, of breaking up tenacious synechiæ.

It is best to observe certain precautions in the use of atropine and mydriatics generally, especially in the treatment of infants. Instances of general intoxication, produced by these substances, are not rare, and may take on a most alarming character. Kugel[2] has published a case, of a child, in which death resulted from atropine instillations. Several other authors have cited unpleasant experiences, whose issue was, however, more favourable, and almost every specialist has encountered them in his practice. A very weak solution (0·1 - 0·4 per cent.) is often sufficient to provoke these unpleasant accidents[3] in certain subjects who are evidently idiosyncratic, relative to the alkaloids of solanaceæ. It is well to be warned of this complication, which may be effectively treated by subcutaneous injections of morphine (0·01 - 0·03) (Kugel, Hedler,[4] Beauvais), or of pilocarpine (0·02 *pro dosi*) (Juhasz).[5]

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[1] Pettorelli, *Compte rendu du Congrès internat. de Milan*, 1881, p. 203.

[2] Kugel, *Arch. f. Ophth.*, xvi., 1, p. 345, 1870.

[3] Beauvais, *Ann. d'hyg. publ. et de méd. lég.*, Janvier 1881.

[4] Hedler, *Berl. klin. Woch.*, p. 471, 1875.

[5] Juhasz, *Klin. Monatsbl.*, p. 82, 1882.

Another inconvenience, though one less to be feared, in connection with the use of atropine, consists in its injurious effect on the conjunctiva. After a prolonged use, sometimes even after a few instillations of this drug, there appears a conjunctivitis, which, from a simple catarrhal inflammation, may sometimes increase until it resembles trachoma, with corneal pannus. Certain persons, whom a special predisposition renders peculiarly sensitive to the influence of atropine, are liable to be attacked by an eczema of the lids, or even by a phlegmonoid swelling of the entire periorbital region.

The genesis of these accidents has been the subject of much discussion. They have been attributed to chemical impurity of the alkaloid, to its acid or alkaline reaction, or to micro-organisms (fungi) which are developed in solutions of it. Starting with this last idea, Kroemer[1] even advises that these solutions be saturated with boric acid, and that, moreover, a small quantity of carbolic acid (1 : 1000) be added.[2] The liquid, thus treated, is said to remain transparent for an indefinite length of time, and to be inoffensive to the mucus covering of the eye. But Schenkl[3] opposes to this assertion the statement that ointments of atropine with vaseline, although giving rise to no parasitic vegetation, nevertheless produce the conjunctivitis mentioned.[4]

However this may be, we should always assure ourselves, by testing it with litmus paper, of the absolute neutrality of a collyrium of atropine that we are going to use, and we ought to be careful to use only fresh solutions. If the conjunctiva becomes irritated, the employment of this preparation would better be suspended, and duboisine substituted, which is generally better tolerated, although it, too, is far from being perfectly innocuous.[5]

*Homatropine* is a product obtained by causing hydrochloric acid to act on the cyanate of tropine. It has the appearance of an oleaginous liquid. The readily crystallisable salt which it forms with hydrobromic acid is the only one used in medicine. Its effect is quite similar to that of atropine. Its action on the accommodation and on the pupil is as energetic as that of the latter; only it manifests itself more promptly and disappears sooner.

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[1] Kroemer, *Corr.-blatt f. Schweizer-Aerzte*, No. 19, 1881.

[2] A small quantity of salicylic acid (1 : 2000) may be added to solutions of atropine, care being taken to use, for the solution, water which is chemically pure, and, above all, free from ammonia.

[3] Schenkl, *Prager med. Wochenschr.*, vii., p. 61, 1882.

[4] Green (*Trans. Amer. Ophth. Soc.*, p. 375, 1875), advises the solution of atropine in castor oil. He says that it keeps very well in this liquid without acquiring irritant properties.

[5] The salts of atropine and of the various mydriatics, and the bases themselves, as found in trade, are far from being as constant as might be desired. The preparations sold are often only a mixture of several of these substances together. Consult, with regard to this matter, the investigations of Ladenburg, *loc. cit.*, those of Regnault and Valmont (*Archives gén. de méd.*, Jan. 1881), and the review of these works in Nagel's *Jahresbericht*, p. 261, 1881. It will be seen that it is important to know whence comes the product which we use.

Tweedy found that a solution in the strength of 1:125 caused mydriasis in twenty minutes. This had disappeared entirely at the expiration of twenty-four hours, as had also the total paralysis of the accommodation, which the medicament had produced.[1] Schäffer[2] and Goetz[3] obtained the same results; the action of a half per cent. solution of atropine lasts six or eight times as long as that of a one per cent. solution of homatropine.

According to Segura,[4] homatropine did not increase intra-ocular tension, in several cases where atropine produced this annoying symptom.

Schell[5] thinks homatropine ill adapted to continued use, on account of the irritation of the conjunctiva which it provokes, while he considers it recommendable when it is a question of simply examining the eye. A three per cent. solution, which is well borne, is said to bring about a maximum mydriasis in twenty minutes, paralysis of the accommodation in half an hour, and its influence disappears in from ten to thirty hours.

2. The sulphate or the salicylate of *duboisine* possesses a more energetic action than atropine. According to Jaarsma, a solution in the strength of 1:1,200,000, containing 0·000,000,054 gr.—*i.e.*, 54 billionths of a gramme of this substance—produces, in an hour and a quarter, a mydriasis which lasts twenty-four hours.

The same quantity of a solution in the strength of 1:3000 determined a paralysis of accommodation, which persisted twenty-four hours. The accompanying mydriasis ceased only at the end of seventy-two hours.

Norris has found that a four per cent. solution of duboisine causes mydriasis in from twelve to eighteen minutes, while atropine, in equal concentration, required from twenty-five to thirty minutes to produce the same effect. Paralysis of the accommodation is complete, for duboisine, at the end of from thirty to fifty minutes, and at the end of eighty to a hundred for atropine.

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[1] Tweedy, *Lancet*, 1, p. 795, 1880.

[2] Schäffer, *Arch. f. Augenheilk.*, p. 186, 1880.

[3] Goetz, *Kiel Thesis*, 1880.

[4] Segura, *Clinica de Malaga*, iii., p. 7, 1882.

[5] Schell, *Specialist and Intellig.*, Philad., i., p. 32, 1880.

On the whole, all authors agree that the action of duboisine is prompter and more energetic than that of atropine. The former has, moreover, the great advantage of irritating the conjunctiva less than the latter. Hence this drug is of great service in cases where atropine is not well borne. On the other hand, it seems to cause general toxic symptoms more easily than the latter.[1]

The sulphate of duboisine is the preparation of it that is generally used: but here again the salicylate is preferable, because of its greater stability and its comparative innocuousness for the conjunctiva.

3. *Hyoscyamine*, and the compound with which it is isomeric, are as yet little known, and not much used in therapeusis. The extract, employed by the first experimenters, seems to have been very unstable or very impure, since the results obtained differ with different observers, and even from one experiment to another.

In the thesis of Miss Simonowitch,[2] who very nicely summarises the history of this medicament, adding also new researches and reflections, she says that the gelatine discs saturated with hyoscyamine (prepared by Messrs. Morson, Merk, Sittel and Heberlin) have proved less efficacious than atropine; but that the former may be advantageously substituted for the latter in chronic iritis, when atropine ceases to be well borne.

The case seems to be different with the sulphate and the hydriodate of *hyoscine* lately prepared by Merk. Risley[3] finds, for the former of these salts, an action which is quite similar to that of duboisine. A drop of a one-third per cent. solution will paralyse the accommodation for from seventy-seven to one hundred hours. Emmert,[4] who has experimented with the second of these salts, thinks its action prompter and more energetic than that of the sulphate of atropine or duboisine, used in like doses. A solution in the strength of 1:1000 is declared to suffice, in most cases, to cause mydriasis and paralysis of the accommodation. The conjunctiva, as well as the general system, bears it well. Hirschberg[5] contests its innocuousness, although he admits that the action of this drug is more powerful than that of the other mydriatics.

The sulphate of *daturine* has been specially examined by Doijer and Jaarsma. They find that a drop of a solution of 1:160,000, *i.e.*,

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[1] Ringer, *Pract.*, p. 247, 1879.

[2] Rosa Simonowitch, Berne Thesis, 1874.

[3] Risley, *Amer. Journ. of the Med. Sciences*, No. 161, 1881.

[4] Emmert, *Corr.-blatt f. Schweizer-Aerzte*, No. 2, 1882.

[5] Hirschberg, *Centralblatt f. pract. Augenheilk.*, Feb. 1881.

0·000,000,04 of a gramme of this substance, suffices to dilate the pupil at the end of an hour, and that this effect persists twenty-four hours.

A drop of a solution, in the strength of 1:2000, paralyses the accommodation for twenty-four hours, while the accompanying mydriasis lasts ninety-six hours.

We have seen, moreover, that daturine and atropine must be regarded as identical.

Oliver,[1] on comparing the action of the *sulphates of daturine and hyoscyamine*, finds that 0·00135 of a gramme of either of these salts, instilled in a healthy eye, suffices to dilate the pupil to the maximum and to paralyse the accommodation. Both these effects are of tardier appearance and disappear sooner for daturine than for hyoscyamine. According to this author, hyoscyamine very easily produces general troubles, even after a single instillation.

4. The action of *gelsemine* has also been studied by Jaarsma.[2] He used the sulphate, seven and ten per cent. solutions of which were instilled into the eye. The effect of this drug proved very variable in different persons. In one case the position of the *punctum proximum* was scarcely changed, while in another the accommodation was entirely paralysed at the expiration of two hours and a half, under the influence of a drop of the same solution (20 per cent.).

The same is true of the other symptoms accompanying the instillation of this mydriatic. Some of the persons examined hardly felt more than a slight smarting of the conjunctiva after the instillation, while, in others, it provoked a very marked congestion, which was, however, only transient; others, again, whose conjunctivæ had borne a 20 per cent. solution very well, had, at the expiration of fifteen minutes, a very disagreeable bitter taste in the mouth, accompanied by nausea.

The uncertainty of its action and the facility with which it provokes general symptoms, render gelsemine unsuitable for practical use. Perhaps it could be utilised in some other form or in a state of greater purity.

## THE ANTAGONISM BETWEEN MYDRIATICS AND MYOTICS.

*Atropine* and *eserine*, as well as the substitutes for them, having directly opposite physiological actions, it is interesting to inquire how these two agents behave when used together; whether it happens that they are mutually neutralised; whether the action of one totally effaces that of the other; or whether both manifest their power, although in unequal degrees.

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[1] Ch. A. Oliver, "The Comparative Action of Sulphate of Daturia and of Sulphate of Hyoscyamia upon the Iris and Ciliary Muscle" (*Amer. Journ. of the Med. Sciences*, July 1882).

[2] Jaarsma, *loc. cit.*, p. 61.

The opinions of experimenters hardly agree on this point. Rossbach seems to be alone in claiming the absolute predominance of paralysing substances over the drugs which act on the nervous system as excitants. He holds, especially, that a pupil dilated by a mydriatic can be contracted, only under exceptional circumstances, by a myotic; as when, atropine having been subcutaneously injected in small quantities, physostigmine is applied directly to the conjunctiva in considerable quantity.[1]

Although this opinion is contrary to that of almost all other writers, it remains nevertheless established that the action of atropine and its congeners, on the nervous system, greatly outweighs that of eserine and similar substances. A partial neutralisation of the effect of the former has, however, always been found upon the administration of the latter, not only as regards the phenomena of secretion (Luchsinger, Heidenhain,[2] Vulpian[3]), but especially with reference to the changes in the pupil and accommodation.

The best experiments, in this connection, we still owe to Donders and his pupils,[4] and to von Graefe. It has been found, by these authors, that, when the instillations of the mydriatic and myotic are simultaneous, the latter is first to develop its action, but is soon overcome by its antagonist, which thereafter maintains the ascendency. Von Graefe noticed that the action of atropine could be momentarily interrupted by the instillation of a solution of Calabar, but that, after this period of arrest, it resumed its normal course.

It also results, from trials made by Donders, that the Calabar bean is still capable of lessening the diameter of a pupil which has been dilated to its maximum by atropine, and of causing the *punctum remotum* to approach the eye during the complete paralysis of the ciliary muscle. The amplitude of accommodation is then re-established in part; but this influence is of short duration, and the diagrams show that the effect of the atropine again becomes predominant at the close of the experiment. Krenchel and Schäffer have ascertained the same facts for the pupil,[5] and Schäffer[6] has observed that eserine was capable of totally effacing the mydriasis which homatropine tends to provoke. On the contrary, duboisine, and especially atropine, combat the effect of their antagonist with greater advantage.

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[1] Rossbach and Froehlich, *Verhandl. d. physic.-medic. Ges. in Würzburg*, Neue Folge Band. V., pp. 1·79, 1873. Rossbach, *Arch. f. d. gesammte Physiol.*, vol xxi., p. 1, 1879.

[2] Heidenhain, *Pflueger's Archiv*, t. ix., p. 335.

[3] Vulpian, *Gaz. des hôp.*, No. 47, 1875.

[4] Donders, *loc. cit.*, p. 617.

[5] Krenchel, *Arch. f. Ophth.*, t. xx., 1, pp. 127-134, 1874.

[6] Schäffer, *Arch. f. Augenheilk.*, t. x., p. 186, 1880.

We have already said that preparations of pilocarpine may be successfully employed to relieve the symptoms of intoxication produced by atropine and other substances of the same group.

COCAÏNE.—Let us mention, finally, cocaïne as the most recent mydriatic we possess. Its influence upon accommodation is, it is true, quite secondary to its anæsthetic effect. Altogether, it is little used as a mydriatic, and its action as such is not yet well known.

The mydriasis of cocaïne is not so decided, nor is it as lasting, as that of atropine. Nevertheless it has the property of increasing the effect of the latter, so that the pupil dilates more energetically after atropine, when cocaïne has previously been instilled into the eye, than when atropine alone is used.

Eserine and pilocarpine easily overcome a cocaïne mydriasis.

According to some authors, the cocaïnized pupil is said to be irregular (v. Reuss,[1] Emmert[2]). We have found, with other observers, that the iris, under the influence of cocaïne, reacts less to light than it does when in its normal condition; again, other authors dispute this influence of cocaïne upon the contractility of the pupil.

Some writers maintain that paralysis of accommodation does not set in until after repeated instillations of a concentrated solution of cocaïne, whereas, in experimenting upon ourselves, we have found that two drops of a weak solution (2 per cent.) would diminish our amplitude of accommodation from 6 D to 2·5 D.[3] The observations of Koller[4] and Knapp[5] agree with ours.

Besides, there is no doubt but that there are different kinds of cocaïne, or, more correctly speaking, the salts of cocaïne, which have until now been used, contain different alkaloids, which influence the action of the drug accordingly. For instance, the first preparation we had occasion to use when this drug had just been introduced into practice, called forth an undeniable and prompt mydriasis, whereas that which we employed afterwards was much less marked in its influence upon the pupil, though its anæsthetic properties seemed to be the same.

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[1] V. Reuss, "Ophthalmologische Mittheilungen aus der $2^{ten}$ Universitäts Augenklinik in Wien," *Wiener Med. z. r. sh. Presse*, 1885.

[2] E. Emmert, *Cocaïn Correspondenz.-Blatt für Schweizer Aerzte*, May 1885.

[3] Landolt, "La Cocaïne," *Archives d'ophthalmologie*, Nov. and Dec. 1884.

[4] R. Koller, "Ueber die Verwendung des Cocaïn zur Anästhesirung am Auge," *Wiener Med. Wochenschrift*, 1884; and *Bericht über die $16^{te}$ Versammlung der Ophth. Gesellschaft:* Heidelberg, 15th September 1884, Stuttgart, 1884.

[5] H. Knapp, "Hydrochlorate of Cocaïne: Experiments and Application," *The Medical Record*, Oct. 1884. Let us still mention Königstein, "über das Cocaïn. mur. in seiner Arwendung in der Okulistik.," *Wien Mediz. Presse*, 1884; Pflüger, "Cocaïn," *Centralblatt für pracktisch. Augenheilkunde*, Juli 1884; Everbusch, *Ueber die Anwendung und Wirkung des Coacïn. mur. als locales Anestheticum; Aerztlich. Intellig. Blatt*, 1885, and *Physiologische Wirkung des Cocaïn*, Oct. 22, 1885; Walter H. Jessop, "On the Mydriasis produced by the Local Application of Cocaïne in the Eye," from the *Proceedings of the Royal Society*, No. 238, 1885; Henry W. Bradford, *Rapid Cocaïnization*, Nov. 1884; Dr. Pollak and Dr. Alt, *The Weekly Medical Review*, 1885; Prof. Keyser, "Cocaïne in Ophthalmolgic Surgery," from *The Therapeutic Gazette*, Jan. 1885; Joseph White, "The New Local Anæsthetic, Mur. of Cocaïne," *Virginia Medical Monthly*, Nov. 1884.

The solution of cocaïne may be used as concentrated as 10 per cent. Our practice, however, has taught us that a 2 per cent. solution will answer all purposes, and that one or two drops will suffice to render the cornea anæsthetic.

The salts of cocaïne, which are principally used, are the *murias cocaïni*, the solution of which is easily decomposed if it is not made antiseptic (sublimat. 1 : 5000 or 10,000), *cocaïnums alicylicum*, *hydrobromicum*, and *citricum*.

We frequently use the drug in the form of an ointment composed of *vaseline* or *lanoline*; the latter being more easily absorbed than any other fatty substance.

## PRESBYOPIA.

Should it have been my good fortune to meet with a reader patient enough to follow me thus far, it is probable that he will now feel inclined to close the book; and yet I have still another paragraph to add to the many gone before, and, indeed, one which until now has never been wanting in a text-book on refraction and accommodation. This is "presbyopia."

It was not forgetfulness which prevented me from touching upon the subject sooner; but I looked in vain for a proper place for it, and so we have come to the end, and have treated of the most important and practical features of refraction and accommodation, without having found an opportunity to speak of presbyopia.

One would be inclined to think that this subject ought to find a conspicuous place in a book of this kind. But, as we shall see, our conception of it renders the notion of presbyopia, as a special condition, so to say, superfluous.

The term *presbyopia* is evidently derived from the Greek πρεσβύς, old, and ὤψ, the eye. The natural translation of it would therefore be "the vision of old age." This is, indeed, the primitive signification of the word. It had been noticed, by those who first used it, that old people, though able to see well at a distance, often lose the ability to see near at hand, that they are obliged to hold a book abnormally far from them when they read, and that, to facilitate vision, they must have recourse to convex glasses. This phenomenon is easily explained, since Donders investigated it, by the decrease of accommodation and consequent lack of positive refraction for eyes whose static refraction is not of very high degree, viz., persons slightly myopic, emmetropes and hyperopes.

This condition must have been considered as a sort of disease incident to old age, at a period when physiological optics were almost unknown. Whenever a man could not see well at a short distance, he was called "presbyopic." The word was even made to cover the cases of young people, and every one who needed glasses for either near or distant vision was classed with the "presbyopes." In other words, hyperopes, as well as those who, in consequence of diminution of their range of accommodation and weakness of refraction, needed glasses for near vision, were called "presbyopes."

This term—"presbyopia"—is very familiar to the general public, which knows even its etymology.[1] A proof of this is found in the fright and indignation encountered nearly every time we dare to propose the use of reading-glasses to a lady. "Why, doctor, I'm not yet presbyopic!" How often do we hear this objection raised!

The word "presbyopia" had long taken firm root before Donders wrote his masterly work on accommodation and refraction. It had become so thoroughly fixed in people's minds that the illustrious teacher in ophthalmology thought he could neither eradicate nor disregard it. He has tried to compromise by giving a precise definition of the term, and restricting its meaning. According to Donders, "the term 'presbyopia' is to be restricted to the condition in which, as the result of the increase of years, the range of accommodation is diminished, and the vision of near objects interfered with."

But this definition still lacked precision. It was necessary to specify what was meant by "vision of near objects." Donders gives, as a measure of near vision, a distance of 8 inches.

The following is, therefore, the signification of the word "presbyopic," according to Donders' definition:—The condition of any eye which, in consequence of physiological weakening of its refraction, can no longer see at a distance of 8 inches, *i.e.*, 216·5 millimetres, or, in round numbers, 22 centimetres. In still other terms, every eye whose *punctum proximum* is more than $\frac{1^{\text{m}}}{4\cdot5}$ distant, or which, by bringing all its refraction into play, does not attain a positive refractive power of more than 4·5 dioptries.

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[1] This statement is truer of France than of this country. But the disinclination of young ladies to wear glasses seems universal, and the author's explanation of this is doubtless, for many cases, the proper one.—TRANSLATOR.

Let us take, again, Donders' diagram (p. 174). We shall find it indispensable to a clear comprehension of the following considerations. According to this diagram, and all that we have hitherto learned, we see at once that no *myope*, whose static refraction or degree of myopia exceeds 7 dioptries, can ever become presbyopic. For, as Donders' diagram indicates, even without accommodation, they see at a distance of $\frac{1000}{7} = 142$ millimetres up to the age of fifty years, and up to the age of sixty-four, by adding to it their dynamic refraction. Even at the age of eighty, when there is no longer any dynamic refraction, and the static has diminished 2·5 dioptries, they still have remaining $7 - 2{\cdot}5 = 4{\cdot}5$ dioptries with which to see at the required distance of $\frac{1^{m}}{4{\cdot}5}$, or 8 inches.

Myopes, whose ametropia is of lower degree than in the preceding case, become presbyopic, but earlier or later, according to the degree of their myopia. Thus, with 4·5 D, one will not need convex glasses to see at 22 centimetres before the age of sixty-three years. This is the age at which the curve *pp* of maximum refraction crosses the zero-line,—that is to say, falls below the primitive value of the static refraction.

The *emmetrope* becomes presbyopic at forty years of age. The scheme shows directly that, at this age, the curve *pp* is at 4·5 dioptries. Hence, from this time on, the amount of positive refraction which the eye needs, in order to have the requisite 4·5 dioptries, must be supplied by convex glasses.

The *hyperope* becomes presbyopic sooner in proportion as his hyperopia (his negative *static* refraction, $-r$) is of higher degree. Thus, a hyperope of four dioptries is presbyopic already at the age of twenty-five years, because, in order to attain the required 4·5 dioptries of refraction, he must have $4 + 4{\cdot}5 = 8{\cdot}5$ dioptries of dynamic refraction, and it is only up to the age of twenty-five that his accommodation is so strong as that. When the hyperopia is of higher degree, presbyopia manifests itself still earlier.

Hence, presbyopia is an infirmity of age, which spares some old persons until the age of eighty, but may affect young persons of twenty-five years or even less. This contradiction, of itself, clashes with logic, yet, but the notion of presbyopia has inconveniences of greater importance than this.

To invest it with accuracy, it has been necessary to define this word presbyopia. The definition necessitates the admission of a single specified distance for near vision. But a uniform distance for near work is manifestly inadmissible, and with this falls the legitimacy of the notion of presbyopia.

Not all people work at the same distance. While many persons hold work 22 centimetres from the eyes, there are many others who remove it much farther, and others who bring it nearer than that. Thus, a tall man, who is not myopic, generally holds his newspaper much more than 22 centimetres from his eyes, on account of the length of his arms. He would hardly thank us for adapting him to 22 centimetres.

It is but proper, however, to say that, in giving the eye 4·5 dioptries of positive refractive power, we do not insist that it shall work at this distance of 22 centimetres. It could work farther off, for instance at 33 centimetres, and reserve 1·5 dioptries of accommodation, if it still have that amount, with which longer to continue its work at this greater distance. These 1·5 D may, however, be too much in certain cases, and not sufficient in others.

But even a distance of 33 centimetres is too short for many people. Painters, book-keepers, and others, who, with a single glance, must be able to take in a more extended surface, and those, like violinists, carpenters, as well as laundresses and many others, whose work requires to be held at some distance, demand a weaker positive refraction. Still others, on the contrary, such as goldsmiths, engravers, and embroiderers, work with such fine objects that they must bring them very near the eye in order to obtain larger retinal images. The same is the case with those who work by insufficient light, or whose acuteness of vision is poor. They endeavour to make up for these defects by increasing the size of the retinal images. Hence, such people require a refraction greater than 4·5 dioptries for their work. In short, the distance of near vision varies considerably, according to the exigencies of the work and the habits of the individual.

Finally, we ought not to forget that, however wonderfully Donders' diagram of the amplitude of accommodation agrees in general with the reality, exceptions are occasionally met with. Paresis and paralysis of accommodation, especially, constitute such exceptions. Therefore, he who is satisfied to take into consideration only the age of the patient and his static refraction, when prescribing glasses for presbyopia, incurs a two-fold risk. He will pass over so important a fact as a defect of accommodation, and then the glasses he has ordered will evidently not suit the patient.

It was for these reasons that we proposed, in a former work,[1] to banish the term "presbyopia" from ophthalmology, and allow ourselves to be guided, in the choice of spectacles, solely by the requirements and desires of those who consult us, without holding ourselves to a given distance or refraction, which has a *raison d'être* only in a very limited number of cases.

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[1] Landolt, *A Manual of Examination of the Eyes*, pp. 141-145, Philadelphia, 1879.

Knowing the static and dynamic refraction, we are, as has been shown, in a position to tell at once what is the proper working-glass for any specified distance.

*Working-glass* (*Arbeitsglas, verre de travail*) is, in fact, the term we apply to the lens which adjusts the refraction of an eye for work, whether it be convex or concave, cylindrical or combined. This way of looking at the matter is, at the same time, more practical and physiological, and in nowise more complicated, than the method which consists in seeking the degree of presbyopia.

Is it necessary to give examples? We think not. The reader will have already found more than he requires, for the adaptation of the eye for near vision has been discussed on many a page of this book. And he who has done us the honour of studying the chapters in question will, we do not doubt, be able to determine accurately and logically the proper kind of glasses in every case.

It would have been astonishing if Donders, a man of so clear and so bright a mind, had not perceived how little logical is the term presbyopia. He questions, in fact,[1] whether it be really necessary to retain the term presbyopia, and if it would not be better to limit ourselves, in each case, to determining the amplitude of accommodation, as well as the degree of myopia or hyperopia. Only he thinks that the suppression of a term so generally known and universally used would meet with little favour. Nor would we, either, wish to entirely expunge this term from ordinary language. It is, indeed, often convenient when we wish to express, in an abridged form, that it is simply age which produces a certain asthenopia. But we should like to see the word presbyopia disappear from ophthalmology, in which field Donders has accustomed us to such clear ideas and to methods of determination at once so precise and so practical.

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[1] Donders, *loc. cit.*, p. 211.

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